SIMULATION OF MULTIBUNCH MOTION WITH THE HEADTAIL CODE AND APPLICATION TO THE CERN SPS AND LHC

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Abstract

Multibunch instabilities due to beam-coupling impedance can be a critical limitation for synchrotrons operating with many bunches. It is particularly true for the LHC under nominal conditions, where according to theoretical predictions the 2808 bunches rely entirely on the performance of the transverse feedback system to remain stable. To study these instabilities, the HEADTAIL code has been extended to simulate the motion of many bunches under the action of wake fields. All the features already present in the single-bunch version of the code, such as synchrotron motion, chromaticity, amplitude detuning due to octupoles and the ability to load any kind of wake fields through tables, have remained available. This new code has been then parallelized in order to track thousands of bunches in a reasonable amount of time. The code was benchmarked against theory and exhibited a good agreement. We also show results for bunch trains in the LHC and compare them with beam-based measurements.

INTRODUCTION

Transverse coupled-bunch instabilities occur in general when several bunches interact with their surroundings, creating wake fields that act back on the bunch train in such a way as to give rise to an exponentially growing oscillation. To evaluate the rise times of such instabilities, several theories exist, such as Sacherer's [1], Laclare's [2] and Scott Berg's [3], as well as a macroparticle simulation code, MTRISM [4]. The latter has several limitations, in particular the absence of quadrupolar impedance – quite significant in e.g. the LHC due the high-impedance flat collimators [5], the impossibility to load wake functions tables which can reflect the complexity of the machine impedance, and the rigid bunch approximation for the wake field computation. The cited theories also rely on strong assumptions: Sacherer's formula assumes certain predefined intra-bunch oscillation modes; both Sacherer's and Laclare's formalisms assume a machine entirely filled with equidistant bunches, which is not the case in the LHC and even less in the SPS where only about 30% of the machine is filled to produce the nominal LHC beam. These two formalisms also assume no transverse mode coupling and a weak chromaticity. On the other hand, Scott Berg's theory is much more general and includes in principle lattice nonlinearities, chromaticity, mode coupling, and any kind of bunch filling scheme. Still, wake fields from a cylindrical structure are assumed there, which is a strong asumption, in particular in the case of the LHC.

Therefore, to be able to study multibunch instabilities in the LHC and the SPS, we chose to extend the single-bunch macroparticle code HEADTAIL [6]. A first simplified version, accounting only for rigid bunch oscillations with no longitudinal motion, was already developed in Ref. [7]. We present here a new extension that can handle both multibunch and intrabunch motion.

DESCRIPTION OF HEADTAIL MULTIBUNCH

HEADTAIL is a macroparticle simulation code where each individual macroparticle i is tracked through a ring subdivided into several kick sections. After initialization with Gaussian (or uniform) distributions, with the possibility to enforce longitudinal matching, macroparticles are tracked in mainly three steps per kick section: 1) the bunches are sliced longitudinally, 2) wake fields kicks are applied to each macroparticles, and 3) their transverse phase space coordinates are linearly transported to the next kick section. Once per turn, the synchrotron motion update is applied, separately for each bunch. For the second step, the kicks $\Delta x'_i$, $\Delta y'_i$ and $\Delta\delta_i$ are computed as

$$\Delta x'_i = \mathcal{C} \sum_{z_S > z_{S_i}} n_S W_x \left(z_{S_i} - z_S, x_S, y_S, x_{S_i}, y_{S_i}\right),$$

$$\Delta y'_i = \mathcal{C} \sum_{z_S > z_{S_i}} n_S W_y \left(z_{S_i} - z_S, x_S, y_S, x_{S_i}, y_{S_i}\right),$$

$$\Delta\delta_i = \mathcal{C} \sum_{z_S \geq z_{S_i}} n_S W_{||} \left(z_{S_i} - z_S\right),$$

where $\mathcal{C} = -\frac{e^2}{E_0\beta^2\gamma}$, γ being the Lorentz factor, $\beta = \sqrt{1-\gamma^{-2}}$, E_0 the rest mass of the elementary particles (protons or electrons) and e the elementary charge. S_i is the slice containing the macroparticle i, and n_S, x_S, y_S, z_S are the number of particles, and the transverse and longitudinal positions of each slice S (z decreases when going toward the tail of the bunches). In the above expressions the sums run over all slices and bunches before the slice of the macroparticle considered, neglecting thus any wake emitted in the forward direction. The sums continue up to a certain number of turns, i.e. the wakes of preceding turns are taken into account. $W_{||}(z)$ is the longitudinal wake function, while $W_x(z)$ and $W_y(z)$ are given by

$$W_x(z, x_S, y_S, x_{S_i}, y_{S_i}) = W_x^{dip}(z)x_S + W_{xy}^{dip}(z)y_S + W_x^{quad}(z)x_{S_i} + W_{xy}^{quad}(z)y_{S_i}, \quad (1)$$

$$W_y(z, x_S, y_S, x_{S_i}, y_{S_i}) = W_y^{dip}(z)y_S + W_{xy}^{dip}(z)x_S + W_y^{quad}(z)y_{S_i} + W_{xy}^{quad}(z)x_{S_i}, \quad (2)$$

where dip stands for "dipolar" and $quad$ for "quadrupolar". Note that coupled terms – i.e. linear wakes in the x direction but proportional to the y position and vice versa, are taken into account. The wake functions above ($W_x^{dip}(z)$, $W_x^{quad}(z)$, etc.) are provided in a table given in input.
The code has been parallelized over the bunches, which is quite efficient since all bunches can be treated independently, the only requirement being that after each slicing the processors exchange for all the bunches the positions and number of particles of each slice, such that the wakes can be computed in all bunches. This represents a limited amount of data since the number of slices usually does not exceed a few hundreds.

