

LONGITUDINAL BEAM DYNAMICS OF A LASER SLICED BUNCH *

P. Kuske, Helmholtz-Zentrum-Berlin, BESSY, Berlin, Germany

Abstract

Slicing of a bunch of electrons with a fs-laser is quite common in storage rings for creating short VUV- and soft X-ray light pulses or pulses of coherently emitted THz-radiation. In this paper the longitudinal dynamics of the sliced bunch is studied numerically. The calculations are based on the one dimensional solution of the Vlasov-Fokker-Planck-equation (VFP) assuming that the shielded longitudinal CSR-wake is dominating the dynamics of the 100 fs-long slice. It is found, that the density modulation survives longer and that the CSR-spectra extend to higher frequencies at later turns even below the corresponding instability threshold. This simple model seems to support experimental observations at BESSY and the Swiss Light Source.

INTRODUCTION

Laser slicing [1] is used routinely for producing 100-200 fs short VUV and soft X-ray light pulses [2] or coherent THz-radiation [3] emitted by the density modulated bunch of electrons. This radiation can be observed over many turns and dies out due to diffusion introduced by the incoherent synchrotron radiation, filamentation in the longitudinal plane, and coupling with the transverse planes [4].

Laser bunch slicing works as follows: a short and intense (typically 50 fs long and 1-2 mJ) laser pulse interacts resonantly with the co-propagating bunch inside of an appropriately tuned undulator. This leads to a ~1% energy modulation of the electrons in temporal overlap with the laser pulse. In this slice as many electrons loose as gain energy. Dispersion quickly turns the energy modulation into a longitudinal density modulation. In this paper the impact of coherent synchrotron radiation (CSR) on the bunch is studied. A similar study has been performed earlier [5] using an analytical coasting beam approach. In this paper a numerical analysis of a more realistic bunch of electrons is presented. In both cases a simplified and purely longitudinal wake for the CSR is used which ignores the transverse beam dynamics.

MODEL

All calculations have been performed for the BESSY II electron storage ring (Table 1). Typically 10^9 electrons are contained within one bunch. The ensemble of electrons is described by a distribution function, $\psi(q,p,\tau)$, in terms of normalized phase-space coordinates: $q=z/\sigma_z$, $p=(E_0-E)/\sigma_E$, and τ in units of the synchrotron period, ω_s . Low-current rms bunch length, ω_s , and natural energy spread, σ_E , are related by: $\omega_s\sigma_z/c=\alpha\sigma_E/E_0$. The Vlasov-Fokker-Planck (VFP) equation describes the evolution of the distribution [6] which depends on the radiation excitation

and damping and the interaction of the particles with themselves [7]. The interaction with the radiation has two contributions. The wake for the free space is used in its approximated form [7, 8] which contains a singularity. In order to ease the calculation of the induced voltage one can perform two integrations by parts by using the correct form of the wake. Now the numerical challenge is shifted to the accurate determination of the second derivative of the longitudinal density function $\lambda(q,\tau)=\int dp\psi(q,p,\tau)$. The wake for the free space acts only in the forward direction. The second contribution is given by the image charges in the parallel plates representing the vacuum chamber. The corresponding wake acts in the forward and backward direction and is a smooth function not causing any numerical problems for the calculation of the induced voltage.

Table 1: BESSY II-Parameters

Parameter	Value
energy, E_0	1.7 GeV
Bending radius, ρ	4.35 m
momentum compaction, α	$7.3 \cdot 10^{-4}$
cavity voltage, V_{rf}	1.4 MV
accelerating frequency, ω_{rf}	$2\pi \cdot 500$ MHz
revolution time, T_0	800 ns
natural energy spread, σ_E	$7.0 \cdot 10^{-4}$
zero current bunch length, $\sigma_z=c\cdot\sigma_0$	$c \cdot 10.53$ ps
longitudinal damping time, τ_l	8.0 ms
synchrotron frequency, ω_s	$2\pi \cdot 7.7$ kHz
height of the dipole chamber, $2h$	3.5 cm

