SPECTRAL ANALYSIS WITH AN ARBITRARY STRENGTH PARAMETER FOR VARIOUS INSERTION DEVICES

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Abstract

An insertion device (ID) with medium strength parameter (K value) is difficult to define as a wiggler or undulator. Once the kind of ID was defined, the formula for calculating spectrum was selected. The formulae for wiggler or undulator differ appreciably, consequently implying a large difference of flux density corresponding to the same strength parameter. It is impossible that the spectrum calculated in both ways is correct. A universal formula should thus be developed for the spectral analysis of insertion devices of various kinds that have disparate strength parameters. A modified formula to calculate the spectrum of an ID was hence derived on reviewing the various existing spectrum formulae. The familiar formula to calculate an undulator spectrum was modified so that it can be used for an ID of arbitrary strength parameter. An algorithm for the formula modification is described. Some related issues, including the effect of phase error and the energy spread, and a tapered undulator are also discussed herein.

INTRODUCTION

Researchers in the field of insertion devices employ a basic logic for a design of an ID for a specified spectral range: a smaller strength parameter is typically preferable because of the interference characteristics, whereas a large strength parameter is applied for the generation of high energy photons [1]. One aspect is always perplexing: if the strength parameter be neither large nor small, should the ID be treated as a wiggler or an undulator? Which formula should we use to predict the spectrum? Here we propose a method to solve this problem.

A spectrum formula that was originally applicable for only an undulator was modified so as to be useful for the evaluation of the spectrum of an ID of arbitrary strength parameter. This formula can serve for a wiggler (ID with large *K* value) and the result is exactly the same as for the spectrum calculated by the formula for a wiggler, but the calculation is protracted for a case of a large strength parameter. The modified spectrum formula can calculate the spectrum precisely for an ID of medium strength parameter, when we might be unable to distinguish the type of ID – wiggler or undulator. The algorithm of this modification is described clearly in this paper, and we discuss some concepts about the characteristics of a large strength parameter, the properties of high harmonics and constructive interference.

ALGORITHM OF THE MODIFICATION

The formula for the density of photon flux from an undulator is [1]

$$\frac{dN_{ph}(\omega)}{d\Omega} = \frac{d^2W(\omega)}{d\omega\, d\Omega} \times \frac{1}{\hbar\omega} \times \Delta\omega \qquad (1)$$

In which

$$\frac{d^2W(\omega)}{d\omega\, d\Omega} = \text{Const}\ \hbar\, N_p^2 \sum_{i=1}^{\infty} i^2 \text{SINC}^2(N_p, \omega, \omega_i) \times \text{Spatial}(K, i) \qquad (2)$$

Some simplifications are made; the SINC-function term indicates the line shape of harmonics, and the spatial function describes the spatial distribution also with the harmonic line intensity varying with frequency. The spectrum formula (Eq. 1) of an undulator is accepted widely to be applicable for only a small strength parameter and is typically used for harmonics 1 – 13. This formula can become valid for higher harmonics if some modification be made as follows.

Modifications of the calculus concept

The first modification is for the bandwidth; the value of $\frac{\Delta\omega}{\omega}$ is generally set as 0.1%, which indicates the width of a window for a collection of photons during measurements. In the case of $\frac{\Delta\omega}{\omega} = 0.1\ \%$, the photons of frequency from $(1-0.05\ \%)\omega$ to $(1+0.05\ \%)\omega$ are all collected. For the condition of a line width, $\frac{\delta\omega}{\omega}$, of a harmonic much larger than $\frac{\Delta\omega}{\omega} = 0.1\ \%$, there is no problem in substituting constant 0.1% for $\frac{\Delta\omega}{\omega}$, but when this condition fails, Eq. (1) should be modified to

$$\frac{dN_{ph}(\omega)}{d\Omega} = \int_{\omega'=\omega-0.5\Delta\omega}^{\omega+0.5\Delta\omega} \frac{d^2W(\omega')}{d\omega\, d\Omega} \times \frac{1}{\hbar\omega} \times d\omega' \qquad (3)$$

which retains the concept of collecting photons inside the bandwidth; this formula yields a more precise result. This modification increases the accuracy of the spectrum formula for an undulator in the high harmonic range, for which the line width is equal to, or smaller than, $\frac{\Delta\omega}{\omega} = 0.1$ %. Substituting Eq. (2) into Eq. (3), we obtain the following formula with some adjustment and approximation:

