BEAM BREAKUP SIMULATION FOR THE PEP-X ERL*

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Abstract

The transverse beam breakup (BBU) is one of the dominant factors in energy recovery linac (ERL) for the available beam current. A tracking code built in Matlab is developed and benchmarked by comparing with the analytical solutions with the simple model. Studies on the threshold current and emittance growth due to the transverse BBU for the proposed PEP-X ERL are presented in this paper.

INTRODUCTION

Energy recovery linacs (ERLs) can in principle provide intense, high quality electron beams for a diverse range of applications, such as spontaneous and FEL radiation production, electron cooling of ion beams and ERL-based electron-ion collider [1]. A radiation light source based on ERL is being planned as one part of the PEP-X project [2]. The planned PEP-X ERL will accelerate the beam with current of 100 mA up to 5GeV. Each bunch receives energy when passing the cavities at the first time, and returns the energy to the cavities during the second pass by shifting the RF phase by π. The main parameters of the PEP-X ERL are shown in Table 1 and the optics functions of one of the designed lattices [3] are presented in Fig. 1.

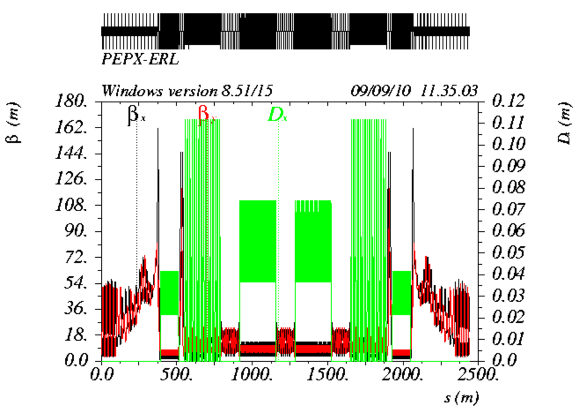

Figure 1: The optics functions of the PEP-X ERL, one recirculation.

Beam breakup (BBU), especially the transverse BBU are one of the generic issues for the ERL, which limits the available current that can be maintained with stable performance. The theoretical and simulation study of the BBU instability in recirculating linacs with continuous wave operation (the bunches of the different turns are at about the same accelerating RF phase) started early from 1980s [4, 5]. Recently, the accelerator physicists in Cornell University made extensive efforts to extend the theoretical study directly applicable to the ERL operation where the bunches do not have to be at the same RF phase during each recirculation turn [6, 7], and developed several programs (BI [8] and BMAD built-in library [9]) to simulate the beam breakup effects in ERL. The proportionate relationship of the calculated threshold currents due to transverse dipole high order modes (HOM), longitudinal HOMs and transverse quadrupole HOMs for the Cornell ERL is approximately found to be 1 : 3 : 7 [10, 11, 12].

Thus the transverse BBU is the dominant factor in ERL for the envisioned beam current. We therefore performed the simulation study of the transverse BBU for PEP-X ERL based on the tracking code developed ourselves.

Table 1: Main parameters of the PEP-X ERL

Parameters	Flux	Coherence	Short pulse	Unit
Energy	5	5	5	GeV
Current	100	25	100	mA
Bunch charge	77	19	77	pC
Rep. rate	1300	1300	1300	MHz
Norm. emittance	0.3	0.08	1	mm-mrad
Geom. Emittance	31	8.2	103	pm
RMS bunch length	2000	2000	100	fs
Energy spread	0.2	0.2	1	10^{-3}
Beam power	500	125	500	MW
I_{peak}	16.3	4	327	mA

THEORETICAL ASPECTS

In the simplest model of beam breakup, only one cavity with one dipole higher order mode (HOM) is assumed. Bunches are injected into the cavity, accelerated, and then recirculated to pass the cavity at a second time before they are ejected in to the beam dump. Assuming a bunch passing through the cavity with one dipole HOM with voltage of $V(t) = V_0 e^{-iwt}$, it will experience a transverse kick $\Delta p_x = eV(t)/c$, and return to the cavity after one recirculation loop with a transverse offset $\Delta x = T_{12}\Delta p_x$, where T_{12} is the element of the transfer matrix that relates initial transverse momentum p_x before and x after the recirculation loop. If this offset leads to an increase of the HOM voltage, the subsequent bunch will experience larger transverse kicks and lead to a further growth of the HOM voltage. The HOM voltage and transverse offset will continuously increase with respect to the bunch index, and therefore instability develops.

