FITTING FORMULAS FOR SPACE-CHARGE DOMINATED FREE-ELECTRON LASERS

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Abstract

A simple power-fit formula for calculating the gain length of the fundamental Gaussian mode of a free-electron laser having strong space-charge effects in the 3D regime has been obtained [1]. This tool allows for quick evaluation of the free-electron laser performance in the presence of diffraction, uncorrelated energy spread, and longitudinal space-charge effects. Here, we use it to investigate high-gain FEL amplifiers considered candidates as high average power light sources. Results are compared with detailed numerical particle simulations using the free-electron laser code Genesis [2].

INTRODUCTION

The free-electron laser (FEL) is a device that transforms the kinetic energy of a relativistic electron beam (e-beam) into electromagnetic radiation. High gain FELs operate on the principle that tunable, narrow bandwidth light pulses can be emitted and amplified many orders of magnitude by the passage of a relativistic e-beam through a periodic magnetic undulator. The power gain length - the distance along the undulator it takes for the power of the emitted light to increase by a factor of e during the exponential growth regime - is an extremely important parameter to consider when designing a FEL because it dictates the size of the undulator system needed for the light to reach saturation. It is given in a one-dimensional system as

$$L_{1D} = \frac{1}{2\sqrt{3}k_u\rho}, \quad (1)$$

where ρ is the well-known Pierce parameter [3], given by

$$\rho = \left[\frac{K\,[JJ]}{4\sqrt{2}}\frac{\theta_p\gamma_z}{k_u\gamma}\right]^{\frac{2}{3}}. \quad (2)$$

Here, k_u is the undulator wavenumber, $K = eB_u/mck_u$ is the dimensionless undulator parameter, with peak magnetic field B_u. The Lorentz factor γ is the electron beam energy in units of the rest energy mc^2, while the Lorentz factor relating the average longitudinal beam motion to the laboratory frame is $\gamma_z = \gamma/\sqrt{1+\frac{K^2}{2}}$. The factor accounting for the coupling of the electron motion to radiation emission in the case of a planar undulator is $[JJ] = J_0\left[K^2/\left(4+2K^2\right)\right] - J_1\left[K^2/\left(4+2K^2\right)\right]$, where J_0 and J_1 are Bessel functions of the first kind. The relativistic plasma wavenumber is

$$\theta_p = \sqrt{\frac{2I_e}{I_A\gamma\gamma_z^2\sigma_x^2}}, \quad (3)$$

where σ_x is the rms electron beam size, I_e is the electron beam peak current and $I_A \simeq 17$kA is the Alfvén current.

There are many instances, however, where one-dimensional theory fails to capture the sensitive and complex dependence of the FEL gain length on various three-dimensional effects. These effects include diffraction (the tendency of the light to spread while propagating), longitudinal space-charge, emittance, detuning from resonance, and the uncorrelated energy spread of the e-beam. The three-dimensional effects conspire to lengthen the gain from the nominal one-dimensional case and can be described using a set of scaled parameters that individually represent the features of the FEL system [4]. Here, we focus on the essential parameters that best represent infrared-optical regime FELs: the diffraction parameter $\eta_d = L_{1D}/2k\sigma_x^2$, the scaled energy spread parameter $\eta_\gamma = 2k_u\sigma_\gamma L_{1D}$, and the space-charge parameter $\overline{\theta}_p = 2\theta_p L_{1D}$. The FEL wavelength is given by $\lambda = 2\pi/k$ while the rms uncorrelated e-beam energy spread is σ_γ. In [4] Xie provided a useful power fit formula for the gain length of short-wavelength FELs where emittance effects play an important role and where space-charge can be neglected. However, many FELs of current interest do not operate in this regime. For instance, infrared-optical self amplified spontaneous emission high-gain FELs that are based on extremely high brightness e-beams operate at much lower energies than their short wavelength counterparts and are thus susceptible to strong space-charge effects. These FELs have long been considered candidates as high average power light sources [5] since the e-beam, which acts as the gain medium, supports the amplification of all frequencies and does not suffer from thermal loading constraints. Recent experiments at Jefferson Lab using an FEL oscillator have demonstrated a power of 10 kW emitted in one second long pulses [6]. The resonator optics in the oscillator, however, become a limiting factor as the power is scaled to MW levels because they quickly become vulnerable to thermal damage attributable to the emitted light's small transverse beam size in the undulator. This challenge can be removed by employing a high-gain FEL amplifier driven by a high-brightness e-beam that possesses a large peak current and a small normalized transverse emittance. Many of the proposed amplifiers are driven by a $\sim$ 100 MeV e-beam where longitudinal space-charge will strongly

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affect the FEL performance [7, 8].

