INTERFERENCE OF CSR FIELDS IN A CURVED WAVEGUIDE

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Abstract

CSR fields generated by a bunched beam passing through a series of bending magnets may interfere with each other due the reflections of outer chamber wall. This kind of multi-bend interference causes sharp peaks and long-range tail in the CSR impedance and wake potentials, respectively. Using a dedicated computer code, CSRZ, we calculated the longitudinal CSR impedance in the SuperKEKB positron damping ring for purpose of demonstration. It was found that multi-bend interference may enhance the CSR fields within a distance comparable to the bunch length, which is typically in the order of several millimeters. A simple instability analysis was performed and it suggested that multi-bend interference might play a role in the single-bunch instabilities of small electron/positron rings.

INTRODUCTION

A new dedicated code called CSRZ was introduced in Ref. [1]. The code calculates CSR impedance by solving the parabolic equation in the frequency domain [2]. In Ref. [1], we presented the numerical calculations of the longitudinal CSR impedance for a bunched beam passing through a single bending magnet. It was found that when the magnet length is long enough, the CSR impedance exhibits sharp peaks due to outer wall reflection. This observation agrees with the steady-sate theory of shielded CSR discussed in Ref. [3]. From the viewpoint of wake potential, a kind of multimode interference exists due to the reflection, namely shielding, of the outer chamber wall. That is, radiation waves superpose each other to form high-amplitude wakefield after the bunch tail. A simple optical approximation model was introduced to explain this interference phenomenon [4].

In a realistic storage ring, the bending magnets are arranged consecutively along the beam orbit. There is a concern that interference may happen among the CSR fields generated in a series of bending magnets. This kind of multi-bend interference is similar to the multimode interference that happens in a long magnet. The code CSRZ can treat this case straightforwardly. In this paper, we continue the investigations in Ref. [1] and study the CSR generated by a bunched beam passing through a series of bending magnets.

There exists another kind of multi-bunch interference in CSR fields. It occurs when a train of bunches pass through a bending magnet following the same trajectory, and was first observed in Ref. [5]. This kind of interference is beyond the scope of this paper.

INTERFERENCE IN A SERIES OF BENDING MAGNETS

To elucidate the interference effect considered in this paper, we start from an ideal example of 4 identical hard-edge magnets assembled along a curved vacuum chamber. The bending magnets guide the beam and generate the curved beam orbit. Straight drift chambers connect the 4 magnets and are also assumed to be identical. The parameters for the magnets and chamber are: bending radius $R = 5$ m, magnet length $L_b = 2$ m, chamber width $a = 6$ cm and height $b = 3$ cm. The length of the drift chambers is $L_d = 2$ m. The longitudinal CSR impedance is calculated and plotted in Figs. 1(a) and 1(b) and compared with that of single-magnet model. The corresponding wake potentials with a short bunch of rms length $\sigma_z = 0.5$ mm are given in Fig. 1(c). One sees that the impedance spectrum becomes spurious when the multi-bend interference is taken into account. It indicates that specific eigenmodes can satisfy the phase matching conditions and will be excited by the beam. The impedance based on single-magnet model looks to be a broad-band approximation of that of a series of magnets. The narrow spikes in the impedance lead to long-range wake fields far behind the bunch as seen in the wake potential.

Naturally one can suspect that if there are many bending magnets placed along a long curved chamber, many high peaks may exists in the CSR impedance. This is proved by a more practical case of the SuperKEKB positron damping ring [6]. In the SuperKEKB damping ring, there are two arc sections. In each section, there are in total 16 cells. Each cell contain two reverse bending magnets [7], of which the bending radius as a function of orbit distance is shown in Fig. 2. The drift between consecutive cells is around 1 m. The vacuum chamber cross-section is approximated by a square with $a = b = 3.4$ cm. The CSR impedances of one arc section (in total 16 cells), of 6 consecutive cells, and of 1 cell are calculated. The results are compared with that of single-bend model as shown in Figs. 3(a) and 3(b). It is not surprising that many high sharp peaks appear in the impedance spectrum when the number of cells grows. And the CSR impedance calculated using single-magnet model again provides a broad-band approximation.

