# SIMULTANEOUS LONG AND SHORT BUNCH OPERATION IN AN ELECTRON STORAGE RING – A HYBRID MODE BASED ON NONLINEAR MOMENTUM COMPACTION

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*Abstract*

The generation of short pulses in electron storage rings is motivated by users interested in time resolved X-ray spectroscopy or coherent synchrotron radiation. The required optics and operation conditions to generate this short bunches are reducing the average photon flux for the regular user. Therefore short bunch operation is usually limited to dedicated user shifts. By controlling higher orders of the momentum compaction factor $\alpha$ by higher order magnetic multipoles it is possible to introduce a hybrid mode and simultaneously supply long and short bunches [1]. The Metrology Light Source (MLS) has the means to control $\alpha = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2$ [2], therefore it is an ideal machine to investigate the feasibility of such a hybrid mode. First measurements will be shown.

## INTRODUCTION

Several electron storage rings offer a quasi-isochronous mode of operation by reducing $\alpha$. As $\alpha$ approaches zero, higher order terms become more important for the resulting bucket structure. Buckets dominated by higher orders of $\alpha$ have been observed at various facilities like NSLS, Soleil, MLS, Diamond or BESSY II [3, 4, 5, 6]. The MLS [2, 7] by design includes additional sextupole and octupole magnets with the possibility to tune $\alpha_1, \alpha_2$ independently. $\alpha$-buckets are one approach to bunch tailoring, offering multiple buckets simultaneously [8].

## $\alpha$-BUCKETS

The longitudinal Hamiltonian $H$ of the particle variables $(\phi, \delta)$ with $\delta = \Delta p/p$ can be written as [9]

$$H(\phi,\delta) = 2\pi q f_{\text{rev}} \int \alpha(\delta)\,\delta \mathrm{d}\delta + \frac{eU_0 f_{\text{rev}}}{\beta^2 E_0}\cos(\phi)\,, \quad (1)$$

where $\phi$ is the phase of the particle in reference to the rf-phase, $U_0$ the rf-voltage, $E_0$ the energy of the reference particle, $f_{\text{rev}}$ the revolution frequency and $q$ the harmonic number. For reasons of simplicity all amplitude dependent orbit lengthening effects, particle energy loss and wakefield interaction were neglected. Additionally only the highly relativistic limit will be treated, $\Delta f_{\text{rev}}/f_{\text{rev}} = -\alpha\delta$, where $\alpha(\delta)$ can be expanded in the power series

$$\alpha(\delta) = \frac{\Delta L}{L}\frac{1}{\delta} = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 \ldots \quad (2)$$

Taking $\alpha_0, \alpha_1, \alpha_2$ into account, the Hamiltonian yields qualitatively different situations depending on which parameter $\alpha_i$ dominates in the given $\delta$-range.

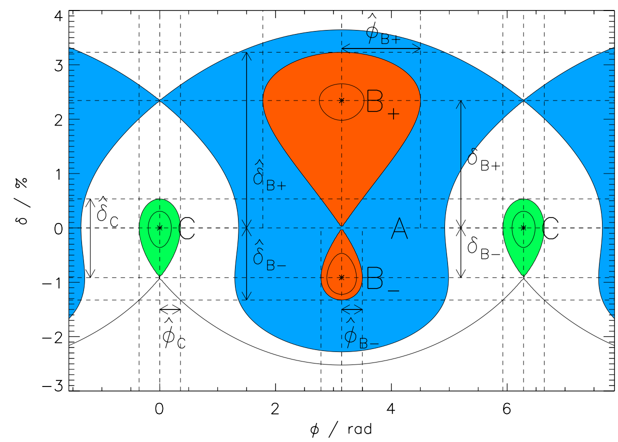


Figure 1: Longitudinal phase space featuring alpha buckets dominated by $\alpha_2$ ($\alpha_0, \alpha_1 < 0$ ; $\alpha_2 > 0$) and characteristic parameters (see text).

