SHIMMING OF THE DYNAMIC FIELD INTEGRALS OF THE BESSY II U125 HYBRID UNDULATOR

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Abstract

Within a continuous program the BESSY II undulators are prepared for Topping-Up operation. The U125 planar hybrid undulator has a period length of 125 mm and a pole width of only 60 mm. The horizontal defocusing of the 1.7 GeV e-beam may result in a significant reduction of the horizontal dynamic aperture, reducing the injection efficiency when injecting into the closed gap. The dynamic field integrals are derived from a 2D-Fourier decomposition of the 3D-field. An analytic description of the dynamic field integrals based on the Fourier coefficients is presented. Magic fingers have been installed in order to minimize the dynamic field integrals and to enlarge the good field region of the device.

INTRODUCTION

The three dimensional magnetic fields of undulators operated in low or medium energy storage rings produce so-called dynamic kicks which may have a significant impact on the electron beam dynamic. Though the straight line integrals are small the integrals along the wiggling trajectory can be large, in particular for APPLE II type undulators [1] or high field, long period wigglers [2]. The effects scale inversely with the square of the electron energy. Careful tracking studies are required and a reduction of the dynamic effects is an important issue for top-up operation (injection into closed gaps).

Recently, a particle tracking scheme which is based on an iterative solution of a Taylor series expanded Hamilton-Jacobi-equation has been published [3]. This method yields a direct transformation of the particle coordinate variables from a generating function, leading to a symplectic variable transformation. Analytic fields are needed for this tracking algorithm. In [3] field expressions for APPLE II undulators are given. Furthermore, analytic expressions for the dynamic effects of APPLE II type undulators operating in arbitrary modes of polarization are derived.

It is worth noting that the formalism can be applied also to other 3D-magnetic fields which are not periodic in the direction of electron beam propagation.

In this paper we apply the formalism to the wiggler U125-2 which is installed in the storage ring BESSY II. The 4m- undulator has a period length of 125mm and a pole width of only 60mm, including two 4mm chamfers in transverse direction. The pole shape causes dynamic field integrals of up to 4.5Tmm. Permanent magnet shims have been installed which reduce the dynamic field integrals within a transverse region of ±18mm from 0.58Tmm down to 0.02Tmm.

AN ORTHOGONAL BASIS FOR PERIODIC FUNCTIONS OF THE MAXWELLIAN TYPE

The symplectic tracking algorithm of [3] requires an analytic magnetic field description. We are looking for a complete set of functions describing arbitrary 3-dimensional undulator, wavelength shifter or other accelerator magnet fields. Once, a single component is known the orthogonal components are derived with Maxwell's equations. In a first step we search for an expression of the vertical component $B_y(x,z)$ in the accelerator midplane (*x*-*z*-plane). *z* is the propagation direction of the electron beam.

We start with the functional properties in the midplane. It can be shown that a set of functions $F_{ik} = \varphi_i(x)\psi_{ik}(z)$ forms a basis for all continuous functions *f(x,z)* on the interval [a,b]x[c,d] if $\varphi_i(x)$ is a basis on the interval [a,b] and if for each *i* $\psi_{ik}(z)$ is a basis on [c,d]. In particular, $\tilde{F}_{ik} = \varphi_i(x)\varphi_k(z)$ is a basis for all continuous functions on [a,b]x[a,b] if $\varphi_i(x)$ is a basis on [a,b].

The trigonometric functions *sin(nx)* and *cos(mx)*, $n,m = 0 \ldots \infty$ form a basis on the interval $[-\pi,+\pi]$ for all continuous functions *g(x)* with $g(\pi) = g(-\pi)$ as can be concluded from the Weierstrass approximation theorem. Thus, the functions $cos(k_{xn}x)cos(k_{zm}z)$, $sin(k_{xn}x)cos(k_{zm}z)$, $cos(k_{xn}x)sin(k_{zm}z)$, $sin(k_{xn}x)sin(k_{zm}z)$ form a basis on the interval $S=[-\lambda_{x0}/2,\ \lambda_{x0}/2]\mathrm{x}[-\lambda_{z0}/2,\ \lambda_{z0}/2\}$ and the general expression for B_y is:

$$\begin{aligned}B_y(x,z) = \sum_{i=0}^{n}\sum_{j=0}^{m}(&cc_{i,j}\cos(k_{xi}x)\cos(k_{zj}z) + sc_{i,j}\sin(k_{xi}x)\cos(k_{zj}z) + \\ &cs_{i,j}\cos(k_{xi}x)\sin(k_{zj}z) + ss_{i,j}\sin(k_{xi}x)\sin(k_{zj}z))\\ k_{zj} &= jk_{z1} = j2\pi/\lambda_{z0} \\ k_{xi} &= ik_{x1} = i2\pi/\lambda_{x0}\end{aligned} \tag{1}$$