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For this simple model, one can obtain explicit dispersion relation between the beam current I_0 and HOM voltage oscillating frequency w [6],

$$I_0 = \frac{2}{KT_{12}} e^{-iwn_r t_b} \frac{e^{(w_r/2Q_r)\delta t_b}[\cos(w^+ t_b) - \cos(w_r t_b)]}{e^{-iw^+ t_b}\sin(\delta w_r t_b) - \sin([\delta - 1] w_r t_b)}, \quad (1)$$

with

$$K = t_b (e/c^2)(R/Q)_r (w_r^2/2),$$

$$w^+ = w + i(w_r/2Q_r).$$

where w_r, $(R/Q)_r$, Q_r are the HOM frequency, shunt impedance, quality factor of the HOM, respectively; e is the elementary charge; c is the speed of light; t_r is the recirculation time; t_b is the time interval between two adjacent bunches; n_r and δ are the parameters describing the relationship between t_r and t_b,

$$t_r = (n_r - \delta) t_b, \quad (2)$$

with n_r as integer and $\delta \in [0,1)$.

The threshold current I_{th} is the smallest current I_0 for which there is a real $w \in [0, \pi/t_b]$ that satisfies the dispersion relation Eq. (1). For the beam current above I_{th}, the w satisfying Eq. (1) will have a positive imaginary part and the corresponding beam motion becomes unstable. Figure 2 presents the $I_0(w)$ in complex plain for one specific case, $n_r = 7$, $\delta = 0.27$.

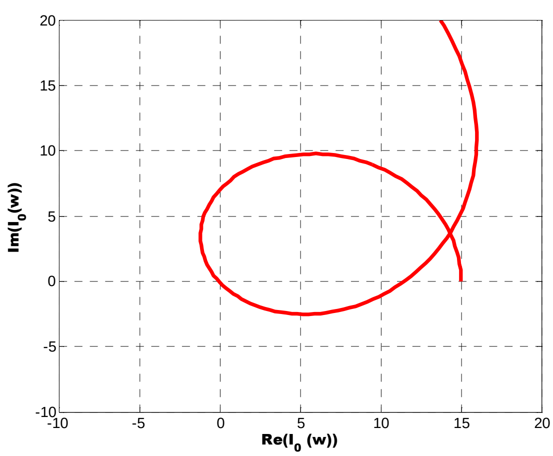

Figure 2: Complex plain of $I_0(w)$ with respect to w obtained from the dispersion relation Eq. (1) for one specific case, $n_r = 7$, $\delta = 0.27$. The threshold current is obtained to be 11.5 A.

SIMULATION CODE

Analytical results can be easily obtained for simple models, like that shown in the above section. However, for a practical ERL design which consists of many cavities and many HOMs, it is difficult to calculate the threshold current analytically and calls for detailed numerical simulations.

The tracking code is developed with c language implanted in Matlab environment, which takes both the advantage of high calculation speed of the c language and the flexible Matlab built-in functions. Each bunch is treated as one micro particle in simulation, with constant nonzero transverse offset at the injection point, and its coordinate is calculated element by element, including the interaction with HOMs at each cavity. The transverse offset of each bunch at the exit of the linac and the voltage the bunch experiences are recorded. The obtained data are used to determine the threshold current for which both the HOM voltage and the transverse offset with respect to the bunch index start increasing exponentially. The tracking can be done within 1 minute, for 0.1 ms beam duration in PEP-X ERL, which has several thousand of elements and 256 cavities in the recirculation loop.

With the simplest model, i.e. one dipole HOM and one recirculation, we compare the threshold currents using the tracking simulation and numerical solving the dispersion relation, as shown in Fig. 3. The two approaches agree very well.

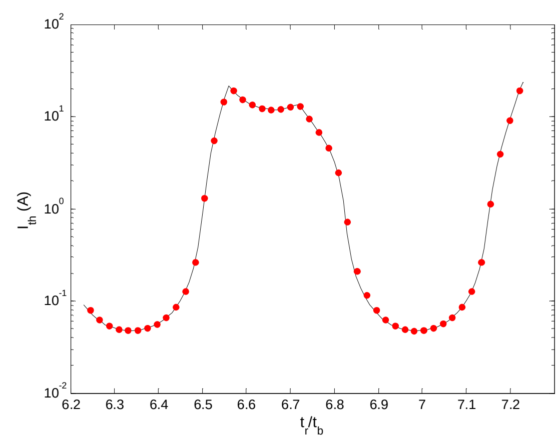

Figure 3: Threshold current obtained by tracking simulation (red dots) and numerical solution (black line) of the dispersion relation Eq. (1).

