DEVELOPMENT OF THE PULSED MAGNETIC KICKE FOR THE SPIRAL INJECTION TEST EXPERIMENT *

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Abstract
A new muon g – 2 / EDM experiment at J-PARC (E34) is under preparation in order to resolve a 3 sigma discrepancy of muon anomalous magnetic dipole moment between the measurement and the standard model prediction. The E34 experiment will employ a unique three-dimensional spiral injection scheme in order to store the muon beam in a small storage orbit. In order to demonstrate the feasibility of novel injection scheme, the Spiral Injection Test Experiment (SITE) with the electron beam is under construction at KEK Tsukuba campus. In SITE, 80 keV DC electron beam was injected at forty degree into the storage magnet, and detected as a fluorescent light due to the de-excitation of the nitrogen gas. The pulsed electron beam, and a pulsed magnetic kicker are developed in order to keep the pulsed beam to the very centre of the storage magnet. The magnetic kicker mainly produced the radial field to reduce the pitch angle of the injected beam to keep the beam at storage region. In this paper, the development of magnetic kicker, tracking studies in kicker field and two designs for the SITE’s kicker is presented.

INTRODUCTION
The muon’s anomalous magnetic moment is one of biggest discrepancies in elementary particle physics and extremely sensitive to the new physics. The most recent measurement of muon g – 2 results in 3 σ [1] discrepancy between measured and standard model prediction. The J-PARC new muon g – 2 / EDM (E34) experiment is aiming to measure muon g – 2 to the precision of 0.1 ppm and EDM down to the sensitivity of 10^-21 e.cm [2].

In the E34 experiment the polarized muon beam will be stored in the magnetic field and the evolution of spin precession vector will be measured with respect to time. In E34 a low emittance ultracold muon beam of 300 MeV/c from muon accelerator will be injected into a 3-T MRI (Magnetic Resonance Imaging) type solenoid magnet in order to store the muon beam in 0.66 m diameter orbit. In order to enhance injection efficiency and overcome technical challenges related to small storage orbit a three-dimensional spiral injection scheme is under development to inject the muon beam into a MRI type storage magnet. The details and recent updates of three-dimensional spiral injection scheme can be found in [3, 6 and 7].

The three-dimensional spiral injection scheme is an unproven idea, therefore, a demonstration experiment to prove the feasibility of this unique scheme is inevitable.

A scale down Spiral Injection Test Experiment (SITE) by the use of electron beam is under development at KEK Tsukuba campus (see Figure 1 (a) and b). SITE is consist of a DC electron electron gun and a 2 m long straight beam. The electron beam from the straight beamline was bent by the bending magnet I towards storage magnet at 40 degrees. At the injection pipe region near to the injection duct of the storage magnet a bending magnet II is also installed. The purpose of this addiational bending is to increase the acceptance of the injection angle into the storage magnet. The details of this will be described in next sections. In Figure 1 (b) the region outside the storage magnet is hightled in yellow color and injection region and and storage regions are hightled as blue and green colors respectively. In the SITE a DC electron beam spiral track has been confirmed in the storage chamber as a fluorescent light from the nitrogen gas de-excitation along the electron beam path (Figure 2). In Figure 2 bright yellow lines show the electron beam as a fluorescent light. The number of turns in the storage magnet depends upon injection angle, in Figure 2 the beam was injected into the storage magnet at around 40 degrees. The distance between two turns of the spiral trajectory was 190 mm. The final goal of the SITE is to store the pulsed electron beam [4, and 5] in the median plane of the storage magnet for the order of few milliseconds. In order to store the pulsed electron beam in the storage region, a pulsed magnetic kicker is being developed to reduce the vertical momentum of the beam to nearly zero. This paper will describe our recent progress towards the development of kicker.