The wake potentials with rms bunch length of 0.5 mm corresponding to the previous impedances are plotted in Fig. 3(c). The figure clearly shows that the tail parts are strongly modulated due to interference effect. It is note-

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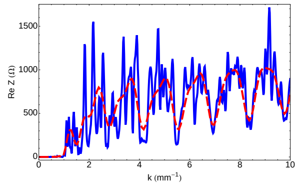
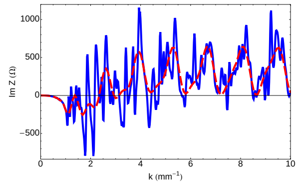
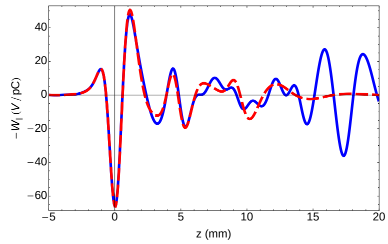


(a) Real part



(b) Imaginary part



(c) Wake potentials

Figure 1: CSR impedance and wake potential for 4 bending magnets interleaved with equidistant drift chambers. The blue solid and red dashed lines denote the cases of with and without interference, respectively. The wake potentials have been normalized by the number of magnets. The bunch head is to the left side.

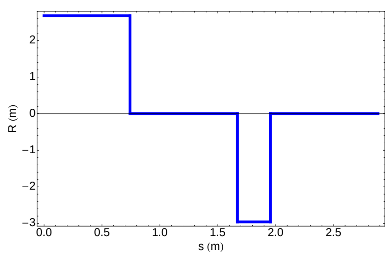
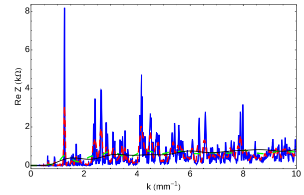


Figure 2: Bending radius as a function of orbit distance for one cell in the arc sections of SuperKEKB positron damping ring.

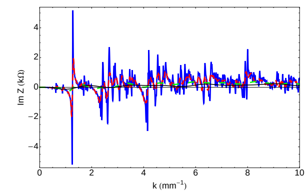
worthy that the shape of wake potential due to 6 cells is already close to that of 16 cells at the distance less than 20 mm to the beam center. This is because that the CSR fields generated at the first cells keeps far behind the beam after the beam traverses several arc cells.

MICROWAVE INSTABILITY THRESHOLD DUE TO INTERFERED CSR

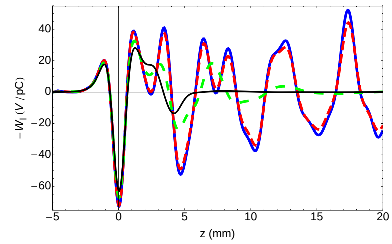
As shown in Fig. 3, in a practical ring like SuperKEKB positron damping ring, CSR impedance including interference effect contains sharp narrow peaks. The peaks indicates field resonances generated by consecutive bending magnets. There is concern that if such resonant CSR impedance is considered, beam will become unstable. The threshold of microwave instability due to CSR can be estimated by solving the dispersion relation [8, 9] or by solving Vlasov-Fokker-Planck equation [10]. For a simple instability analysis, one can apply the Keil-Schnell criterion [11]



(a) Real part



(b) Imaginary part



(c) Wake potentials

Figure 3: CSR impedance and wake potentials of the arc section in the SuperKEKB positron damping ring. The impedances and wake potentials have been normalized by the number of cells for convenience of comparision. Blue solid line: 16 cells; red dashed line: 6 cells; Green dashed line: 1 cell; Black solid line: single-bend model.

to a bunched beam [12]

$$\left|\frac{Z_{\|}}{n}\right| < F Z_0 \frac{\gamma \alpha_p \sigma_\delta^2 \sigma_z}{N_0 r_e}, \tag{1}$$

where $Z_{\|}/n$ is the longitudinal impedance driving the instability, α_p is the momentum compaction factor, σ_δ is the relative energy spread, σ_z is the rms bunch length, N_0 is the bunch population, r_e is the classical radius of electron. F is a form factor, for a gaussian bunch, we take it as $F = \sqrt{\pi/2}$. $n = \omega/\omega_0$ is the harmonic and ω_0 is the revolution angular frequency. The above equation is also called Keil-Schnell-Boussard criterion. It's apparent that this criterion can only provide a crude estimate of the instability threshold in a storage ring. As pointed out in [13], this criterion is only correct for a broad-band impedance wider than the frequency spectrum of the bunch. In principle, sharp resonances can also drive microwave instabilities [14]. But, as derived from [13], it's more appropriate to use a modified criterion as follows

$$\left|\frac{\sqrt{2\pi}\omega_0 c \sigma_z}{4} \frac{R_s}{Q}\right| < F Z_0 \frac{\gamma \alpha_p \sigma_\delta^2 \sigma_z}{N_0 r_e}, \tag{2}$$