where λ_{x0} and λ_{z0} are the ranges for the transverse and longitudinal Fourier decomposition of the field. In the general case, λ_{x0} and λ_{z0} are chosen such that the field is zero at the boundaries of the interval *S*. Concentrating on this area we can ignore the finite field values outside which are due to the translational field symmetry in *x* and *z*. In an undulator or wiggler structure with many periods the periodicity within the device can be included to simplify the expressions (neglecting endpole effects): Now, we have $B_y(z_0)=B_y(z_0,z_0+\lambda_0)$ where λ_0 is the undulator period. Without loosing generality we assume $B_y(z)=B_y(-z)$ and we get the simplified fields:

$$B_y(x,z)=\sum_{i=0}^{n}\sum_{j=1}^{m}(c_{i,\tilde{j}}\cos(k_{xi}x)\cos(k_{\tilde{j}}z)+s_{i,\tilde{j}}\sin(k_{xi}x)\cos(k_{\tilde{j}}z))$$
$$\tilde{j}=1+l(1-j) \tag{2}$$
$$k_{\tilde{j}}=\tilde{j}k_1=\tilde{j}2\pi/\lambda_0$$

l is the number of magnets per period (four for a conventional pure permanent magnet undulator). Using the magnetic field properties of zero rotation and zero divergence the ansatz $\hat{B}_{y,ij}(x,y,z)=B_{y,ij}(x,z)\cdot H_{ij}(y)$ yields the differential equation: $(k_{xi}^2+k_{\tilde{j}}^2)H=\partial^2 H_{ij}/\partial y^2$. which is solved by the general expression:

$$H_{ij}=c_1\exp(-k_{yi,\tilde{j}}y)+c_2\exp(+k_{yi,\tilde{j}}y) \tag{3}$$

Undulators which are lacking a midplane symmetry (e.g. HELIOS at the ERSRF [4]) can be described in this way. In case of a midplane symmetry, $\hat{B}_y(y)=\hat{B}_y(-y)$, we get $H_{ij}=c\cdot\cosh(k_{yi,\tilde{j}}y)$ which leads to the well-known Halbach expression. Equation (4) describes all Maxwellian functions which have midplane symmetry, translational symmetry in x- and mirror symmetry in z-direction with arbitrary accuracy.

$$\hat{B}_y(x,y,z)=\sum_{i=0}^{n}\sum_{j=1}^{m}(c_{i,\tilde{j}}\cos(k_{xi}x)+s_{i,\tilde{j}}\sin(k_{xi}x))\times \tag{4}$$
$$\cosh(k_{yi,\tilde{j}}y)\cos(k_{\tilde{j}}z)$$
$$k_{yi,\tilde{j}}=\sqrt{k_{\tilde{j}}^2+k_{xi}^2}$$

The sin-terms are required only if the fields are not symmetric with respect to the z-y-plane. This is the case for wigglers with transversely displaced poles or APLPE II undulators in the inclined mode. The other field components are derived from B_y using Maxwell's equations.

GENERAL EXPRESSIONS OF DYNAMIC FIELD INTEGRALS

In the following we concentrate on fields of the form of Eq. (4) though the arguments can be extrapolated to the more general form of Eq. (1) as well. A vector potential function (A_x, A_y, A_z=0) is derived from Eq (4):

$$A_x(x,y,z)=\frac{1}{B\rho}\int B_y dz \tag{5}$$
$$=\frac{1}{B\rho}\sum_{i=0}^{n}\sum_{j=1}^{m}(c_{i,\tilde{j}}\cos(k_{xi}x)+s_{i,\tilde{j}}\sin(k_{xi}x))$$
$$\times\frac{1}{k_{\tilde{j}}}\cosh(k_{yi,\tilde{j}}y)\sin(k_{\tilde{j}}z)$$
$$A_y(x,y,z)=-\frac{1}{B\rho}\int B_x dz$$
$$=\frac{1}{B\rho}\sum_{i=0}^{n}\sum_{j=1}^{m}(c_{i,\tilde{j}}\sin(k_{xi}x)-s_{i,\tilde{j}}\cos(k_{xi}x))$$
$$\times(k_{xi}/(k_{yi,\tilde{j}}k_{\tilde{j}}))\sinh(k_{yi,\tilde{j}}y)\sin(k_{\tilde{j}}z)$$