Numerical Solution of the VFP-Equation

The proposed algorithm for finding the solution for the distribution function on a grid [6,9] can lead to unphysical negative values for $\psi(q,p,\tau)$. Therefore, a kind of wave function, $g(q,p,\tau)$, is introduced so that the probability to find electrons is given by $\psi(q,p,\tau)=g^2(q,p,\tau)$. In the VFP equation for $g(q,p,\tau)$ the Fokker-Planck-term needs to be modified:

$$\frac{\partial g}{\partial \tau} + p \frac{\partial g}{\partial q} - [q + I_c F] \frac{\partial g}{\partial p} = \frac{1}{\omega_s \tau_l} \left(\frac{g}{2} + p \frac{\partial g}{\partial p} + \frac{1}{g} \left(\frac{\partial g}{\partial p} \right)^2 + \frac{\partial^2 g}{\partial p^2} \right)$$

This equation is solved with an algorithm proposed by Venturini, et al. [9] but instead of the cubic, a fourth order Hermite interpolation for off-grid points is used. The interpolation is between the grid points $g(x-\Delta)$, $g(x)$, and $g(x+\Delta)$ and the slopes at these points and thus is more symmetric relative to the central point. x stands for the p or q coordinate and Δ is the distance between grid points. The diffusion term with g in the denominator is only included in the calculation for sufficiently large values of g , $|g| > 10^{-6}$. Usually a 128 by 128 mesh with a spacing of $\sigma/10$ is used. Stability of the algorithm requires a certain number of steps per synchrotron period in proportion to

*Work supported by the BMBF and the Land Berlin

the damping and excitation $\omega_s \tau_i$. For BESSY II, 512 to 1024 steps are taken and the distribution is followed over 200 to 400 synchrotron periods corresponding to up to 6 damping times. If the resolution is increased also the number of steps must go up. The code was compared successfully to the weak instability created by a purely resistive δ -function wake, $w(q) = R \cdot \delta(q)$ [10], the wake of a broad band resonator, and to the results obtained with a shielded CSR-wake [8].

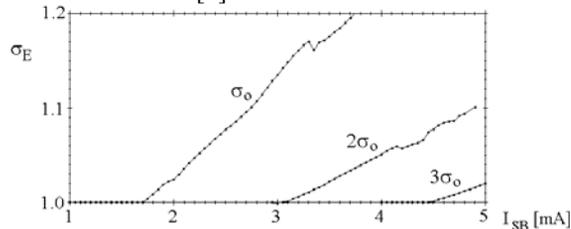


Figure 1: Current dependent normalized energy spread for different values for the bunch length. A harmonic cavity is used in order to double and triple the bunch length.

Instability Threshold

For the nominal BESSY II bunch length of 10.53 ps the CSR-driven μ -wave instability is calculated to set in at 1.72 mA. According to the approximate formula given in [8] one would have expected a slightly lower threshold current of 1.53 mA. The direct simulation with Bane's damping factor, $1/\omega_s \tau_i = 0.00125$, also leads to a threshold of 1.72 mA. Higher harmonic cavities for bunch lengthening are included in the code. Longer bunches have higher threshold currents. The calculated energy spread for these cases is presented in Fig. 1. Turbulent bunch lengthening associated with energy widening is clearly visible in the plot. The result is the same with the resolution doubled.

For the BESSY II storage ring a number of observations in the THz spectral region are available for a comparison [11] with theoretical predictions. The experimental observations with a fast InSb-Bolometer are modelled in the following way: After each iteration $g(q, p, \tau)$ is used to calculate the line density, $\lambda(q, \tau)$. The instantaneous spectral power, $P_{csr}(\omega, \tau)$, of detected coherent synchrotron radiation would be:

$$P_{csr}(\omega, \tau) \propto \left| \int_{-\infty}^{+\infty} \lambda(q, \tau) \cdot e^{-i\omega\sigma_0 q} dq \right|^2 \cdot \frac{\text{Re} Z_{shielded}(\omega)}{\text{Re} Z_{free_space}(\omega)} \cdot D(\omega)$$

The first term is the form factor followed by the impedance ratio of the real parts of shielded and free space impedance [13]. This takes into account the impact of shielding on the emission process, and $D(\omega)$ accounts for the frequency response of the detector. The output of the InSb-detector is proportional to the frequency integrated signal and usually observed with a spectrum analyser. For simplicity just the form factor is integrated beginning at $2\pi \cdot 100$ GHz which corresponds to the on-set of the emission process in case of the BESSY vacuum chamber. The detector response is assumed to be linear up to 1 THz where the form factor usually falls to zero

already. Finally the Fourier transform of $P_{csr}^{tot}(\tau)$ is calculated. The theoretical result of the current dependent Fourier transform for the nominal bunch length is presented in Fig. 2.