Up until recently, no simple formulation similar to Xie's has been developed for quickly predicting important FEL characteristics when space charge effects become important even though there has been historical interest and recent experimental investigations into FELs that operate under these conditions. However, a simple fitting formula for the gain length of the fundamental Gaussian mode of a FEL in the presence of diffraction, uncorrelated energy spread, and longitudinal space-charge effects was recently developed in [1] in the limit that $\eta_\epsilon = 2kL_{1D}\epsilon_x^2/\sigma_x^2 \to 0$, or $\eta_\epsilon \ll 2\eta_\gamma$ and $\sqrt{\eta_\epsilon \eta_d} \ll 1$. Here, we use this formula to quickly evaluate the performance of the high power FEL described in [7] and compare the results to numerical particle simulations using the 3D FEL code Genesis 1.3 [2].

ANALYTIC TREATMENT

The general three-dimensional high-gain integro-differential FEL field equations in the presence of uncorrelated energy spread and space-charge effects have been solved in the single, fundamental mode limit using a variational approach, which we quote here without derivation, to yield the equations

$$\left(\overline{S_1} - \frac{\overline{\theta_p}^2}{1+\alpha}\right)\left(\overline{\delta k} + \frac{\eta_d}{\alpha}\right) + \left(\frac{2}{\sqrt{3}}\right)^3 \frac{1}{1+\alpha} = 0,$$
$$-\left(\overline{S_1} - \frac{\overline{\theta_p}^2}{1+\alpha}\right)\frac{\eta_d(1+\alpha^2)}{\alpha^2} + \dots$$
$$\dots + \overline{\theta_p}^2\left(\overline{\delta k} + \frac{\eta_d}{\alpha}\right) = \left(\frac{2}{\sqrt{3}}\right)^3. \tag{4}$$

The energy spread contribution to the gain is given by the term,

$$\overline{S_1} = -2\sqrt{2\pi}\eta_\gamma^3\left[\int d\overline{\eta}\frac{\overline{\eta}\,\exp\left(-\overline{\eta}^2/2\eta_\gamma^2\right)}{\overline{\delta k} - \overline{\theta} + 2\overline{\eta}}\right]^{-1}, \tag{5}$$

where $\overline{\theta} = 2L_{1D}\theta_0$, $\alpha^2 = w^2/4\sigma_x^2$ is the complex spot size parameter, and where $\overline{\delta k} = 2L_{1D}\delta k = 2L_{1D}(\delta k_r - i\delta k_i)$ is the scaled complex wave number associated with the FEL process. The gain length of the three-dimensional mode L_g can be expressed as

$$\overline{\delta k_i} = \frac{L_{1D}}{L_g} = \frac{1}{1+\Lambda_{0,0}}. \tag{6}$$

Here, $\overline{\delta k_i}$ is maximized at the optimal detuning to yield the shortest possible gain length and $\Lambda_{0,0}$ is expressed as a power fitting formula that is a function of η_d, η_γ and $\overline{\theta}_p$.

Power Fit Formula

The fit formula was obtained from numerical solutions to the variational equations [1] and is found to be

$$\Lambda_{0,0} = 0.450\eta_d^{0.570} + 3.00\eta_\gamma^2 + 0.196\overline{\theta}_p^{1.91} + 51.0\eta_d^{0.950}\eta_\gamma^3$$
$$+ 0.0988\eta_d^{0.230}\overline{\theta}_p^{1.21} + 0.0375\eta_\gamma^{0.875}\overline{\theta}_p^{12.7}$$
$$+ 2.35\eta_d^{11.9}\eta_\gamma^{14.9}\overline{\theta}_p^{11.4} \tag{7}$$

EVALUATION OF HIGH AVERAGE POWER FEL DESIGN

We use the representative parameters for a megawatt class FEL amplifier based on the VISA undulator similar to those given in [7] (see Table 1).

Table 1: MW class FEL parameters

Parameter	Symbol	Value
e-beam energy	E	65 MeV
normalized emittance	ϵ_n	2.0 mm-mrad
current	I	600 A
rms uncorrelated energy spread	σ_γ	5×10^{-4}
rms e-beam size	σ_x	$100\mu m$
undulator period	λ_u	1.8 cm
undulator parameter	K	1.26
radiation wavelength	λ	$1\mu m$

Using the parameters listed above we find $\eta_d = 0.641$, $\eta_\gamma = 0.028$, and $\overline{\theta}_p = 0.400$. Inserting these parameters into the analytic formula (7), we find that the effects of diffraction, energy spread and longitudinal space-charge collude to extend the gain length from $L_{1D} = 0.081m \to L_g = 0.114m$.

