# HIGH INTENSITY TRANSIENT BEAM DYNAMIC STUDY IN TRAVELLING WAVE ELECTRON ACCELERATORS WITH ACCOUNTING OF BEAM LOADING EFFECT

T.V. Bondarenko, E.S. Masunov, S.M. Polozov, V.I. Rashchikov, A.V. Voronkov
National Research Nuclear University - Moscow Engineering Physics Institute, Moscow, Russia.

*Abstract*

The beam loading effect is one of the main problems limiting the beam current. The methods of the beam dynamic simulation taking into account the beam loading effect were discussed previously. The simulation methods and the especial code version BEAMDULAD-BL were described in the paper [1]. The beam loading effect was considered only for travelling wave linacs and for the stationary beam acceleration regime only. Now it is important to study the beam dynamics of short current pulses, i.e. for transient mode. We can consider only one beam bunch (or a bucket of bunches) in a long external RF field pulse in the stationary case. The beam radiation and wake fields can be calculated in the quasi-statically approximation this case. This approximation cannot be used for transient mode. The methods of beam dynamics simulation were discussed in this paper for transient mode. New code version BEAMDULAC-BLNS is described. The test simulations are carried out.

## INTRODUCTION

At the present time charged particles linear accelerators are useful in experimental physics and in some areas of technology. The main advantages of linear accelerators are: high rate of energy gain, high limit beam intensity, simple beam extraction. The linear accelerator can get a well-collimated beam with relatively high pulse intensity. Linear accelerators are often used as injectors for high power synchrotrons.

Accurate treatments of the beam own space charge field and its influence on the beam dynamics is one of the main problems for designers of high current RF accelerators. Coulomb field, beam radiation and beam loading effect are the main factors of the own space charge. Typically, only one of the components takes into account for different types of accelerators. It is the Coulomb field for low energy linacs and radiation and beam loading are studying for higher energies. But both factors should be treated in modern low and high energy high intensity linacs. The mathematical model should be developed for self consistent beam dynamics study taking into account both Coulomb field and beam loading influence. That is why three-dimensional self-consistent computer simulation of high current beam is very actually.

Let us describe the beam loading effect briefly. The beam dynamics in an accelerator depends not only on the amplitude of the external field but on the beam own space charge field also. The RF field induced by the beam in the accelerating structure depends on the beam velocity as well as the current pulse shape and duration in general. The influence of the beam loading can provide the irradiation in the wide eigen frequency mode and decrease the external field amplitude. Therefore we should solve the motion equations simultaneously with Maxwell's equations for accurate simulation of beam dynamic.

The method of kinetic equation and the method of large particles are the most useful methods for self-consistent problem solving. The Maxwell equation solving can be replaced by solving of the Poisson equation if we take into account only Coulomb part of the own beam field. This equation can be solved by means of the well-known large particles methods as particle in cell (PIC) or cloud in cell (CIC). There is no easy method of beam dynamics simulation that takes into account the beam loading effect.

The methods of beam dynamic simulations taking into account the beam loading effect in linacs, working on a traveling wave mode in a stationary case were considered in the previous articles [1-2]. Traditionally it was considered that the main influence on the dynamic has the Coulomb field charge at small energies of the beam. The radiation field is not taken into account. The radiation field is taken into account only at high energies and for well-bunched beams. The joint account of both parts of the own field of the beam does not usually occur before our previous works [1-2].

It is important to know what beam loading influences the beam dynamics in the transient mode. We will study short current pulses which are inputting into resonator with steady-state field and with the transient field mode. We must take into account both process: gradually beam input to the resonator or waveguide and transient mode for external RF field. The second process is more easily for account. We can consider external field as in stationary case if the beam is input into cavities when RF field transient process is finished.

Let us consider the algorithm of beam dynamics simulations taking into account the beam loading effect in accelerators, working on a traveling wave in the transient mode.

## THE EQUATION OF MOTION IN SELF CONSISTENT FIELD AND SIMULATION METHODS FOR TRANSIENT MODE

In the stationary case the beam dynamics can be calculated for only one beam part that has the phase length equal to one period of the external RF field. It is necessary to calculate the dynamics of all beam particles in the transient case and we must to take into account all particles of short current pulse which are into accelerating structure in the time moment. In this case, the analyzed beam can be represented in 2D or 3D phase space as a number of the large particles. These large particles would have the torus form (a ring with finite-size) with a rectangular cross-section for 2D simulation due to the axial symmetry of the task. The parallelepiped form large particles are conveniently use in 3D case.

The charge of any large particle is:

$$Q = J_{pulse} \cdot \tau_{pulse}/N, \tag{1}$$

where $J_{pulse}$ – the pulse beam current, $\tau_{pulse}$ - the duration of the current pulse, $N$ – the number of large particles.

