HIGHER-ORDER MULTIPOLE ANALYSIS OF BEAM-INDUCED ELECTROMAGNETIC FIELDS USING A STRIPLINE-TYPE BEAM POSITION MONITOR

T. Suwada*, KEK, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

Abstract

A measurement of multipole moments of an electromagnetic field generated by single-bunch electron beams with a pulse width of 10 ps was performed using stripline-type beam-position monitors at the KEKB injector linac. A theoretical multipole analysis agrees well with the experimental results within the measurement errors. The experiment enables one to measure the transverse spatial profile of charged beams; especially, the variations of the higher-order moments were very consistent with those measured by wire scanners.

1 INTRODUCTION

The KEK B-Factory (KEKB) project[1] is in progress of testing CP violation in the decay of B mesons. KEKB is an asymmetric electron-positron collider comprising 3.5-GeV positron and 8-GeV electron rings. The KEKB injector linac[2] injects single-bunch positron and electron beams directly into the KEKB rings. The beam charges are designed to be of 0.64 nC/bunch and 1.3 nC/bunch, with a maximum repetition rate of 50 Hz for the positron and electron beams, respectively. High-current primary electron beams (~10 nC/bunch) are required to generate sufficient positrons. Stable control of the beam positions and energies through beam-position and energy-feedback systems[3,4] using beam-position monitors (BPMs) is essential in daily operation. On the other hand, the spatial beam profile is generally measured using fluorescent screen monitors (SCs)[5] and wire scanners (WSs)[6]. However, these monitors have several drawbacks in measuring the beam sizes in real time. The SCs destroy the beam, and the WSs cannot obtain any pulse-by-pulse beam sizes, although they can measure precise transverse beam sizes by detecting high-energy γ-rays generated from thin wires. Miller *et al.*[7] showed that a stripline-type BPM with four electromagnetic pickups could be utilized as a nonintercepting emittance monitor by theoretically developing a multipole-moment analysis of an electromagnetic field generated by a charged beam. They also experimentally demonstrated that the transverse emittances of electron beams could be derived from the second-order moment of the electromagnetic field. In this report the author not only presents a clear experimental verification of this method based on a similar technique used by Miller *et al.*, but also demonstrates that the second-order and third-order moments of the electromagnetic field can be properly measured using two kinds of stripline-type BPMs.

2 MULTIPOLE-MOMENT ANALYSIS

The electromagnetic field generated by relativistic charged beams inside a conducting duct is predominantly boosted in the transverse direction to the beam axis due to the Lorentz contraction. Thus, for a conducting round duct, the image charges induced by a line charge can be solved as a boundary problem in which the electrostatic potential is equal on the duct. The formula for the image charge density (j) according to a similar treatment by Miller[7] is given by

$$j(r,\phi,R,\theta)=\frac{I(r,\phi)}{2\pi R}\left[1+2\sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^n\cos n(\theta-\phi)\right], \quad (1)$$

where I is the line charge, (r,ϕ) and (R,θ) are the polar coordinates of the line charge and the pickup point on the duct, respectively, and R is the duct radius. If a transverse distribution of a traveling beam obeys a Gaussian function inside the duct, the image charge (J) is formulated by integrating the image charge density with the weight of the Gaussian distribution inside the duct area. Assuming that the widths of the charge distribution are sufficiently small compared to the duct radius,

$$\begin{aligned} J(R,\theta)\approx\frac{I_b}{2\pi R}\Bigg\{&1+2\left[\frac{x_0}{R}\cos\theta+\frac{y_0}{R}\sin\theta\right] \\ &+2\left[\left(\frac{\sigma_x^2-\sigma_y^2}{R^2}+\frac{x_0^2-y_0^2}{R^2}\right)\cos 2\theta+2\frac{x_0y_0}{R^2}\sin 2\theta\right] \\ &+2\left[\frac{x_0}{R}\left(\frac{3(\sigma_x^2-\sigma_y^2)}{R^2}+\frac{x_0^2-3y_0^2}{R^2}\right)\cos 3\theta+\frac{y_0}{R}\left(\frac{3(\sigma_x^2-\sigma_y^2)}{R^2}+\frac{3x_0^2-y_0^2}{R^2}\right)\sin 3\theta\right] \\ &+\text{higher orders}\Bigg\}, \end{aligned} \quad (2)$$