This vector potential is used in a symplectic, precise and fast tracking algorithm as described in [3]. The vector potential of Eq. (5) defines a potential function [3], $f_{002}=-\int(A_x^2+A_y^2)dz/2$, which is used to derive explicit expressions for the transverse kicks of particles, passing a section z_f of the undulator parallel to the central axis. The kicks are given as $\theta_x=\partial f_{002}/\partial x$ and $\theta_y=\partial f_{002}/\partial y$. For the BESSY II U-125-2 undulator the B_y fields are symmetric in x and we drop the sin-terms in x. Averaging over an integer number of periods we get the transverse kicks:

$$\theta_x=-\frac{z_f}{2(B\rho)^2}\sum_{i1=0}^{n}\sum_{i2=0}^{n}\sum_{j=1}^{m}c_{i1,\tilde{j}}c_{i2,\tilde{j}}\frac{k_{xi1}}{k_{\tilde{j}}^2}\sin(k_{xi1}x)\cos(k_{xi2}x)$$
$$\times(\frac{1}{k_{yi1,\tilde{j}}}\frac{k_{xi2}^2}{k_{yi2,\tilde{j}}}\sinh(k_{i1,\tilde{j}}y)\sinh(k_{i2,\tilde{j}}y)-\cosh(k_{i1,\tilde{j}}y)\cosh(k_{i2,\tilde{j}}y))$$
$$\theta_y=-\frac{z_f}{2(B\rho)^2}\sum_{i1=0}^{n}\sum_{i2=0}^{n}\sum_{j=1}^{m}c_{i1,\tilde{j}}c_{i2,\tilde{j}}\frac{1}{k_{\tilde{j}}^2}\cosh(k_{yi1,\tilde{j}}y)\sinh(k_{yi2,\tilde{j}}y) \tag{6}$$
$$\times(k_{yi2,\tilde{j}}\cos(k_{xi1}x)\cos(k_{xi2}x)+k_{xi1}\frac{k_{xi2}}{k_{yi2,\tilde{j}}}\sin(k_{xi1}x)\sin(k_{xi2}x))$$

The kicks of Eq. (6) are useful for the dimensioning of compensating shims and for the estimation of tune shifts. The dynamic kicks are sometimes evaluated from the similar but generally incorrect formula:

$$\tilde{\theta}_{x/y}=-\frac{1}{(B\rho)^2}\int_0^{z_f}\left\{\int_0^z B_y dz\int_0^z\frac{\partial B_y}{\partial x/y}dz'+\int_0^z B_x dz\int_0^z\frac{\partial B_x}{\partial x/y}dz'\right\}dz \tag{7}$$

For illustration we apply a simple planar field expansion with an arbitrary, longitudinal phase term ϕ, as derived from a scalar potential $V=-\bar{V}(x,y)\cos(k_z z+\phi)$ where $\bar{V}$ is a function of the type $B_0\cos(k_x x)\sinh(k_y y)$. Derivatives of $\bar{V}$ with respect to x or y are indicated by the index. Integration over an integer longitudinal period z_f yields,

$$\tilde{\theta}_x=-\frac{z_f}{2(B\rho)^2k_z^2}(\bar{V}_y\bar{V}_{yx}+\bar{V}_x\bar{V}_{xx})(1+2\sin^2(\phi)) \tag{9}$$

and similar for $\tilde{\theta}_y$. The achieved integrated kick per period is dependent on the arbitrary phase value ϕ, introduced by the specified integration limits. The transverse focussing, however, should be independent on the phase ϕ. For a planar undulator the validity of Eq. (7) is limited to the specific case of $\phi=0$ where it delivers the same result as Eq. (6). In contrast, the potential function f_{002} will result in phase independent θ_x- and θ_y-kicks without any constraints.

THE U125-2 WIGGLER

In the following we consider the dynamic field integrals of the BESSY II U125-2 wiggler. We use Eq. (4) for the fields neglecting the *sin*-terms. The Fourier coefficients are determined from RADIA [5] simulations. The transverse profile of the vertical field is evaluated at m z-positions. For each profile a Fourier decomposition is performed yielding the quantities $C_{i,j}$. The Fourier coefficients $c_{i,j}$. which are related to the harmonics in longitudinal direction are derived via solving a set of linear equations (Eq. (10)). In order to minimize

$$C_{0,1} = \sum_{j=1}^{m} c_{0,\tilde{j}} \tag{10}$$
$$\ldots$$
$$C_{n,1} = \sum_{j=1}^{m} c_{n,\tilde{j}}$$
$$\ldots$$
$$C_{n,m} = \sum_{j=1}^{m} c_{n,\tilde{j}} \cos(k_{\tilde{j}} z_m)$$

numerical noise the z-positions of the transverse profiles have been chosen such that the harmonics j have their maxima at the position z_j. The analysis of the BESSY II U125-2 transverse field profiles shows a strong 3rd harmonic (Fig. 1).