Similar to the argumentation in reference [2] for the $\alpha_2$ dominated case with $\alpha_1 \neq 0$ the quantities defining the $\alpha$-buckets as shown in Fig. 1 can be derived. For $\alpha_0\alpha_2 < 0$ there are three bucket types further referred to as buckets $A$, $B_\pm$ and $C$. One defining characteristic of the $\alpha$-buckets is the position of the fixed points (FP), which can be calculated from Eq. (1). The FP in $\phi$ are $\phi_{B,C} = \pi, 0$ , whereas the stable FP in $\delta$ become

$$\delta_{B_\pm} = -\frac{\alpha_1}{2\alpha_2} \pm \frac{1}{2\alpha_2}\sqrt{{\alpha_1}^2 - 4\alpha_0\alpha_2}, \quad (3)$$

$$\delta_C = 0. \quad (4)$$

The corresponding full bucket heights can be calculated by

$$\hat{\delta}_{B_\pm} = -\frac{2\alpha_1}{3\alpha_2} \pm \frac{1}{3\alpha_2}\sqrt{4{\alpha_1}^2 - 18\alpha_0\alpha_2}, \quad (5)$$

whereas $\hat{\delta}_C$ yields a non-trivial solution. Using the substitution $F = 2\pi q\beta^2 E_0/(eU_0)$, the phase acceptances of buckets $B_\pm$ can be written as

$$\hat{\phi}_{B_\pm} = \cos^{-1}\left[1 - \frac{F}{\cos(\phi_B)}\sum_{i=0}^{2}\frac{\alpha_i}{i+2}{\delta_{B_\pm}}^{i+2}\right], \quad (6)$$

where $\hat{\phi}_C$ is equal to the smaller one of $\hat{\phi}_{B_\pm}$. With $\hat{\delta}_{B_\pm}$ and $\delta_{B_\pm}$ given, the bucket momentum acceptances around the FP can be derived. Tune characteristics of the $\alpha$-buckets can be obtained from small oscillations around the FP [2],

$$ {\omega_C}^2 = \frac{\omega_{\text{rf}}^2}{F}\underbrace{\alpha_0}_{\alpha_{\text{eff}}}, \quad (7)$$

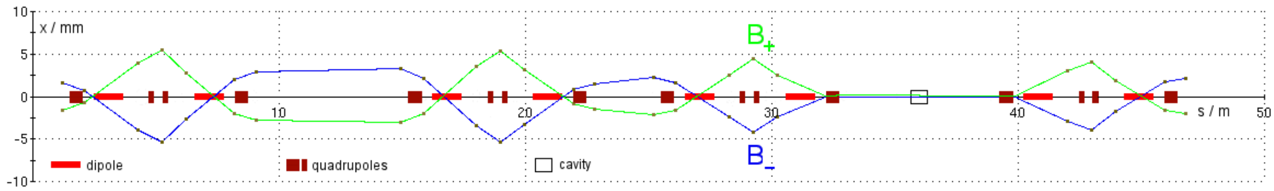


Figure 2: Measured dispersive orbit of the $B_\pm$ electron beams with $\alpha_1 = 0$, $x$ describes the horizontal displacement, $s$ the longitudinal position. The dispersion in the straight of the cavity (at $s = 36$ m) is zero.

$$\omega_{B_\pm}^2 = \frac{\omega_{\text{rf}}^2}{F} \underbrace{\left(2\alpha_0 - \frac{\alpha_1{}^2}{2\alpha_2} \pm \frac{\alpha_1}{2\alpha_2}\sqrt{\alpha_1{}^2 - 4\alpha_0\alpha_2}\right)}_{\alpha_{\text{eff}}}, \quad (8)$$

where the sign of $\omega_{B_\pm,C}^2$ is connected to $\cos(\phi_{B,C})$ indicating, at which phase the oscillation is stable. $\alpha_{\text{eff}}$ can be interpreted as the effective momentum compaction factor around the FP. It is worth noting that $\omega_C$ is independent of $\alpha_1, \alpha_2$. By tuning $\alpha_1 \to 0$ the Hamiltonian becomes symmetric in $\delta$ and $\omega_B$ becomes independent of $\alpha_2$, which can be used as an experimental control mechanism. For $\alpha_1 = 0$ the tune $\omega_B$ is a factor of $\sqrt{2}$ larger than $\omega_C$. For the symmetric case of even higher orders $\alpha(\delta) = \alpha_0 + \alpha_{2n}\delta^{2n}$ this tune factor is $\sqrt{2n}$. Bunch lengths can be calculated using the small oscillations approach as above yielding the known relation:

$$\sigma_{B_\pm,C} = \frac{\alpha_{\text{eff}}}{\omega_{B_\pm,C}}\delta_0 = \frac{\alpha_{\text{eff}}}{2\pi f_{B_\pm,C}}\delta_0, \quad (9)$$

where $\delta_0$ is the natural momentum spread (rms). When the momentum acceptance of the $\alpha$-bucket is comparable to the momentum spread to be stored, the nonlinear bucket has to be taken into account for the calculation of the bunch length.