Table 1: Comparison of Parameters between E34 and SITE

<table>
<thead>
<tr>
<th>Parameters</th>
<th>E34</th>
<th>SITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage magnet field strength</td>
<td>3 T</td>
<td>0.0083 T</td>
</tr>
<tr>
<td>Momentum</td>
<td>300 [MeV/c]</td>
<td>0.296 [MeV/c]</td>
</tr>
<tr>
<td>Cyclotron Period</td>
<td>7.4 nsec</td>
<td>5.0 nsec</td>
</tr>
<tr>
<td>Storage Orbit diameter</td>
<td>0.66 m</td>
<td>0.24 m</td>
</tr>
</tbody>
</table>

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the injection angle of the beam to some extent. In order to store the beam at the center of the storage magnet, the pitch angle of the beam must reduce to the zero as it reaches storage region. A pulsed magnetic kicker is used to decrease the pitch angle ($\psi$) of the beam. Figure 3 is illustrating the character of the magnetic kicker. The black trajectory is showing the motion of the beam inside the storage magnet without kicker, the injection angle of the beam reduced due to radial fringe field but do not decrease to zero. Beam passed through the middle region of the storage magnet without getting stopped. Whereas, when an appropriate kick is applied to the beam as shown in the pink trajectory, the pitch angle reduced to zero. The curved region in Figure 3 right depicts the weak focusing region (next subsection).

The pulsed magnetic kicker have to produce the radial field which will be perpendicular to the vertical component of the momentum of the beam. The required radial magnetic field and a stopping volume (region needed to stop the beam) may calculate as following

$$B_r = \frac{m_e}{2q} \theta_{pitch} \omega$$  \hspace{0.5cm} (1)$$
$$Z_0 = \frac{c}{\gamma e m_e} T P_{z0}$$  \hspace{0.5cm} (2)$$

Where $B_r$ is the required radial field to kick the beam, $\theta_{pitch}$ is the pitch angle of the beam, $\omega (= \frac{2\pi}{T})$ is the angular frequency, $m_e$ is the mass and $q$ is the charge of electron. $Z_0$ gives the region in which beam will stop (stopping volume), $T$ is the time period and $P_{z0}$ is a vertical momentum of the beam. In SITE, stopping volume have been chosen 100 mm due to the good field region (the region in which pitch angle remains constant) and the realistic number of required kick field and time period from the point of view of the electronic circuit of the power supply.

**Tracking in the kicker field and weak focusing field**

The tracking of the single particle with the different initial condition in the kicker field has been done by using the Runge-Kutta 4th order method. The kicker field has been defined as follows

$$B_{kick} = B_{peak} \sin\left(\frac{2\pi}{T_{kick}} t\right)$$  \hspace{0.5cm} (3)$$

**PULSED MAGNETIC KICKER**

The electron beam is injected into the storage magnet at a vertical angle, therefore the beam has a component parallel to the axial magnetic field of the storage magnet and starts spiraling through the storage magnet. The radial fringe field of the solenoidal storage magnet will reduce the injection angle of the beam.
Where $T_{kick}$ is the duration of the kick and $B_{peak}$ represent the peak kicker field. The half-sine shape has been considered for the kick, therefore, $T_{kick} = T/2$, where $T$ is total duration of the kick. Even after the kick, there will be residual pitch angle, in order to compensate the effect of the residual pitch angle the weak focusing has been introduced in the storage volume. In the weak focusing field the beam motion will be oscillatory in the storage volume. The amplitude of the oscillation depends upon the residual pitch angle from the kicker field. The weak focusing magnetic field can be defined as follow

$$B_z = B_0 \left(1 - n \frac{\delta r}{r} + n \frac{z^2}{2r^2}\right)$$  \hspace{1cm} (4)

$$B_\rho = -n \frac{z}{r} B_0$$  \hspace{1cm} (5)

An additional bending magnet II is also placed near the storage magnet injection duct ($Z = -450$), in order to enhance the acceptance of the injection angle. The grey line in Figure 5 shows injection pipe. The injection pipe is narrow in bending magnet I region, if we change the injection angle to large or small values at this point then beam just hit injection pipe instead of going into the storage magnet. In order to increase the acceptance of the injection angle a large diameter beam is used, after bending magnet I region, and one additional bending magnet II is installed in order to change the injection angle. In Figure 5 Type (I - V) shows the different injection angle (see Table 2) adjusted by bending magnet II.