COMPARISON WITH THEORY

The code has been compared to the formalism of Ref. [2], in which the complex angular frequency shifts $\delta\omega$ of all possible modes are found as the eigenvalues of an infinite matrix. The most critical instability is then the one whose $\delta\omega$ has the most negative imaginary part. The formalism has been implemented in a code that automatically checks that the necessary matrix truncation still gives accurate eigenvalues (within 0.1%) by testing convergence with respect to the matrix size. The formalism assumes no linear coupling, a completely filled machine with equidistant bunches, as well as dipolar transverse impedances and a linear longitudinal bucket without distortion. We used therefore the same conditions in HEADTAIL. Also, longitudinal Gaussian distributions cut at $2\sigma_z^{rms}$ were used in both the theory and HEADTAIL. Note that in the simulations the longitudinal parameters were initially matched.
To obtain the rise times from the simulations, we compute thanks to SUSSIX [8] the highest spectral lines of the beam average transverse position and momenta, on a sliding window along the simulation, and fit the amplitude of the highest spectrum line as a function of time by an exponential.

Case of 924 bunches in the SPS

We study first the SPS filled with a 25 ns beam. The impedance of its vacuum pipe is computed from Zotter's theory for an infinitely long axisymmetric cylindrical structure [9], assuming the pipe has 2 cm radius, 2 mm thickness and is made of stainless steel surrounded by vacuum [10]. The wake function is then obtained thanks to a Fourier transform with an uneven sampling [11]. Since the beam pipe is actually of elliptical cross section with the horizontal semi-axis significantly larger than the vertical one, to obtain the final dipolar impedances and wake functions we multiply by the Yokoya factors [12] for a flat chamber, i.e. $\frac{\pi^2}{24}$ in x and $\frac{\pi^2}{12}$ in y.
Table 1 shows the beam parameters used in both the simulation and the theory, and in Fig. 1 we can see the excellent agreement, with a slight discrepancy only for the highest positive chromaticity considered.

Table 1: Beam parameters for the SPS.

Nb. of bunches	N_b	924
Bunch population	N	$5 \cdot 10^{10}$
RMS bunch length	σ_z^{rms}	0.19 m
Momentum spread	σ_δ	0.002
RF voltage	V_{rf}	3 MV
Harmonic number	h	4620
Bunch spacing	ΔT_b	25 ns
Circumference	C	6911 m
Tunes	$Q_{x,y,s}$	26.13, 26.16, 0.0073
Beta functions	$\beta_{x,y}$	42, 42 m
Norm. emittances	$\varepsilon_{x,y}$	4, 4 μm.rad
Mom. compaction	α_p	$1.92 \cdot 10^{-3}$
Lorentz factor	γ	27.7

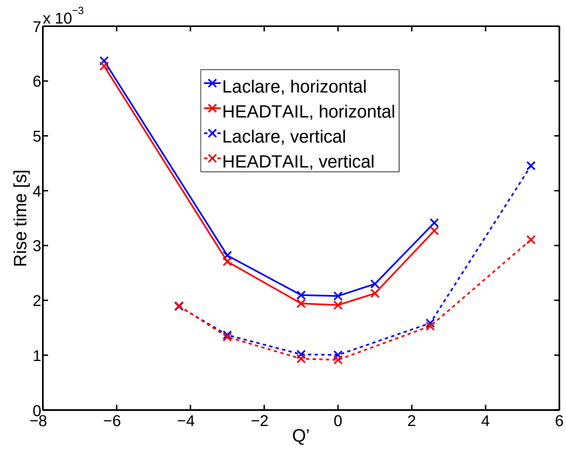

Figure 1: Rise times vs. Q' for 924 bunches in the SPS: Laclare's theory and HEADTAIL multibunch.

