

ELECTRON CLOUD EFFECTS IN COASTING HEAVY-ION BEAMS*

F. Petrov, O. Boine-Frankenheim, Th. Weiland, TU Darmstadt, TEMF, Germany

Abstract

During slow extraction of intense ion beams electron clouds (EC) can accumulate in the circulating coasting beam and reduce the extraction efficiency. This is a concern for the existing SIS-18 heavy ion synchrotron at GSI and for the projected SIS-100 as part of the FAIR project. For medium energy heavy-ion beams the production of electrons from residual gas ionization is very effective. The electron density is limited due to Coulomb scattering by the beam ions. Above a threshold beam intensity the two-stream instability and the resulting coherent beam oscillations limit the electron density. Below this threshold the electron cloud can lead to observable deformations of the Schottky side-bands. To avoid EC build-up one can introduce a gap in the beam using barrier rf bucket. The reduction of the build-up efficiency caused by the gap is studied in details based on the solution of the Hill's equation for electrons. Finally we estimate the saturation level for the electron cloud density.

INTRODUCTION

The main working machine in the initial phase of FAIR project will be SIS100. Existing SIS18 will accelerate the beam and inject it further into the SIS100. Before the rf capture in SIS18 and during slow extraction from SIS100 the beam is coasting. Residual gas pressure in ion accelerators is usually several orders lower than in proton machines. At the same time ionization by the heavy ions is very effective [1]. Previous studies [2] using rigid slice model [3] showed that under the conditions relevant to FAIR two-stream instability may take place. One of the cures is to apply a barrier rf bucket introducing a gap into the coasting beam. The efficiency of this measure depends on the gap length and penetration depth of the beam particle into the gap. However, there was no detailed analytical model up to now which could help to understand the effectiveness of the rf barrier. In this note we present the analytical model and benchmark it with the PIC code.

SURVIVAL RATIO OF ELECTRONS IN KV-BEAM WITH GAP

Basic methods to study the stability of the cloud in linear approximation are given in [4]. One can construct the transfer matrix M_{tr} for electrons over one beam period and study its trace. This matrix describes the motion of electrons in time for one physical point z_0 along the accelerator. In completely linear system if the trace satisfies

stability condition then all produced electrons survive gaining however different maximal amplitude depending on the moment of production (beam center, beginning or end). In realistic case when the field is decaying outside the beam electrons exceeding the border of the beam will be lost because the focusing is not enough. This will be confirmed later in this paper. But even with these losses some of electrons may survive. The ratio of the number of surviving electrons to the number of produced we call survival ratio.

For simplicity we will investigate the case of KV-beam. The transverse motion of the electron near the beam center at z_0 is described by the Hill's equation

$$x'' = K(t, z_0) \cdot x = -\omega_e^2(t, z_0) \cdot x \quad (1)$$

where electron trapping frequency $\omega_e(t, z_0)$ is the periodic function of time and longitudinal coordinate. Solving this equation one finds electron Twiss parameters α_e, β_e and γ_e . These parameters depend on time and repeat themselves with the beam revolution period P forming the lattice for electrons in time. In our case to find the lattice we construct the transfer matrix M_{tr} for one revolution period by slicing the beam and multiplying the matrices of each slice. Knowing the elements of M_{tr} for one period it is easy to find the electron Twiss parameters [5] using Eq. 2.

$$M_{tr} = \begin{bmatrix} \cos(\psi) + \alpha_e \sin(\psi) & \beta_e \sin(\psi) \\ -\frac{1+\alpha_e^2}{\beta_e} \sin(\psi) & \cos(\psi) - \alpha_e \sin(\psi) \end{bmatrix} \quad (2)$$

Plugging these Twiss parameters and electron initial conditions into Eq. 3 one can find a single electron emittance.

$$\gamma_e x^2 + 2\alpha_e x V_x + \beta_e V_x^2 = \epsilon_e \quad (3)$$

In our case we assume that electrons are produced due to the residual gas ionization with zero initial velocity. The maximal deviation is then given by Eq. 4.

$$R_{max} = \sqrt{\epsilon_e \beta_{e,max}} = x_0 \sqrt{\gamma_{e,0} \beta_{e,max}} \quad (4)$$

where $\gamma_{e,0}$ is the electron parameter at initial moment t_0 .

Let δ_{z_0, t_0} be the differential survival ratio meaning the number of surviving electrons divided by the total number of produced electrons at a certain moment t_0 at fixed z_0 . The differential survival rate δ_{z_0, t_0} is given by Eq. 5

$$\delta_{z_0, t_0} = \frac{1}{\beta_{e,max} \cdot \gamma_{e,0}} \quad (5)$$

To obtain the latter expression we have assumed that all the electrons exceeding the beam border become defocused and finally lost. In round beam only those electrons

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will continuously accumulate whose maximal amplitude is lower than the beam radius *i. e.* $a \geq x_0 \sqrt{\gamma_0 \beta_{max}}$. The area of the circle with x_0 radius to the beam cross section gives the differential survival rate.