to detect the instability threshold driven by sharp resonances, of which the frequency width is much narrower than that of bunch spectrum. In the above equation, $\omega_0 = 2\pi c/C$ where C is the circumference of the storage ring. One should notice that Eq. (2) is written in the form that it has the same right side as Eq. (1). In the above equation, R_s is shunt impedance, Q is quality factor and k_r is the resonant impedance. These parameters define the well-known resonator impedance model of

$$Z_{\parallel}(k) = \frac{R_s}{1 + iQ(k_r/k - k/k_r)}. \qquad (3)$$

As an example, let us examine the resonant peak at $k_r = 1.264\,\mathrm{mm}^{-1}$ in the CSR impedance shown in Fig. 3. This peak is close to the first mode defined by Eq. (4) in Ref. [1] and exhibits large amplitude and relatively large width. As shown in the figure, R_s and Q should be functions of number of cells N_{cell}. We fit this peak using Eq. (3) and get the corresponding parameters of R_s and Q. The results are plotted in Fig. 4. The figures show that the quality factor is almost a linear function of N_{cell}. And $R_s/Q/N_{cell}$ converges to a constant of around 130 Ω when $N_{cell} > 5$, this agree with the shape of calculated wake potentials in Fig. 3(c). Larger values are observed at $N_{cell} < 5$ and we believe it is due to fitting errors. It is seen that for small number of cells, the CSR impedance spectrum becomes broad-band and neighboring resonant peaks overlap with each other.

Next we check the instability threshold using a set of machine parameters of SuperKEKB positron damping ring as shown in [6]. The parameters are listed as follows: $C = 135.5\,\mathrm{m}$, $\gamma = 2153$, $\alpha_p = 0.0141$, $\sigma_\delta = 5.5 \times 10^{-4}$, $\sigma_z = 7.74\,\mathrm{mm}$, $N_0 = 5.0 \times 10^{10}$. Using these parameters, the threshold is calculated as $\left|Z_{\parallel}/n\right| < 0.24\,\Omega$ according to Eq. (1).

The bunch length of $\sigma_z = 7.74\,\mathrm{mm}$ corresponds to a critical bandwidth $0.13\,\mathrm{mm}^{-1}$ in unit of wavenumber. With $k_r = 1.264\,\mathrm{mm}^{-1}$, the critical quality factor is $Q_{th} = 9.8$. Comparing with Fig. 4(a), we conclude that when $N_{cell} > 5$, Eq. (2) should be applied. Choosing $R_s/Q/N_{cell} = 130\,\Omega$ and taking into account the total number of cells $N_{cell} = 32$ for two arc sections, we obtain an impedance of $0.95\,\Omega$ for the left side of Eq. (2). This value is quite above $0.24\,\Omega$ which is defined by machine parameters. It implies that interfered CSR might be of importance in the SuperKEKB positron damping ring. More careful studies via standard numerical simulations have been done to understand the CSR induced microwave instability. The readers are referred to Ref. [15].

SUMMARY

In this paper we presented the numerical calculations of the longitudinal CSR impedance for a bunched beam passing through a series of bending magnets. The multi-bend interference in CSR fields was demonstrated. The calculations revealed that the multi-bend interference can be significant in a small ring. In such a ring, the bending radius may be in the order of several meters, and the CSR fields reflected from the outer chamber wall may reach the bunch tail.

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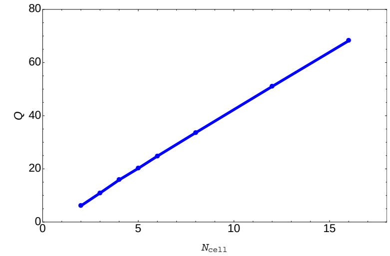
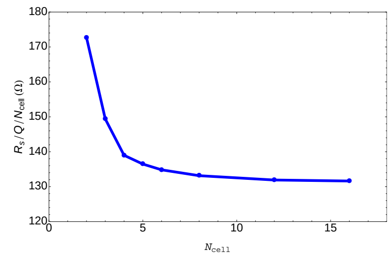


(a) Quality factor



(b) Shunt impedance

Figure 4: Quality factor and shunt impedance as a function of number of cells at the resonant peak of $k_r = 1.264\,\mathrm{mm}^{-1}$. CSR impedances are calculated using different number of cells in one arc section of SuperKEKB positron damping ring dipole magnets. The shunt impedances have been normalized by the number of cells and quality factor.

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