## MEASUREMENTS AT THE MLS

To confirm the above results, buckets of the type shown in Fig. 1 were created at the MLS. Different bucket characteristics were measured while varying $E_0$, $\alpha_0$, $\alpha_1$ and $\alpha_2$. Measurements were performed at standard operation energy of $E_0 = 629$ MeV with $\delta_0 = 4.4 \cdot 10^{-4}$ and $-14 < \alpha_2 < 14$. Additionally a smaller energy $E_0 = 250$ MeV was chosen to provide a smaller momentum spread of $\delta_0 = 1.7 \cdot 10^{-4}$ and higher octupole impact $-35 < \alpha_2 < 35$.

### *Orbit Separation*

The beam stored in bucket $C$ stays on the reference orbit. Whereas the two beams stored in the buckets $B_\pm$ travel along paths, that are defined by the dispersion function $D$ and the FP $\delta_{B_\pm}$. Fig. 2 shows two consecutive orbit measurements by BPMs, where either $B_+$ or $B_-$ buckets were populated. The symmetry of the $B_+$ and $B_-$ orbits breaks, when $\alpha_1 \neq 0$ is included (Eq. (3)). A measurement by a source point imaging system can be seen in Fig. 3 showing simultaneously populated $B_\pm$ and $C$ buckets (triple beam) while varying the longitudinal chromaticity $\xi_s = -\alpha q \frac{\Delta f_s}{\Delta f_{\text{rf}}}$, whereas $\xi_s \propto \alpha_1$.

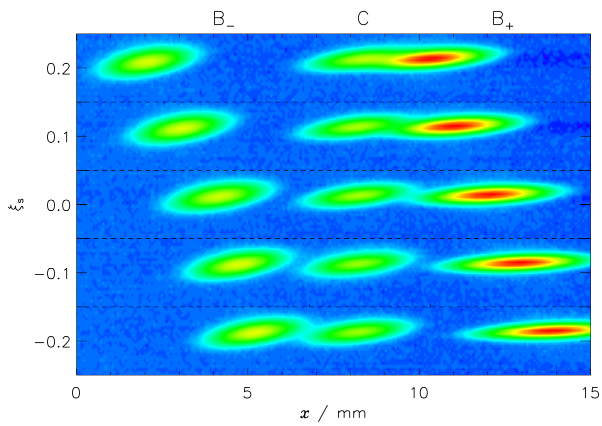


Figure 3: Triple beam measured by a source point imaging system at a $D > 0$ location, while varying $\xi_s$.

### *α-Bucket Tunes*

To verify the bunch shortening / lengthening a streak camera measurement was attempted, but the orbit separation of the two buckets was too large to be able to measure both bunch lengths at the same time without a complete realignment of the beamline setup. However, a tune measurement was conducted to conclude on the bunch lengths in the corresponding buckets. Fig. 4 shows the ratio $f_{B_\pm}/f_C$, which corresponds to the bunch length ratio $\sigma_{B_\pm}/\sigma_C$ (Eq. (7,8,9)) depending on $\xi_s$. The measured ratio $f_{B-}/f_{B+}$ increases up to a factor of 2.7 for 250 MeV and is limited by a lifetime decrease.This factor is significantly lower for 629 MeV, as the increased momentum spread requires a larger momentum acceptance around the FP (see next section). The horizontal dashed line shows the starting tunes at $\alpha_1 = 0$ which corresponds to $\sqrt{2}f_C$.

### *Machine Limitations*

The real machine adds additional constraints to the asymmetric $\alpha$-buckets. There is a natural momentum spread, which has to be available to a certain extend around the FP. For the long bunch, that travels along large $\delta$, the

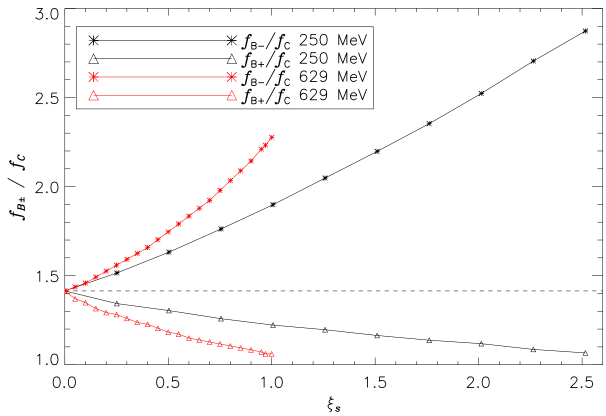


Figure 4: Measured longitudinal synchrotron oscillation frequencies of the buckets $B_\pm$ while varying $\xi_s$.