where I_b is the beam charge, σ_x and σ_y are the horizontal and vertical root mean square (rms) half widths of the beam, respectively, and (x_0,y_0) are the charge center of gravity of the beam. The first to fifth expanded terms correspond to the monopole, dipole (first-order), quadrupole (second-order), sextupole (third-order), and higher order moments, respectively. Four pickups of the BPM are normally mounted at the polar coordinates (θ=0, $\pi/2$, π, $3\pi/2$) or at the polar coordinates ($\theta=\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$). Here, for the sake of simplicity, the former and latter BPMs are called "90° BPM" [see Figs. 1] and "45° BPM", respectively. A beam-size measurement is performed to detect the quadrupole moment (J_{quad}) for the 90° BPM and the sextupole moment (J_{sext}) for the 45° BPM at the least orders. These formulas are defined using

* E-mail address: tsuyoshi.suwada@kek.jp

the following four pickup amplitudes [V_i (i=1-4)] of the BPM:

$$J_{quad} \equiv \frac{(V_1+V_3)-(V_2+V_4)}{\sum_{i=1}^{4} V_i}, \tag{3}$$

$$J_{sext} \equiv \frac{[(V_1+V_2)-(V_3+V_4)]/y_0-[(V_1+V_4)-(V_2+V_3)]/x_0}{\sum_{i=1}^{4} V_i}. \tag{4}$$

Here, for example, the quadrupole moment for the 90° BPM is given by

$$J_{quad} = 2\left(\frac{\sigma_x^2-\sigma_y^2}{R^2}+\frac{x_0^2-y_0^2}{R^2}\right), \tag{5}$$

and the sextupole moment for the 45° BPM is given by

$$J_{sext} = \frac{2}{R\sqrt{2}}\left(\frac{6(\sigma_x^2-\sigma_y^2)}{R^2}+\frac{4(x_0^2-y_0^2)}{R^2}\right). \tag{6}$$

Normalization by summing the four pickup amplitudes must cancel out the beam charge variations due to the beam jitter. It is noted that the sextupole moment cannot be defined if the beams pass through the center of the BPM, and the absolute beam sizes cannot be independently obtained by the BPM because the square difference (σ_x^2- σ_y^2) of the beam sizes is only related to the multipole moments. This is because the equipotential lines are invariant under the condition σ_x^2- σ_y^2 = const if the beam positions do not change. The performance and characteristics of the BPMs are described in detail elsewhere[4]; the design parameters are summarized in Table I for both types of BPMs.

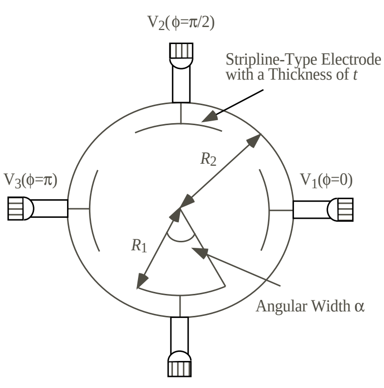

Figure 1: Schematic cross-sectional drawing of the stripline-type 90° BPM.

Table 1: Mechanical design parameters of the BPMs.

Parameters	90°BPM	45°BPM
R_1 [mm]	13.55	16.0
R_2 [mm]	18.5	20.0
a [degree]	60	34
t [mm]	1.5	1.5
L [mm]	132.5	132.5