The motion of the single particle in the kicker and weak focusing field was simulated by the 4th order Runge-Kutta method in Mathematica [8]. Here we will referred injection angle as an angle at which beam is injected in the storage magnet and pitch angle when injection angle reduced to some extent by the radial fringe field of the storage magnet at 100 mm form the center of the storage magnet. In SITE, the pitch angle of the beam is adjustable from the injection point, therefore, we considered five different pitch angles of the beam (see Figure 5). Figure 6 (a) is representing the different pitch angles as a function of the vertical position ($Z$). It can be seen that if we decrease the injection angle then the pitch angle will also decrease accordingly. The value of the field index ($n$) for all tracking studies was set to $10^{-3}$. The starting point of the kick is $Z = -100$ mm from the center of the storage magnet. If a pitch angle of the beam changes then the required kick field ($B_{kick}$) and time period accordingly change.

Figure 6 (b) shows the zoom up of the particle track in the kicker and weak focusing region. Figure 6 (c) is representing the particle trapping in the weak focusing field. The amplitude of oscillation depends upon the residual pitch angle. For all cases in Figure 4, half sine shape ($T = T/2$) was assumed for the magnetic field and spatially kicker angle. In Figure 5 − $\n$ is the field index. Radial focus-

Figure 4: Left: Axial field profile of the storage magnet. Right: Radial field profile of the storage magnet with weak focusing index $n = 10^{-3}$.

Where “$n = - \frac{r_0 \partial B_z}{B_0 \partial r}$” is the field index. Radial focusing can be achieved if $n < 1$ and vertical focusing can be attained if we set $n > 0$. Figure 4 is representing the magnetic field profile of the SITE storage magnet with the field index value of $n = 10^{-3}$.

Figure 5 explains the strategy of the injection angle into the storage magnet. The electron beam from the straight beamline was bent by a bending magnet I at bend at 0.698 rad (~ 40 degree) towards the storage magnet shown as a black line.

Figure 5: The electron beam from the straight beamline was bend at 0.698 rad (~ 40 degree) towards the storage magnet (Black Line) by bending magnet I. At $Z = -450$ mm from the center of the storage magnet an additional bending magnet II is installed in order to enhance the acceptance of the injection angle in the storage magnet.
The magnetic field is symmetric over the circumference of the particle. Table 2 summarizes the required parameter ($B_{kick}$, $T_{ns}$, and $\psi_0$ (residual pitch angle)) for the different pitch angles of the beam. The amplitude of the stored beam in the weak focusing potential depends upon the residual pitch angle of the beam.

A thyratron switch will be used as a switching device in the magnetic kicker power supply. The thyratron switch has a time jitter of a few nanoseconds, therefore, it is more feasible to apply the kick with the long time period.

Table 2: Kicker parameters for the different pitch angle, residual pitch angle and amplitude of the stored beam.

<table>
<thead>
<tr>
<th>Type</th>
<th>Injection angle (Radian)</th>
<th>Pitch angle $\psi$ (Radian) at Z = -100 mm</th>
<th>$T_{ns}/2$ (ns)</th>
<th>$B_{kick}$ (Gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.81</td>
<td>0.19</td>
<td>15/2</td>
<td>2.41</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
<td>0.08</td>
<td>31/2</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>0.67</td>
<td>0.05</td>
<td>48/2</td>
<td>0.27</td>
</tr>
<tr>
<td>4</td>
<td>0.66</td>
<td>0.035</td>
<td>62/2</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>0.64</td>
<td>0.018</td>
<td>95/2</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Therefore, the kick with 62 and 95 nanoseconds is easier to produce. In the case of a longer kick time period, the required kick magnetic field reduce drastically (Table 2), it is very difficult to measure such a small field. Therefore, we are considering alternatives configuration of the kicker. These alternative configurations of the kicker must have a longer time period with the measurable magnetic field.