maximum or minimum storable momentum deviation $\bar{\delta}_\pm$ will be limiting:

$$\left|\bar{\delta}_\pm - \delta_{B_\pm}\right| > n\delta_0. \tag{10}$$

For the short bunch stored in $B$ (and similar for $C$) it is crucial to supply sufficient momentum acceptance of the $\alpha$-bucket:

$$\left|\hat{\delta}_{B_\pm} - \delta_{B_\pm}\right| > n\delta_0. \tag{11}$$

Using the above limitations and given ranges for $\alpha_0, \alpha_1, \alpha_2$ maximum bunch length ratios $\sigma_{B_\pm}/\sigma_C$ and $\sigma_{B_\pm}/\sigma_{B_\mp}$ can be calculated. Fig. 5 shows a MAD-X [10] tracking simulation of a MLS lattice configuration inlcuding damping and quantum excitation, which was performed close to these machine limitations. The beams stored in $B_-$ and

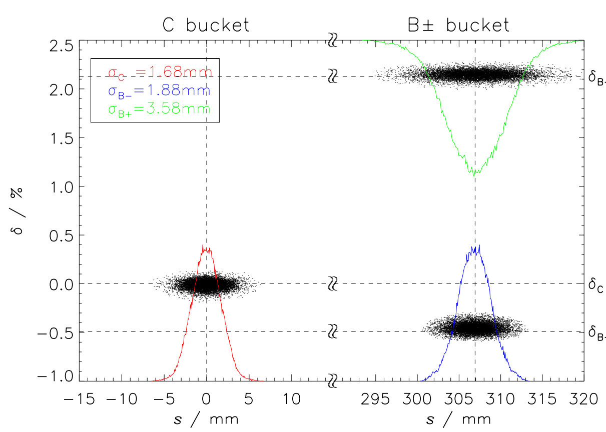


Figure 5: MAD-X simulation of an asymmetric triple beam stored at the MLS, $(\alpha_0, \alpha_1, \alpha_2) = (-0.00145, -0.23, 14)$, $E_0 = 629$ MeV, $U_0 = 250$ kV.

$C$ already feature visibile effects originating from scanning nonlinear bucket areas.

## CONCLUSION

$\alpha$-buckets seem to be a feasible way to introduce bunch tailoring to an electron storage ring. This can be done controlling $\alpha_1, \alpha_2$ with sextupoles and octupoles. The experiments at the MLS show, that the $\alpha$-buckets provide stable operation at currents of up to 175 mA with a lifetime of 7 h, which is comparable to the rf-bucket lifetime. The measured bucket characteristics at the MLS behave according to theory. A bunch length ratio of 2.7 was achieved. For comparison, BESSY II relies on a factor of 5 between standard user and low-$\alpha$ operation. Another application of $\alpha$-buckets could be to make use of the orbit separation providing different photon beams. Users would then be able to select one of three photon beams by switching beamline optics. This could be combined with a strong alternating rf-focussing scheme [11] to generate the different bunch lengths. When bunch length is not a parameter of interest, the $\alpha$-buckets still could supply flexible filling patterns. Another interesting aspect is featured by bucket $A$, which surrounds both $B$-buckets and catches particles that are lost to a certain extend. This can be used to generate a particle transfer from bucket $B_\pm$ to bucket $B_\mp$ by resonantly exciting particles exploiting $\omega_{B_+} \neq \omega_{B_-}$.

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## REFERENCES

[1] D. Robin et al., EPAC'08, Genoa, Italy, p. 2100-2102 (2008).

[2] J. Feikes et al., Phys. Rev. ST Accel. Beams 14, 030705 (2011).

[3] J. B. Murphy and S. L. Kramer, Phys. Rev. Lett, Vol. 48 Num. 24 (2000).

[4] A. Loulergue, Presentation, ESLS XV, (2007).

[5] G. Wüstefeld, "Coherent Synchrotron Radiation and Short Bunches in Electron Storage Rings", slides EPAC'08, Genoa, Italy (2008).

[6] I.P.S. Martin et al., Phys. Rev. ST Accel. Beams 14, 040705 (2011).

[7] B. Beckhoff et al., Phys. Status Solidi B 246, p. 14151434 (2009).

[8] D. Robin et al., Nucl. Instr. and Meth. A 622, p.518-535 (2010).

[9] K.Y. Ng, Nucl. Instr. and Meth. A 404, p.199-216 (1998).

[10] MAD-X 5.00.00, CERN, Geneva, Switzerland.

[11] G. Wüstefeld et al., "Simultaneous Long and Short Electron Bunches in the BESSYII Storage Ring", Proceedings of IPAC 2011, San Sebastin, Spain (2011).