$$\frac{dN_{\mathrm{ph}}(\omega_i)}{d\Omega}\Big|_i = \mathrm{Const}\times \mathrm{Spatial}(K,i) N_p^2 i^2 \times \underline{\int_{\omega'=\omega_i-0.5\Delta\omega}^{\omega_i+0.5\Delta\omega} \sum_{i'=1}^{\infty} \mathrm{SINC}^2(N_p,\omega',\omega_{i'}) \times \frac{d\omega'}{\omega_i}} \quad (4)$$

in which the underlined term is called the *normalized photon number* (NPN). Figure 1 shows the values of NPN (in unit 0.1%) for various harmonics. For a special case of the observation frequency equal to the harmonic frequency and the harmonic line width much larger than $\frac{\Delta\omega}{\omega} = 0.1\%$, NPN is equal to $\frac{\Delta\omega}{\omega} = 0.1\%$, but in the ordinary spectrum formula of an undulator (cf. Eq. (1)) all harmonics are treated as a special case, which means that NPN is set to a constant $\frac{\Delta\omega}{\omega}$ =0.1 %.

Integration by energy spread effect

The abrupt variations in Figure 1(b) are not coincident with our common sense, but this effect is eliminated on adding the effect of the energy spread of an electron bunch. This effect is taken into account on introducing a gaussian distribution into the electron energy of standard deviation $\frac{\delta E}{E}\frac{1}{2}$; $\frac{\delta E}{E}$ is the amount of energy spread (e.g.0.1%), which causes the photon frequency, $\frac{\delta' \omega}{\omega} = 2 * \frac{\delta E}{E}\frac{1}{2}$, of a bunch to vary. The plot of the formula taking into account the energy spread is shown in red in figure 1(a, b).The abrupt changes at harmonic numbers 4001, 8001 and 12001 vanish because the effect of the energy spread broadens the sharp harmonic line.

THE DIFFERENCE OF THE SPECTRUM BETWEEN WIGGLER AND UNDULATOR

The value of NPN_s (in unit 0.1%) becomes constant at $1/2N_p$ (0.016667 for N_p=30) in the high harmonic range; this ratio $1/2N_p$ distinguishes between the spectrum calculated with the ordinary formulae for a wiggler and for an undulator. If interference phenomena for the frequency, which forms sharp harmonic lines, are totally eliminated, the photons of a particular sharp harmonic line can be supposed to become distributed into a range $2\omega_1$. The maximum intensity is decreased to $1/2N_p$.When NPN (in unit 0.1%) decreases to $1/2N_p$, the interference phenomena of the frequency are no longer evident. Under a 0.1% energy spread, the difference between $1/2N_p$ and NPN_s (in unit 0.1%) is less than 1% for a harmonic number greater than 1001.

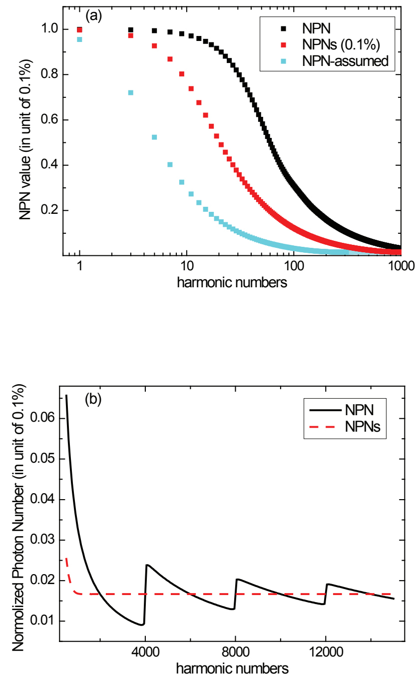

Figure 1. Variation of NPN with harmonic number.(a) The black points denote NPN; the red points (NPN) denote NPN with 0.1% energy spread; the cyan points denote NPN with a serious destructive interference factors. (b) The black line denotes NPN; there are abrupt changes at harmonic numbers 4001, 8001 and 12001 because the flux density of the two adjacent extra sharp harmonic lines are included in Δω/ ω. For the red line with 0.1% energy spread, those abrupt changes vanish.

Flux density calculations of three kinds were compared: the ordinary formula Eq. (1) for an undulator (ordinary-U), the formula for an undulator with a modified bandwidth and an energy-spread effect (mod-U), and the formula for a wiggler (formula-W). An insertion device of period length 48 mm, *K*= 5.38 serves for an example, and three ranges of photon energy are drawn. For the range of the small harmonic number, Figure 2(a), mod-U is near ordinary-U; for the range of large harmonic number, figure 2(b), mod-U overlaps with formula-W; for the intermediate harmonic range, shown in figure 2(c), mod-U varies from ordinary-U to formula-W. Mod-U is useful for the case of calculating a photon energy range with a medium harmonic number, and helps us to recognize that we can feel relieved to use formula-W to calculate the spectrum for a harmonic number greater than about 1001, and reminds us to be careful about the accuracy of the spectrum calculation for a harmonic number greater than 7.