**Alternative Kicker Configurations**

One way to overcome the issue of low magnetic field is to reduce the kick region of the magnetic kicker. The higher magnetic field will be required to kick the beam due to the limited area of the kicker field. Therefore, we considered kicking the beam with $\pi/4$, $\pi/6$ and $\pi/10$ shapes instead of the uniform $2\pi$ kick. Figure 5 represents the tracking with kicker field in case of $2\pi$, $\pi/4$, $\pi/6$ and $\pi/10$ shapes.

For all cases in Figure 7 the initial pitch angle and position of the beam was set to 0.96 degrees and $Z = -100$ mm respectively and pulse shape for the kicker field was half sine. The type I in Figure 7 (a) shows the kick with the uniform kicker field of $2\pi$ shape, and it can be seen clearly that the pitch angle of the beam decreases smoothly.

Whereas type II, III and IV shows the kick with non-uniform kicker shapes i.e. $\pi/4$, $\pi/6$ and $\pi/10$ respectively. In the case of $\pi/4$, $\pi/6$ and $\pi/10$ kicker shape the pitch angle decrease in the steps because the beam sees the kicker field only in the limited region. For the region where kicker field is absent the pitch angle remains constant and when it comes to the kicker field region it starts decreasing again and this pattern keep repeating until the pitch angle decrease to zero.

The types II, II and IV overlap each. Although the area of the kicker reduce in different types but the reduction in the area of the kicker is balanced by the high magnetic field value hence the effect of the kicker will remain same. Table 3 summarize the study of the different kicker configurations. The $\pi/6$ and $\pi/10$ configurations both are good candidates for the new kicker shape due to the reasonable values of magnetic field.

**Kicker Design**

Figure 9 shows kicker coil design in CST [8] to produce the $2\pi$ uniform kick. In this configuration of the kicker the current in upper and lower coils are opposite to each other and they produced cylindrical symmetric radial field. Figure 9 left shows the field profile along the axial direction. The alternative design of the kicker shape can be realised by the rectangular dipole coil. The rectangular dipole coil was design in CST [9]. In the CST simulation coil height was assumed 300 mm, width is 90 mm (correspond to $\pi/6$) and gap between coils is 35 mm. The kicker coil geometry is shown in Figure 10 left and right shows the field profile in X (mm) direction. As mentioned in the previous section localized coil do not produced cylindrical symmetric field but the produced field only in the region covered by coils. In the simulation coil excitation current was set to 30 Ampere. The rectangular kicker coil will produce magnetic field in the radial direction of the storage magnet, the magnetic field of kicker will be perpendicular to vertical component of the beam. In terms of the required magnetic field and time period rectangular kicker coils is better option and it will be realized for the SITE.
The rectangular kicker coils will also have smaller inductance as compared to full \(2\pi\) kicker, therefore, it will ease the kicker power supply construction too.

**Future Plan**

The feasibility of the pulsed magnetic kicker has been shown, and now we are preparing for the construction of a pulsed power supply for the kicker. We will complete the construction of the pulsed kicker power supply and test our pulsed magnetic kicker with the electron beam within Japanese FY2018.

**SUMMARY**

A new three-dimensional spiral injection is under development in order to realize the new J-PARC muon \(g-2\) / EDM experiment. For the establishment of this novel scheme, the Spiral Injection Test Experiment (SITE) by the use of the electron beam is underway at KEK Tsukuba campus. A pulsed magnetic kicker is being under development to guide the pulsed electron beam to the very center of the storage magnet for the SITE. Single particle tracking studies have been carried out with the different initial conditions in the kicker field and weak focusing field in order to find the required magnetic field and time period for a kicker. From tracking studies it has been found out that the localized kicker will fulfil the requirement for the SITE.

**REFERENCES**


Figure 9: (Left) The model of kicker coils to produce uniform \(2\pi\) kick. The direction of the current in upper and lower coils are opposite to each other. These pairs of coils produced cylindrical symmetric field. Right: Field profile of kicker coil along the axial direction.

Figure 10: (Left) Rectangular dipole kicker coil design in CST the size (width) of the coil correspond to \(\pi/6\) shape. Right: Field profile of kicker coil in the X (mm) direction.

The rectangular kicker coils will also have smaller inductance as compared to full \(2\pi\) kicker, therefore, it will ease the kicker power supply construction too.