Integrating δ_{z_0, t_0} over one beam period P gives integral survival ratio (Eq. 6)

$$\delta_{z_0} = \int_0^P \frac{\delta_{z_0, t_0}}{P} \frac{\lambda_i(t_0)}{\bar{\lambda}_i} dt_0 \quad (6)$$

here $\lambda_i(t_0)$ and $\bar{\lambda}_i$ correspond to the local and average beam line charge density. Density weight is introduced because electron ionization rate depends on the beam density.

Fig.1 compares δ_{z_0} obtained from analytical theory and PIC simulations in SIS18. In PIC simulation cloud was generated randomly inside the cylindrical beam. Simulation time was not bigger than 10 μs to skip the initial transition processes and to achieve the linear phase of density growth. The ratio of this growth rate to the actual production rate gives δ_{z_0} . One can observe a very good agreement between linear theory and PIC simulation including the wall processes and exact field solution outside the beam.

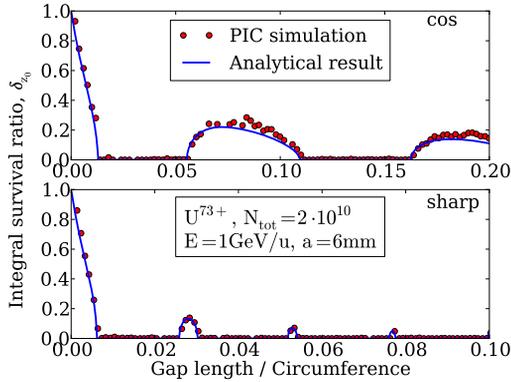


Figure 1: Comparison of δ_{z_0} obtained from PIC simulation with exact solution for transverse E-field outside the beam and analytical results. Two cases are depicted: smooth gap with cosine density profile and perfectly sharp gap.

As soon as the strong dipole magnetic field is applied the axial symmetry is broken and the survival rate increases. The reason is that electrons are bound to the field lines and can cross the beam border only vertically. Corresponding stable region is represented by the ellipse with one semi axis equal to the beam radius and another one flattened by $\sqrt{\beta_{e,max} \cdot \gamma_{e,0}}$. This yields the following (Eq. 7) differential survival ratio.

$$\delta_{z_0, t_0}^B = \frac{1}{\sqrt{\beta_{e,max} \cdot \gamma_{e,0}}} \quad (7)$$

Ellipticity of the beam also breaks the symmetry. Because the focusing force in both planes becomes different

05 Beam Dynamics and Electromagnetic Fields

D05 Instabilities - Processes, Impedances, Countermeasures

the motion of electron in x and y is decoupled. This also leads to the different Twiss parameters in both planes. The stable area is given by ellipse. One of the semi axes is obtained from the beam cross section by flattening in x direction by $X_{max} = \sqrt{\beta_{e,max,x} \cdot \gamma_{e,0,x}}$, another one by flattening in y direction by $Y_{max} = \sqrt{\beta_{e,max,y} \cdot \gamma_{e,0,y}}$

$$\delta_{z_0, t_0}^{el} = \frac{1}{X_{max} Y_{max}} \quad (8)$$

Above we were focused on defining the survival ratio for electrons in one fixed point z_0 in accelerator. The beam transverse size and form change together with z_0 according to the accelerator Twiss parameters. The whole number of electron Twiss parameters also changes making it possible that the accumulation appears in one place in accelerator and disappears in another. The consequence is that it is impossible to completely get rid of the electron accumulation using barrier bucket. The only thing possible is to reduce the accumulation rate. To understand how strong is the reduction it is useful to plot the accumulation diagram (Fig.2) as a function of x and y beam sizes.

From the given example one can see that in the drift sections the accumulation regions are distributed among rhombus like islands in a periodic pattern. In the dipole section not only the peak survival ratio is increased according to Eq. 7 but also the islands dramatically change form and size. As a first guess one can assume that different beam sizes can be met equally often in accelerator. To estimate the total survival ratio δ_{tot} one has to average δ_{z_0} over the whole range of beam sizes. Values averaged over the shown regions are given in Table 1. It is seen that the strong dipole field dramatically increase δ_{tot} especially for "dirty" cosine gap. On the other side the values are several times lower than the peak δ_{z_0} on Fig. 2.

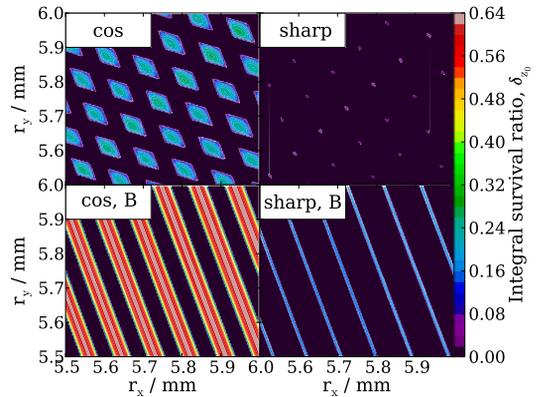


Figure 2: Accumulation diagram in drift and dipole sections for conditions on Fig. 1 with 5% gap length and two different gap forms: rectangular and cosine.