The dynamics of every large particle should be simulated in the external field and in the own space charge field self-consistently. The initial particles distribution is given in the start of simulations with the help of especial algorithm in the 2D or 3D phase space. The initial particles distribution should take into account the delay of each particle input into accelerating structure. Further on, the system will be defined self-consistently.

The beam which is traveling into resonant structure decreases the amplitude and changes the phase of the external RF field. It also excites a number of wake fields for all resonant frequencies of the structure. Let we consider for example waveguide section with $\beta_v > \beta_{gr} > 0$, where $\beta_v$ and $\beta_{gr}$ are phase and group velocities of the wave, respectively. For simplicity we will consider only one RF field harmonic with $\nu=1$. The field acting in the beam cross section with coordinate $z$ differ on value $\Delta\tilde{E}^+ = \tilde{E}(z,\tau_{k+1}) - \tilde{E}(z,\tau_k)$ for the $k$-th and $(k+1)$-th bunches, i.e. during the time equals to pulse length $T_b$ the field is changed to $\Delta\tilde{E}^+$:

$$\Delta\tilde{E}^+ = \frac{E_s^0(z_k)E_s^0(z_{k+1})T_b}{2P_s\left(v_{gr}^{-1}(z) - v_q^{-1}(z)\right)}\tilde{I}_1(t) \tag{2}$$

where $v_q$ – particles velocity, $E_s^0$ - the amplitude of the accelerating field in the $s$-th bandwidth; $P_s$ - the power of the $s$-th bandwidth, $\tilde{I}_1$ - pulse beam current.

In the other hand, if we do not takes into account the attenuation of RF power in the walls and structure dispersion, the field will change at a fixed time $t$ to the same value $\Delta\tilde{E}^+$ on the length equal to:

$$\Delta z = \frac{v_{gr}v_q}{v_q - v_{gr}}T_b \tag{3}$$

Indeed, in accordance with $\tau_{gr}(z) - \tau_q(z) = T_b N_f$, where $N_f$ – number of bunches, which are radiates into the structure and get part of the own space charge field in the coordinate $z$. The own field influence will increase with a displacement $\Delta z$ in case when

$$\tau_{gr}(z+\Delta z) - \tau_q(z+\Delta z) = T_b(N_f+1) \tag{4}$$

and equation (3) can be easily rewritten.

Let we introduce the new variable

$$\tau = t + \tau_{gr}(z) - \tau_q(z) = \sum\left[\Delta t_j + \Delta z_j\left(\frac{1}{v_{gr}} - \frac{1}{v_q^j}\right)\right], \tag{5}$$

where $j$ is the large particle number. According to the above noted

$$\frac{\Delta\tilde{E}^+}{\Delta\tau} = -\frac{vv_{gr}}{v - v_{gr}}R_{sh}\tilde{I}_1. \tag{6}$$

where $R_{sh}$ – shunt impedance of the structure in the base band width. When a large number of bunches are considered and $\Delta\tau \to 0$ we will have

$$\frac{\partial\tilde{E}^+}{\partial\tau} + \frac{vv_{gr}}{v - v_{gr}}\frac{\partial\tilde{E}^+}{\partial z} = -\frac{vv_{gr}}{v - v_{gr}}R_{sh}\tilde{I}_1. \tag{7}$$

It is easy to generalize this equation taking into account the field attenuation in the structure and the structure dispersion [3-4]. It should be remembered that at a fixed time $t$ and at the length $\Delta z$ the field value is additionally reduced by the small amount of $-\left(\alpha - \frac{1}{2R_{sh}}\frac{\Delta R_{sh}}{\Delta z}\right)\tilde{E}^+$.

Here $\alpha$ is the RF power attenuation. As the result we have finally the equation of beam motion in the point of bunch placement taken into account the beam loading effect for transient mode:

$$-\left(\frac{1}{v} - \frac{1}{v_{gr}}\right)\frac{\partial\tilde{E}^+}{\partial\tau} + \frac{\partial\tilde{E}^+}{\partial z} + \left(\alpha - \frac{1}{2R_{sh}}\frac{dR_{sh}}{dz}\right)\tilde{E}^+ = -R_{sh}\tilde{I}_1(z,t). \tag{8}$$

For waveguide system with negative dispersion in the same way we can obtain

$$-\left(\frac{1}{v} + \frac{1}{v_{gr}}\right)\frac{\partial\tilde{E}^+}{\partial\tau} + \frac{\partial\tilde{E}^+}{\partial z} + \left(\alpha - \frac{1}{2R_{sh}}\frac{dR_{sh}}{dz}\right)\tilde{E}^+ = R_{sh}\tilde{I}_1(z,t). \tag{9}$$

There are no limitations on the amount of the group velocity in the derivation of non-stationary equations of excitation (8), (9). So they will be used just like for highly dispersed systems and for the weak dispersion of waveguide systems.

