

# STUDY OF A LARGE PIWINSKI'S ANGLE CONFIGURATION FOR LINEAR COLLIDERS

R. Versteegen, O. Napoly, CEA, Irfu/SACM, Gif-sur-Yvette, France

## Abstract

The application of a large Piwinski's angle configuration to the interaction region of a linear collider is studied. The calculation of the equivalent disruption parameter and beamstrahlung parameter in the presence of a crossing angle are necessary to estimate the beam-beam interaction effects in such a configuration. The reduction of these effects, based on the beam-beam parameters while keeping the same luminosity, is presented for both ILC and CLIC parameters.

## INTRODUCTION

To reach the high luminosity foreseen at the next linear collider, beams have to be very intense and strongly focused at the interaction point. This leads to strong electromagnetic beam-beam interaction during collision, provoking the deviation of particles trajectories, and therefore the emission of beamstrahlung [1]. These phenomena limit the performances of the collider because beams are disrupted after collision, and beamstrahlung generates background in the detector. The nominal design of ILC [2] and CLIC [3] interaction regions are based on a non-zero crossing angle to facilitate the beam extraction and to avoid parasitic crossings. Crab cavities are needed to compensate the luminosity loss induced by the crossing angle. In this paper we explore the possibility to use a large Piwinski's angle configuration instead of the crab cavity. It could help to mitigate the effect of beam-beam interaction and eventually could provide a back-up configuration if the crab cavity is not tuneable as expected [4].

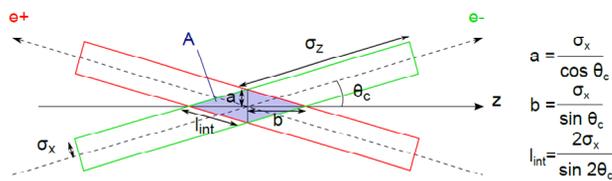


Figure 1: Large Piwinski's angle crossing scheme.

With the notation introduced in Fig. 1, the Piwinski's angle  $\phi$  is defined as:

$$\phi = \frac{\sigma_z}{\sigma_x} \tan \theta_c \quad (1)$$

A large Piwinski's angle (LPA) was first suggested as part of the nano-beam scheme for B-factories [5]. Increasing  $\phi$  makes the beam population in the overlapping area (A) smaller, and the interaction length shorter (ie.  $l_{int}$ , characterized by  $\sigma_z$  for head-on collision). This has an impact on disruption, on beamstrahlung emission, and it moves the limit of the hourglass effect to smaller  $\beta_y^*$ . Moreover  $\phi$  influences the luminosity  $L$  (neglecting the hourglass effect):

$$L \propto \frac{1}{\sqrt{1 + \phi^2}} \frac{1}{4\pi\sigma_x\sigma_y} \quad (2)$$

The disruption parameter characterizes the effect of beam-beam interaction in terms of stability, and the beamstrahlung parameter corresponds to the average energy loss due to the emission of synchrotron radiation. Analytical methods developed to evaluate these beam-beam parameters in the presence of a crossing angle are exposed in the first two parts of this paper. Then the application of the LPA configuration to ILC and CLIC, focusing on the preservation of the nominal luminosity, is described.

## DISRUPTION PARAMETER

If the disruption is small, the beam acts like a focusing lens of focal length  $f$  in the vertical plane. For head-on collision, the disruption parameter is defined as the ratio of the interaction length over  $f$  [1].

The equations of motion of a charged test particle interacting with the opposite beam (assumed unperturbed) taking a crossing angle into account are solved using Mathematica [6]. Since particles are ultra-relativistic, the electromagnetic field ( $\vec{E}, \vec{B}$ ) is emitted transversely to the direction of propagation. Consequently, the test particle is not affected by the electromagnetic field unless it crosses a wavefront (see Fig. 2). The electromagnetic force in the laboratory frame is equal to  $e(1 - \dot{z}/c)\gamma E$ , where  $\dot{z}$  designates the particle's velocity projected on the longitudinal axis,  $e$  the charge of the particle,  $\gamma$  the relativistic factor, and  $c$  the speed of light.