3 BEAM EXPERIMENT

Beam experiments using single-bunch electron beams of about 1 nC/bunch were carried out at two locations of sector B[8]. The nominal beam energy was about 1.7 GeV at the end of sector B. A set composed of the quadrupole, the 90° BPM and the WS was used to measure the quadrupole moment, where the beam energy was 1.53 GeV, and another set composed of the quadrupole, the 45° BPM and the WS was used to measure the sextupole moment, where the beam energy was 1.7 GeV. The transverse beam sizes at the locations of these BPMs and WSs were controlled by changing the quadrupole currents applied to the quadrupole magnets installed upstream of the monitors. Neighboring WSs for each BPM were used to calibrate the transverse beam sizes at the locations of the BPMs. Twenty other BPMs located at sectors A and B monitored the beam positions and charges in order to control the electron beams stably without any beam loss during the experiment. Beam-orbit feedback using steering magnets was added instead of that used for the nominal operation at sector B, particularly to control the beam positions at the quadrupoles. The beam positions were controlled within 0.0 ± 0.1 mm at the quadrupole magnet in order to suppress the dipole moments as much as possible for the quadrupole-moment measurement, and they were controlled within 1.0 ± 0.1 mm at the quadrupole magnet in order to keep the term x^2 - y^2 = 0 for the sextupole-moment measurement. Figures 2 and 3 show the variations of the beam sizes obtained by the 90° and 45° BPMs in the quadrupole- and sextupole-moment measurements, respectively. The moments were obtained by averaging twenty BPM data points with an error of one standard deviation after their beam-position dependence was corrected by the dipole moments.

4 ANALYSIS

A beam phase space matrix (Σ_1) at location 1 is transformed to that (Σ_2) at location 2 by the transport matrices (M)[9] between locations 1 and 2,

$$\Sigma_2 = M\Sigma_1 M^t, \tag{7}$$

where M^t is the transposed matrix of the matrix M. Assuming that beams are adequately described in linear optics by two-dimensional ellipsoidal phase spaces without any x-y couplings, the matrix components, σ_{11} and σ_{33}, give the squares of the horizontal and vertical rms half widths of the beam, respectively. They are related by

$$\sigma_{11}^{(2)} = m_{11}^2\sigma_{11}^{(1)} + 2m_{11}m_{12}\sigma_{12}^{(1)} + m_{12}^2\sigma_{22}^{(1)}, \tag{8}$$

$$\sigma_{33}^{(2)} = m_{33}^2\sigma_{33}^{(1)} + 2m_{33}m_{34}\sigma_{34}^{(1)} + m_{34}^2\sigma_{44}^{(1)}, \tag{9}$$

where superscripts (1) and (2) of the beam phase spaces show locations 1 and 2, respectively. If the beam sizes (σ_{11}^2 and σ_{33}^2) are measured at location 2, other unknown parameters on the beam phase spaces at location 1 can be solved by a least-squares fitting procedure. Thus, the horizontal and vertical beam phase spaces at the BPMs can be transformed by using the beam sizes measured by the WS and the transport matrices. The transport matrices (*M*) were calculated according to the optics database of the linac. Figure 2 shows the variations of the quadrupole moments for the 90° BPM(solid points) and the WS(solid line) by changing the quadrupole current. Figure 3 shows the variations of the sextupole moments for the 45° BPM(solid points) and the WS(solid line) by changing the quadrupole current. Here, for deriving the multipole moments, first, the square differences of the beam sizes (σ_{11}^2 - σ_{33}^2) by the WS are fitted by a parabolic function. The square differences of the beam sizes at the BPM locations are calculated using eqs. (8) and (9). The BPM and WS data are analyzed using the following formulas for the quadrupole-moment analysis,

$$\sigma_x^2 - \sigma_y^2 = \frac{R_{eff}^2}{2} J_{quad} + b, \tag{10}$$

and for the sextupole-moment analysis,

$$\sigma_x^2 - \sigma_y^2 = \frac{R_{eff}^3 \sqrt{2}}{12} J_{sext} + b, \tag{11}$$

where parameter b is the offset due to the gain imbalance and the geometrical position errors of the four pickups, and parameter R_{eff} is the effective radius of the BPM. The effective radius is required to parametrize in this analysis because the simple formulation described in section 2 is insufficient because the BPM radius is not clearly defined for the real configuration of the electrodes. Parametrization using an effective radius means that the electromagnetic-field lines are slightly deformed near the electrodes, and that the BPM radius viewed from the electromagnetic fields is shortened due to this effect, that is, $R_{eff} < R_2$. By using the least-squares fitting procedures, the effective radii and offsets of the BPMs are obtained as R_{eff} = 17.3 ± 0.78 mm and b = 0.0478 ± 9 x 10^{-4} mm^2, respectively, for the quadrupole-moment measurement, and R_{eff} = 21 ± 8 mm and b = -0.067 ± 0.005 mm^2, respectively, for the sextupole-moment measurement. They are in good agreement with the experimental results obtained by a bench[10] within the permissible error specifications.