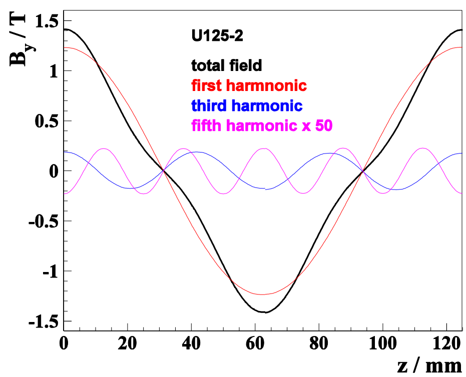

Figure 1: Total B_y and individual field harmonics of the BESSY II U125-2 plotted versus the z-direction.

Though the 3rd harmonic amounts to 14% of the 1st harmonic the contribution to the dynamic kicks is only 0.48% because it is scales with the square of the field and with the factor $1/k_{\tilde{j}}^2$. Off-axis the contributions of the higher harmonics increase faster as compared to the 1st harmonic but they are still small (1.0% for particles 5mm out of midplane). Nevertheless, the higher field harmonics have to be evaluated in order to derive the 1st harmonic (Fig. 2).

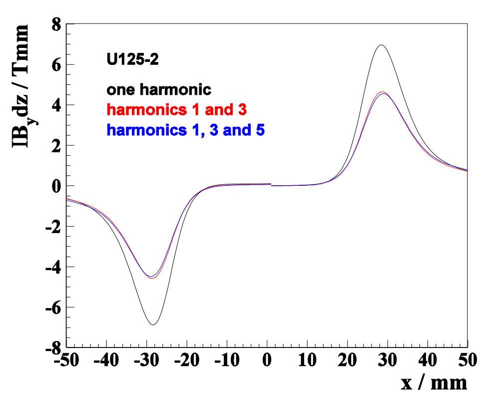

Figure 2: Dynamic field integrals of the U125-2 installed at BESSY II at smallest gap of 15.7mm. The dynamic field integrals have been determined from m=1, 2 and 3 transverse profiles yielding the harmonics 1, 1 and 3 and 1, 3 and 5 (Eq. (6)). The 3rd harmonic contributes only little, however, it must be determined from Eq. (6) with m=2 for a correct evaluation of the 1st harmonic.

The U125-2 is a quasiperiodic device [6] which reduces the integrated dynamic effects. The values in Figure 2 include this effect. The kicks of the dynamic field integrals in Figure 2 have been compensated in the midplane (Figure 3) using an array of permanent magnets with cross sections of 4x4 mm^2 and variable thickness in longitudinal direction (so-called magic fingers). Due to space limitations the magic fingers are installed only at the downstream end (Figure 4). The block at the right hand side is used for a coarse compensation. The length of all magnets is 4mm. The smaller array on the left is a standard BESSY II magic finger housing magnets with thickness variations of 0.1mm for fine tuning.

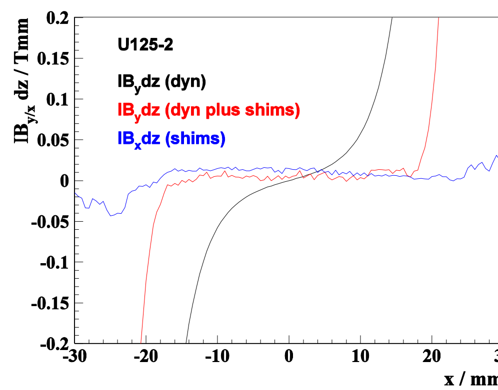

Figure 3: Dynamic vertical field integrals (dyn) before shimming (black) and with shims (red). The horizontal field integrals due to the shims are indicated in blue.

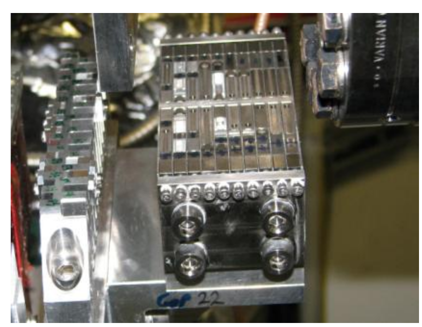

Figure 4: Permanent magnets for the compensation of dynamic field integrals at the downstream end of the BESSY II U125-2 (for details see text).

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