SATURATION LEVEL

To estimate the local saturation level let's assume that EC is distributed uniformly in the beam cross section. Ini-

Table 1: Total Survival Ratio δ_{tot} in Drift and Dipole Sections

Section	Rectangular shape	Cosine shape
Drift	$2.4 \cdot 10^{-4}$	$1.4 \cdot 10^{-2}$
Dipole	$4.8 \cdot 10^{-2}$	$2.8 \cdot 10^{-1}$

tially the space charge of electrons is negligible. Growing electron number will gradually neutralize the beam leading to the decrease in electron trapping frequency. This will last until the trace of the effective M_{tr} do not approach 2. In simulations in case of arbitrary M_{tr} one should gradually decrease the charge of the beam to reach the nearest unstable point. The difference between this decreased and the real beam charge is the upper limit for cloud charge.

One can illustrate this approach using the round beam with a sharp gap. Corresponding trace of the effective M_{tr} for electrons is given as follows.

$$Tr(M) = 2 \cdot \cos(\phi - \phi_+) - \omega_+ \cdot \tau \cdot \sin(\phi - \phi_+) \quad (9)$$

where $\phi = \omega_e T$ - electron phase advance per beam length; ϕ_+ - phase shift caused by electron space charge; τ - gap length. One should find the closest ϕ_+ that satisfies the condition $\|Tr(M)\| = 2$. The solution for phase is

$$\phi_+ = \phi - 2 \cdot \text{atan}\left(\frac{\omega_e \cdot \tau}{2}\right) + \pi \cdot k \quad (10)$$

where integer k is chosen to get the minimal positive ϕ_+ .

On the other hand, electron trapping frequency affected by the EC space charge in linear case is given by Eq. 11.

$$\tilde{\omega}_e = \omega_e - \omega_+ = \frac{\phi_e - \phi_+}{T} = \sqrt{\frac{(\lambda_i - \lambda_e)e}{2\pi\epsilon_0 a^2 m_e}} \quad (11)$$

The solution for local neutralization degree yields

$$\chi = \frac{\lambda_e}{\lambda_i} = 1 - \frac{2\pi\epsilon_0 a^2 m_e}{e\lambda_i} \cdot \left(\frac{\phi - \phi_+}{T}\right)^2 \quad (12)$$

Figure 3 compares saturation level from PIC simulation and result of Eq. 12. The level is overestimated. However, if one takes into account that the average rms cloud size in coasting beam with low electron space charge is $3 \cdot \pi/2$ times smaller then the agreement becomes better. Further decrease in the analytical density can be obtained if one takes into account repelling force of the cloud in the gap but the equation is more complicated.

CONCLUSIONS

In this paper we have presented the development of analytical methods to study EC build-up in long, coasting beam like bunches. This theory works only for relatively small beam charges for which no multipacting appears. This is the case for coasting beams in FAIR.

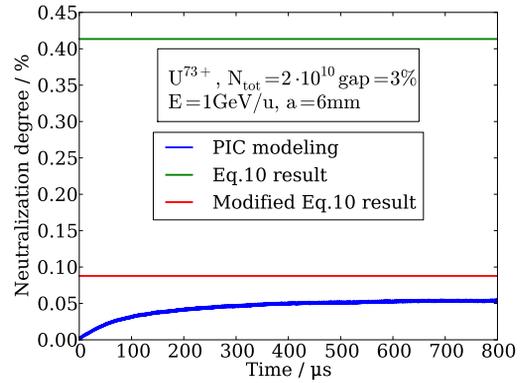


Figure 3: Saturation level obtained from PIC simulations. It is seen that Eq.12 overestimates the saturation level. For realistic average cloud radius the agreement is good.

One important result of these studies is that the cloud accumulation happens only for electrons under the influence of a field growing with distance from the beam center. PIC simulation shows that as soon as the field start to decay the electrons are untrapped.

We have shown that it is not enough to take an average beam radius and study the properties of the corresponding electron transfer matrix. The accumulation take place independently on the gap parameters. However, the hot spots where electrons accumulate and the reduction of accumulation efficiency depend on the gap parameters.

In comparison with adiabatic theory [6] our model gives exact solution for the amount of trapped electrons because it is based on the exact solution of Hill's equation governing the transverse motion of electrons. Adiabatic theory gives approximate solution which can be used, however, in case of smooth beam density distributions with significant leakage in the gap.

Further investigation is needed to find out how strong are the contributions of electron longitudinal drift and collisions with beam ions.

In application to SIS18 and SIS100 one should first additionally verify the danger of EC instability using more sophisticated PIC code than the one used in previous contribution [2] and choose the best clearing solution.

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