In the simulation, we adopt the same type of cavities as that are used in Cornell ERL [13]. The four most dominant HOMs used in simulations are listed in Table 2. Previous study shows that, one can significantly increase the BBU threshold current by introducing a random distribution of HOM frequencies by fabricating each cavity slightly [6, 10]. We simulate four cases, *a*) with one HOM at each cavity, *b*) with one HOM at each cavity, but having a frequency spread with rms width σ_f = 2, 4, 6, 8, 10 MHz, *c*) with four HOMs at each cavity, *d*) with four HOMs and frequency spread with σ_f = 2, 4, 6, 8, 10 MHz. The results are shown in Table 3.

Table 2: The Four Dominant Transverse HOMs for the Cavity

HOMs	f_r (GHz)	Q_r	(R/Q)r(Q)
1	1.87394	20912.4	109.6
2	1.88173	13186.1	27.85
3	1.86137	4967.8	71.59
4	2.57966	1434.2	108.13

It shows that, without HOM frequency spread, the transverse BBU threshold current for the PEP-X ERL is 14.8 mA, after introducing 2 MHz frequency spread, the threshold increases to be 250.0 (± 23.8) mA.

Table 3: Threshold Currents for PEP-X ERL

σ_f (MHz)	I_{th} (mA) mod 1	σ_I (mA) mod 1	I_{th} (mA) mod 1-4	σ_I (mA) mod 1-4
0	14.8	0	14.8	0
2	229.2	29.0	250.0	23.8
4	318.5	33.0	381.6	49.7
6	372.2	47.3	414.4	49.4
8	407.1	51.4	440.4	60.0
10	458.8	71.8	466.5	60.6

EMITTANCE GROWTH BELOW THE THRESHOLD CURRENT

The beam emittance preservation is an important issue in practical operation. Even before the beam current reaches the threshold, the HOM voltage and the transverse offset of the injected bunch will increase to some extent, leading to unexpected emittance growth.

We simulate the case with injected bunches having random initial transverse offsets, which is close to the scenario of the practical operation, and record the variation of the transverse projected emittances with respect to the bunch current, with one HOM at each cavity, as shown in Fig. 4.

The transverse projected emittance is evaluated using successive 10,000 bunches,

$$\varepsilon_p = \sqrt{< x^2 >< x'^2 > - < xx' >^2} \quad (3)$$

It shows that, the emittance can increase 10% at the current I_0 = 12.5 mA, which is 85% of the threshold current I_{th} = 14.8 mA. As a result, to preserve the emittance, one should operate the machine with current sufficiently below the threshold current.

CONCLUSION

A tracking code built in Matlab is developed to simulate the transverse beam breakup in PEP-X ERL. The code is benchmarked by comparing with the analytical solutions with the simple model that consists of one cavity and one HOM. The threshold current with zero and 2 MHz HOM frequency spread for the PEP-X ERL are calculated to be 14.8 and 250.0 (± 23.8) mA, respectively. We also study the emittance growth caused by the HOMs. To preserve the emittance and ensure good performance, one should operate the machine with current sufficiently below the BBU threshold.

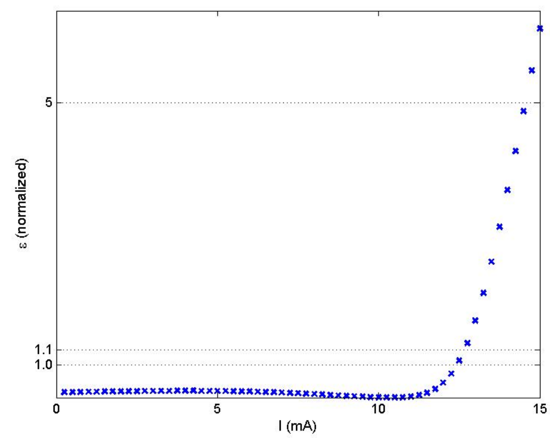

Figure 4: Variation of projected emittance with respect to the beam current.

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