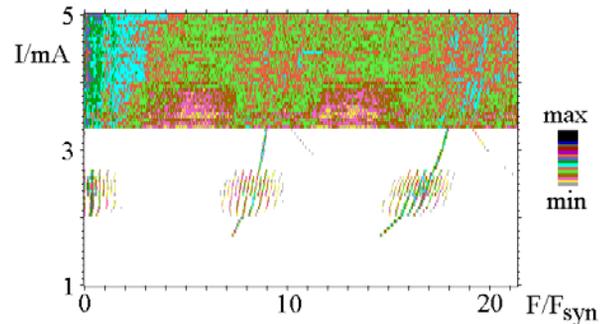


Figure 2: Simulated signal of a fast THz-detector.

The spectra are displayed in units of the unperturbed synchrotron frequency, $F_{syn} = 1/\tau$. The instability sets in with a clear fundamental spectral line and some harmonics. The lowest component at 7.3 times the unperturbed synchrotron frequency corresponding to quite a high azimuthal mode. What usually is associated with regular or random bursts of CSR [13] is nothing but the well known regular or irregular saw tooth-like region of the μ -wave instability at higher intensity [14]. The saw tooth instability leads to sidebands spaced by the repetition frequency and the repetition frequency component shows up directly and appears close to zero. These signals are usually seen and interpreted as the bursting instability threshold. On the way to chaos the bunch goes through different intensity dependent phases of the instability: the number of sidebands is doubled, all sidebands disappear and finally the chaotic regime is reached. Very similar pictures can be obtained with the simpler broad band resonator wakes and in all cases damping plays a significant role where the phase transitions occur. The agreement with the actual observations [11] is not yet perfect; however, this is the way to go.

SIMULATION OF A SLICED BUNCH

The solution of the VFP equation with a 100 fs rms laser sliced bunch requires a substantially larger mesh with much finer resolution. 800 q- and 25-p steps per σ and a mesh with 16384 by 1024 points was used. The initial line density is given by the Haissinski solution. Where the bunch overlaps with the laser pulse the gaussian momentum distribution is folded with the function:

$$f(q, p') = \frac{1}{\pi \sqrt{\Delta P(q)^2 - p'^2}}$$

$\Delta P(q)$ is the momentum transfer due to the laser interaction in units of the natural energy spread. This averages the interaction over the wavelength of the laser and takes into account the initial phase of the electrons with respect to the laser field.

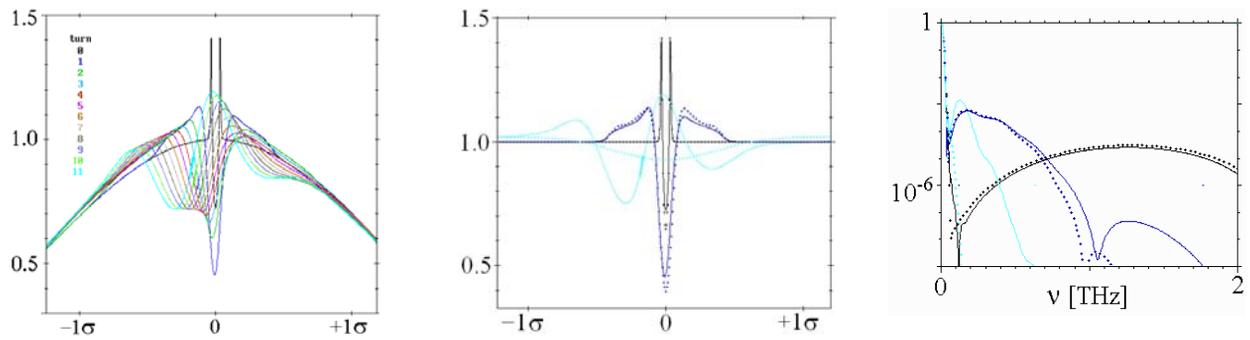


Figure 3: Left - evolution of the 200 fs laser slice over the first 11 turns. Centre - comparison of results with (lines) and without the CSR interaction (dots). Right - normalized form factors of the distributions in the centre.