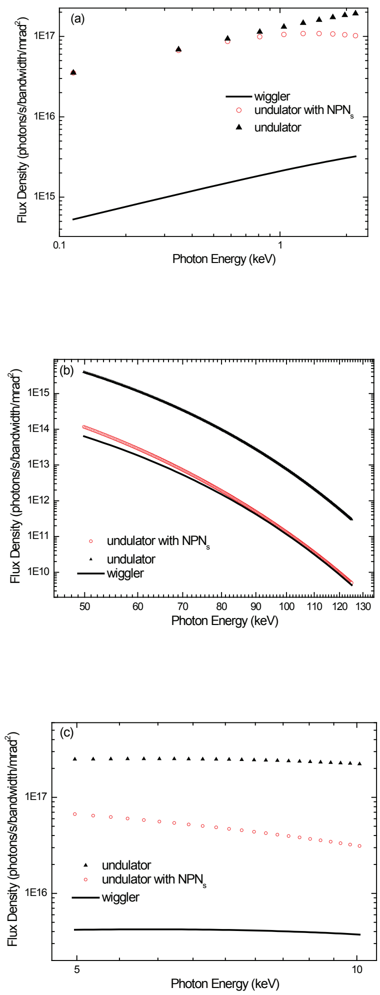

Figure 2 Three spectra with varied harmonic range and the same parameters. Harmonics (a) 1~19. (b) 431~1081. (c) 21~43.

ENERGY SPREAD EFFECT AND OTHER DE-INTERFERENCE EFFECT ON HIGH HARMONICS

The full interference phenomena in spectra appear only when a single electron passes through a perfectly sinusoidal magnetic wave. In a practical case, a non-ideal electron bunch and an imperfect magnetic field are both the cause of destructive interference. Here we consider only the effect of the energy spread; in the future, all known effects that can weaken the interference will be taken into account, such as finite emittance, the phase error of the ID, the tapper undulator, and the deformation of the magnetic field for a large *K*-value. In Figure 1(a), the NPN value and the NPN_s value from harmonic 1 to 1001 were calculated; one situation with a strong destructive effect on interference was assumed and its NPN values (called NPN-assumed) were simulated. The NPN-assumed values decrease to $1/2N_p$ about harmonic 200. If we seek the spectrum performance with a harmonic number greater than that value, we can apply the formula for a wiggler; if we seek the spectrum with a harmonic number less than that value, the modified undulator formula should be used.

The modified calculation is needed for the case of a sharp harmonic line. If the constructive effect of the frequency is seriously destroyed, the line width increases and the modification can be avoided.

THE ISSUE OF DEFINING THE TYPE OF ID WITH A STRENGTH PARAMETER

How to specify the type of ID is always a problem for an ID developer. A few years ago, we claimed that wigglers and undulators were classified according to *K* with a separation by unit. Eventually, we made undulators with a greater field, and now we classify ID with *K* with a separation by ten. A precise way to specify the type of ID is needed. A convention should be accepted that, if the constructive interference effect on photon frequency is obvious, the device is an undulator, whereas if the constructive interference effect is totally absent, the device is a wiggler. The NPN-all value (NPN-all means NPN value calculated on considering all relevant effects) specifies the precise extent of the interference effect. If the harmonic range with NPN-all (in unit 0.1%) value equal to $1/2N_p$is focused, the ID is definitely a wiggler; if smaller harmonics for which the NPN-all value exceeds $1/2N_p$are considered, the ID can be called an undulator. Note that IDs are now classified by harmonic number associated with all effects other than the *K*-value.

CONCLUSION

The type of ID should be specified with a harmonic number and with the de-interference effect considered, not specified with the strength parameter. The *normalized photon number* (NPN) can provide a specific distinction between there cognition of a wiggler and an undulator, but the modified calculation concept is less effective when the harmonic line width greatly increases.

REFERENCE

[1] S. D. Chen, C. S. Hwang, and K. S. Liang, “An Algorithm for the Optimization of Insertion Devices to Increase Useful Flux and to Decrease Useless Radiation Power.” AIP Conference Proceedings (2010)

[2] H. Wiedemann, “Particle Accelerator Physics”