The solution of the equations (8), (9) should be done with the given initial and boundary conditions. For example, if the beam is injected with $t$=0 into the empty waveguide with length $L$, they are agreed with the two parts:

$$E^+(z, t=0) = 0 \text{ and } \left.\begin{array}{l} E^+(z=0,t)=0, v_{gr}>0, \\ E^+(z=L,t)=0, v_{gr}<0. \end{array}\right\} \tag{10}$$

Thus the calculation of RF fields excited in waveguide systems can be done for not relativistic or relativistic beams with the long duration of the current pulse $T_b >> T_f$ and in transient mode $T_b < T_f$ also. Same equations with zero right side (the homogeneous equation) describe the self-consistent beam dynamics taken into account external RF field and own space charge field in the stationary case. The self-consistent electromagnetic field can be founded from equations (7)-(9) using correct initial and boundary conditions. Equations (7)-(9) can be easily rewritten taking into account all own space charge RF field harmonics.

The algorithm of simulation for the transient mode is mainly similar to the algorithm developed for the stationary case [1-2]. The method of Coulomb field treatment used for BAMDULAC-BL code was discussed in [5].

## ELECTRON BEAM DYNAMICS SIMULATION

The results of beam dynamics simulation were compared with the measurement data obtained for the traveling wave electron linac U-28 of Radiation-Accelerating Centre of National Research Nuclear University "MEPhI". The main U-28 characteristics are given in Table 1. Three-dimensional code BEAMDULAC-BL has been used for beam dynamics simulation in U-28 for stationary case and new 3D code BEAMDULAC-BLNS was used for beam dynamics simulation in U-28 for transient mode.

Table 1: Parameters of U-28 linac

| Parameterer | Value |
|---|---|
| Average output energy, MeV | 10 |
| Range output energy, MeV | 2 - 12 |
| Max pulse beam current, mA | 440 |
| Max average beam current, μA | 170 |
| Normalized energy spectrum $(\Delta W/W)_{min}$, % | 3 |
| Pulse duration, μs | 0,5 – 2,5 |
| Pulse repetition rate, 1/s | 400 |

Some results of simulation are presented in Figure 1. It was shown, that beam loading effect is too small for beam with current $I \leq 0.2$ A. The results of numerical simulation are in a good agreement with experimental one. The modeling shows that the results of the particle dynamics in stationary and transient case are in good agreement too.

## CONCLUSIONS

The simulation of the high current electron beam dynamics in the linear accelerator can be carried out taking into account beam loading effect in the transient mode. The mathematical model of self-consistent 3D high current relativistic beams dynamics in linacs has been described for transient mode. The algorithm and the computer code were done using this model. The analysis of an electron beam dynamics in the traveling wave linacs allows us to make the conclusion, that for low beam current (less than $I \leq 0.2$ A) the beam loading effect can be not taken into account. The comparison of the beam loading effect simulations for stationary and transient modes was done. The insufficient influence of transitional processes to the beam bunching in U-28 linac was shown. Proposed methods and algorithms can be used to solve the wide range of accelerator and RF electronics problems.

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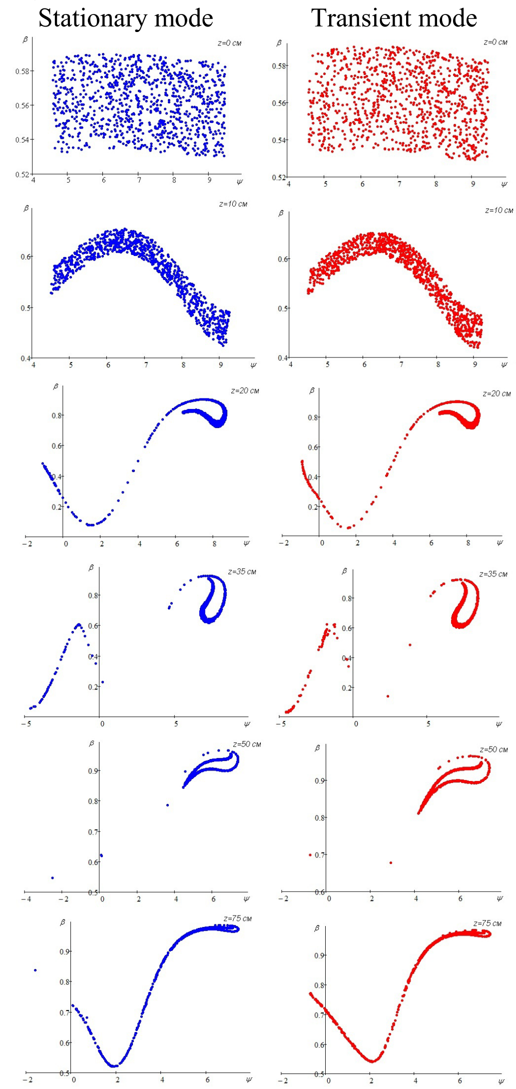


Figure 1: Electron beam bunching in U-28 linac.

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