Case of 1782 bunches in the LHC

We study now the LHC with a 50 ns beam. The impedance model used contains several contributors: the 44 collimators with settings as measured in operation of the machine at 3.5 TeV in 2011, the beam screens covering 86% of the ring which are cold copper coated devices inside the beam pipe, the vacuum pipe for the remainder 14% with various cross sections, and a broad band impedance model to account for most of the smooth transitions around the ring [13]. Zotter's theory [9] and Yokoya factors [12] were used to compute the impedances of each element (except for the broad band model, coming from analytical estimates), then Fourier transformed thanks to the same technique as in the previous section. In the horizontal plane, the beam screens' wakes have been multiplied by an approximative factor 2 [14] to account for the stainless steel weld (note that the exact effect requires a deeper analysis). The full wake and impedance model will be detailed in a later publication [11]. To obtain the wake functions of the full ring to be applied at a single kick location, we weight the wake functions of each element by its beta function [15], except for the coupled terms where the weight is $\sqrt{\beta_x\beta_y}$ [11].
The beam parameters used in both the simulation and the

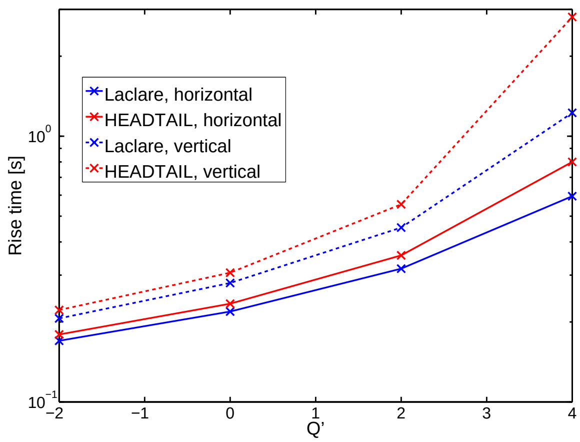

Figure 2: Rise times vs. Q' for 1782 bunches in the LHC: Laclare's theory and HEADTAIL multibunch.

theory are shown in Table 2, and their comparison for various chromaticities in Fig. 2. The agreement is overall good.

Table 2: Beam parameters for the LHC with 1782 bunches.

N_b	1782	N	$1.2 \cdot 10^{11}$	σ_z^{rms}	0.09 m
V_{rf}	16 MV	h	35640	σ_δ	$1.9 \cdot 10^{-4}$
ΔT_b	50 ns	C	26659 m	α_p	$3.2 \cdot 10^{-4}$
Q_x	64.31	Q_y	59.32	Q_s	0.0029
β_x	66 m	β_y	71.4 m	γ	3730.3
ε_x	3.75 μm.rad		ε_y	3.75 μm.rad	

COMPARISON WITH LHC BEAM-BASED MEASUREMENTS

Coupled-bunch instability rise times were measured in the LHC for a beam with one batch of 36 bunches following a batch of 12 bunches, with 50 ns spacing and 925 ns between the two batches [16]. For such small number of bunches and given the scrubbing already performed in the machine, the effect of electron cloud is thought to be negligible and has not been considered. We compare in Fig. 3 one set of measurements, whose parameters are summarized in Table 3, with simulations with the same LHC impedance model as in the previous section, except that the collimators settings were those measured during the experiment, and that all the wake components of Eqs. (1) and (2) were used, including coupled terms due to tilted collimators. The agreement is reasonably good.

CONCLUSION

The wake fields code HEADTAIL is now able to simulate multibunch trains, and was successfully benchmarked with theory in simple cases, for instability rise times. Using the LHC impedance model, the code was also compared to an LHC measurement, giving good agreement.

Table 3: LHC experiment parameters. Momentum spread and synchrotron tune have been deduced from bunch length and RF voltage, assuming perfect matching.

N_b	48	N	$1.21 \cdot 10^{11}$	σ_z^{rms}	0.094 m
V_{rf}	6 MV	h	35640	σ_δ	$3.4 \cdot 10^{-4}$
ΔT_b	50 ns	C	26659 m	α_p	$3.2 \cdot 10^{-4}$
Q_x	64.28	Q_y	59.31	Q_s	0.0049
Q'_x	0.4	Q'_y	0.3	γ	479.6
β_x	66 m	β_y	71.5 m	ε_x	2.5 μm.rad
ε_y	2.3 μm.rad				

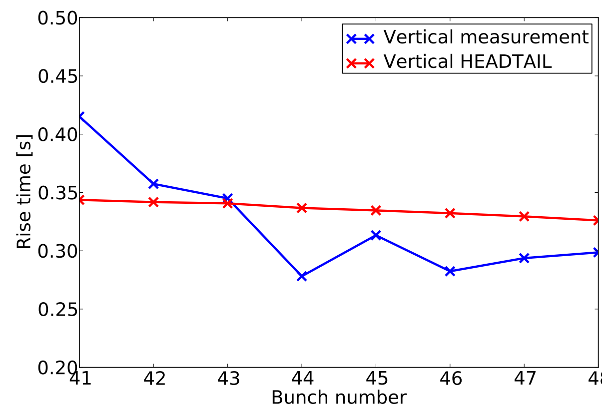

Figure 3: Vertical instability rise times for the 8 last bunches of the train: LHC measurement vs. HEADTAIL.

ACKNOWLEDGMENTS

The authors warmly thank V. Kain, D. Valuch, L. Ponce and W. Höfle for helping with the LHC measurements, A. Rossi, D. Wollmann and R. Bruce for providing collimator information, and B. Salvant for helpful discussions.

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