The initial particle distribution is followed over 11 turns which is 7.4 % of a synchrotron period. At BESSY II the THz-port is located two dipoles downstream of the laser interaction zone. At this port the sliced line density from a 200 fs long laser pulse on the first few turns according to this calculation for the nominal bunch with 3 mA is displayed in Fig. 3. In the centre the comparison of the slice with and without the CSR interaction is shown on a few selected turns. Displayed is the line density relative to the unperturbed bunch. At the end of the simulation and without the interaction the slice has smeared out and has remained at the peak of the bunch. As predicted [5], the perturbation has moved towards the head of the bunch with the CSR interaction, the density modulation becomes sinusoidal with a more and more pronounced characteristic frequency. This is more clearly seen on the right of Fig. 3 where the form factors normalized to 1 at zero frequency are shown for the two cases. The interaction enhances the emission in the THz region of the spectrum on later turns. Maybe not as strongly as expected from the electro optical sampling experiments performed at the SLS [15]. At BESSY II [16] the slice can be followed over as many as 15 turns, probably impossible without a mechanism enhancing and keeping the structure which was initially created by the laser.

According to Fig. 2 the bunch is already unstable at 3 mA. The simulation at this current was started with the potential well distorted line density. After the laser slicing the bunch is on its way to a moderately unstable state which is characterized by a broad spectral component around 100 GHz in the form factor. After the 11th turn this peak shows up clearly, still at a higher frequency and twice as large as in the final steady state. This component is the result of the peak in the real part of the shielded CSR impedance.

Spontaneous bursts of CSR at THz frequencies can occur with regular and irregular time intervals and they are characterized by repetition frequencies below F_{syn} (see Fig. 2). Laser slicing can trigger the emission of bursts. The mechanism is the forced excitation of the 100 GHz-component since the much shorter density perturbation smears out. At high enough bunch current this component will grow through the instability feedback

mechanism until higher harmonics due to over-focussing and finally filamentation of the electrons in the 100 GHz-buckets will limit the process. This is accompanied by the emission of a burst of coherent radiation. Until the next burst can start the density has to recover by natural damping [17].

Modelling the reoccurrence of bursts after about a quarter of a synchrotron period [18] indicates the importance of the longitudinal to transverse coupling [4] and is beyond the model presented in this paper.

REFERENCES

- [1] A.A Zholents and M.S. Zolotarev, *Phy. Rev. Lett.* 76, 912 (1996)
- [2] S. Khan, et al., *Phys. Rev. Lett.* 97, 074801 (2006)
G. Ingold, et al., PAC'01, Chicago, p. 2656
- [3] J.M. Byrd, et al., EPAC'04, Lucerne, p. 2448
K. Holldack, et al., *Phys. Rev. Lett.* 96, 054801 (2006)
M. Shimada, et al., *Jpn. J. Appl. Phys.*, Vol. 46, No. 12, 7939 (2007)
- [4] M. Shimada, et al., *Phys. Rev.* 103, 144802 (2009)
- [5] G. Stupakov and S. Heifets, EPAC'06, Edinburgh, THPLS097.
- [6] R. Warnock and J.A. Ellison, SLAC-PUB-8404 (2000)
- [7] J.B. Murphy, et al., *Part. Accel.* 57, 9 (1997)
- [8] K.L. Bane, et al., *Phys. Rev. ST-AB* 13, 104402 (2010)
- [9] M. Venturini, et al., *Phs. Rev. ST-AB* 8, 014202 (2005)
- [10] K. Oide, *Part. Accel.* 51, 43 (1995)
- [11] P. Kuske, PAC'09, Vancouver, FR5RFP063
- [12] R.L. Warnock, PAC'91, p. 1824
- [13] J.M. Byrd, et al., *Phys. Rev. Lett.* 89, 224801 (2002)
- [14] S.A. Heifets, *Phys. Rev. E*, Vol. 54, 2889 (1996)
- [15] F.L. Müller, PhD-thesis, 2011, Universität Bern
- [16] K. Holldack, et al., PAC'05, Knoxville, p. 2239
- [17] M. Venturini and R. Warnock, *Phys. Rev. Lett.* 89, 224802 (2002)
- [18] J.M. Byrd, et al., PAC'05, Knoxville, p. 3682.