5 CONCLUSIONS

A transverse beam-size measurement using two kinds of stripline-type BPMs has been carried out on the basis of a multipole-moment analysis for the electromagnetic field induced by single-bunch electron beams. The second-order and third-order moments of the electromagnetic field were accurately measured using the 90° and 45° BPMs, respectively. The experimental results show that the analyzed beam sizes obtained by both types of BPMs agree with those measured by the WSs within the estimated errors. The result also shows that the multipole moments must be corrected in this analysis because of the deformation of the electromagnetic-field lines due to the geometrically complex configuration of the stripline-type electrodes, and that the correction can be performed by using the effective BPM radius which was measured by the test-bench calibration for the BPMs.

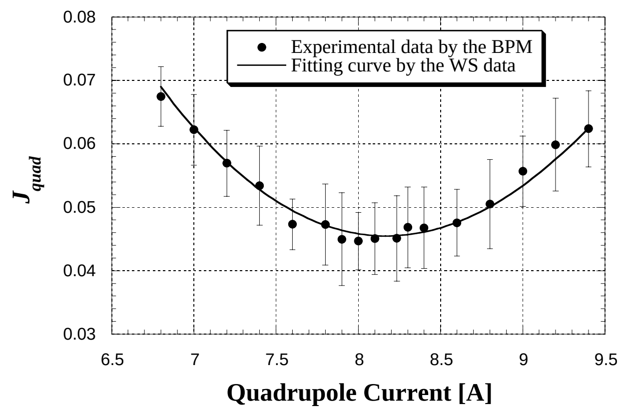

Figure 2: Variations of the quadrupole moment at the 90° BPM by changing the quadrupole current. The solid line shows the fitting curve measured by the WS.

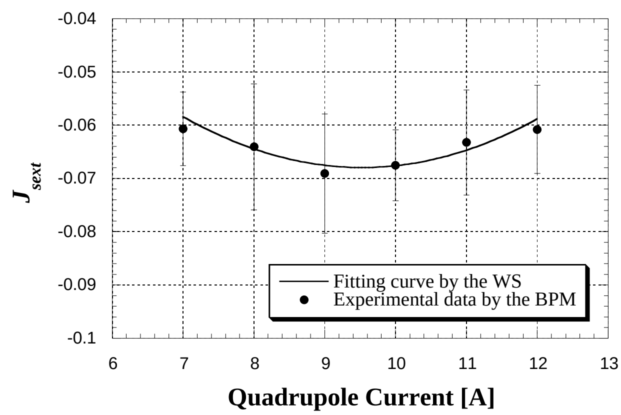

Figure 3: Variations of the sextupole moment at the 45° BPM by changing the quadrupole current. The solid line shows the fitting curve measured by the WS.

REFERENCES

[1] KEKB B-Factory Design Rep., KEK Rep. 95-7 1995.

[2] Design Report on PF Injector Linac Upgrade for KEKB, KEK Rep. 95-18, 1996.

[3] K.Furukawa, *et al., ICALEPCS'*99, Trieste, 1999, p.248.

[4] T.Suwada, *et al.*, NIM A396 (1997) p.1.

[5] T.Suwada, *et al.*, *Proc. the 20th Linear Accelerator Meeting in Japan*, Osaka, 1995, p.245.

[6] N.Iida, *et al., PAC'99*, New York City, 1999, p.2108.

[7] R.H.Miller, *et al.*, *HEAC'83*, Fermilab, 1983, p.602.

[8] T. Suwada, Jpn.J.Appl.Phys. Vol.40 Part 1, No.2A (2001) p.890.

[9] K. L.Brown, *et al.*, SLAC-R-0091-REV-2 (1977).

[10] T. Suwada, *et al.*, *Proc. the 23rd Linear Accelerator Meeting in Japan*, Tsukuba, 1998, p.175.