The 3D FEL code Genesis 1.3 was also used to evaluate the performance of the high average power FEL. Figure 1 shows the evolution of the FEL power along the undulator. The power gain length obtained through simulation is $L_g = 0.115m$. The results from the fitting formula and simulation for this specific case are in agreement to within 0.9%.

Transverse Mode Profile

In addition to establishing the fit formula for the power gain length, the analytic treatment also allows for the calculation of the fundamental mode intensity profile. By virtue of the appearance of the complex spot size parameter α, the mode profile can be extracted along with the gain length from the numerical solutions to the variational equations (4). For the fundamental mode, the field profile depends on α as $E \propto \text{Exp}[-r^2/4\alpha\sigma_x^2]$. The results for the mode profile from the analytic treatment and simulation for the high

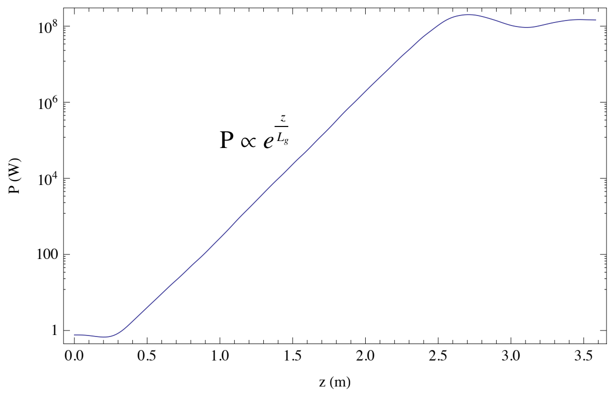

Figure 1: Power evolution along the undulator for the high average power FEL.

average power FEL are compared in Figure 2 and show excellent agreement.

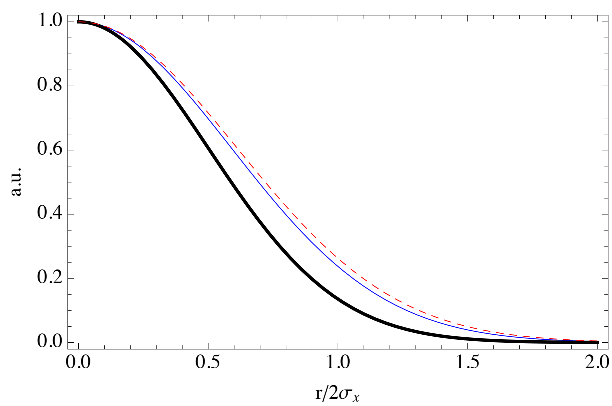

Figure 2: Comparison of the transverse normalized intensity mode profile between simulation and theory for the high average power FEL. The profile from Genesis is represented as the dashed red line while the profile from the theory is represented as the solid blue line. The e-beam is shown in black.

Beam Quality Effects

The fit formula (7) can also be used for optimization purposes in a multidimensional parameter space. Figure 3 shows the effects of e-beam current and uncorrelated energy spread on the gain length of a beam with the parameters listed in table 1 using equation (7). Notice the increase in performance (smaller gain length) as the beam quality increases (larger current and smaller energy spread) and vice versa. The gain length dramatically increases as the energy spread approaches the upper limit $\sigma_\gamma \sim \rho$.

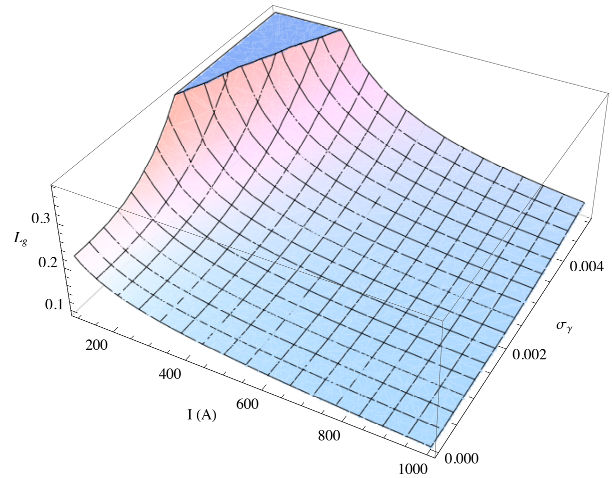

Figure 3: L_g vs. I and σ_γ for the high average power FEL.

CONCLUSION

A handy fit function that can be used to quickly calculate the gain length of the fundamental three-dimensional optical mode of a FEL in the presence of uncorrelated energy spread, diffraction and space-charge was used to quickly evaluate the performance of an FEL amplifier considered for use as a high average power light source. Results have been compared to detailed numerical particle simulations using the three-dimensional code Genesis and show excellent agreement.

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