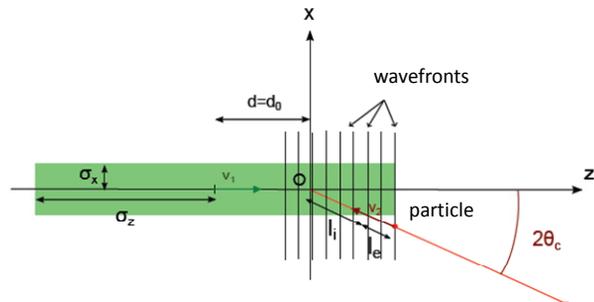


Figure 2: Schematic of the electromagnetic interaction.

The initial conditions are set so that the test particle and the reference particle of the beam meet at the interaction point (O), meaning that the particle undergoes the strongest interaction if the maximum charge density of the beam lies in its centre.

## Focal Length

To calculate the focal length of the equivalent lens supposing small disruption, equations of motion are solved for  $y(s) = \text{constant} = y_0$ . If  $y_0 \ll \sigma_y$  (with  $\sigma_y$ , the

vertical beam size), the integrated vertical kick  $\Delta y'$  after collision is proportional to  $y_0$  and  $f = y_0/\Delta y'$ .

### Interaction Length

The interaction length  $l_{int}$  actually corresponds to the duration while the test particle experiences the field produced by the beam. Considering the integrated electromagnetic force over  $l_e$  or  $l_i$  along the machine axis (see Fig. 2), and considering  $2\theta_c \leq 30$  mrad, we show that:

$$\frac{\int_{l_i}^{l_e} e(1 - z/c)\gamma E}{\int_{l_i}^0 e(1 - z/c)\gamma E} < 1\% \quad (3)$$

The field generated outside of the beam can then be neglected and the interaction length can be calculated as the RMS value ( $\sigma_{zeq}$ ) of the beam distribution seen by the test particle during collision. This means that for uniform distribution:

- $l_{int} = \sigma_{zeq} \approx \sigma_z$  for  $\phi < 1$
- $l_{int} = \sigma_{zeq} \approx \sigma_x / \theta_c \approx \sigma_z / \phi$  for  $\phi \geq 1$

### Vertical Disruption Parameter $D_y$

Fig. 3 represents the variation of the resulting disruption parameter  $D_y = \sigma_{zeq} / f$ , for transverse gaussian beam distributions. Uniform and gaussian longitudinal distributions cases are plotted in blue. Data referred to as ‘Yokoya-Chen’ correspond to the variation law given in reference [1], i.e.  $D_y = D_{y0} / (1 + \phi^2)$ , with  $D_{y0}$  the parameter for zero crossing angle. They are in good agreement with the case of a gaussian longitudinal distribution. As expected, a crossing angle has a strong influence on disruption:  $D_y$  is divided by 10 approximately for  $\theta_c \approx 6$  mrad.

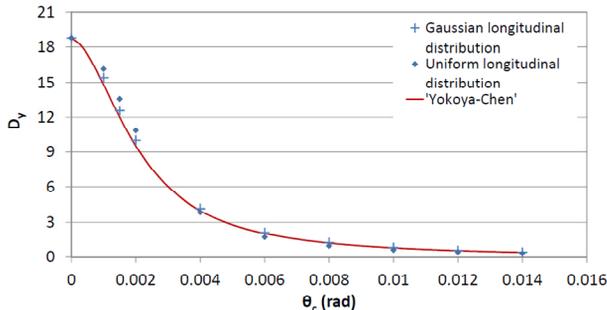


Figure 3: Vertical disruption parameter as a function of  $\theta_c$  for ILC nominal beam parameters.

## BEAMSTRAHLUNG PARAMETER

The beamstrahlung parameter  $\delta_{BS}$  is defined as the average energy loss due to beamstrahlung emitted during collision. A variation law of  $\delta_{BS}/\delta_{BS0}$  as a function of  $\theta_c$  (with  $\delta_{BS0}$  the value obtained for zero crossing angle) is given in reference [1] for a small vertical disruption parameter. In this work,  $\delta_{BS}$  is calculated without any assumption on  $D_y$ . The classical approximation of synchrotron radiation theory is used, implying that this analytical method is not applicable to CLIC parameters, since this approximation is no longer valid at 3TeV. The evaluation of the electromagnetic field experienced by a

particle of given initial conditions enables to calculate the second synchrotron radiation integral  $I_2 \cdot \delta_{BS}$  is then obtained averaging over transverse distributions:

$$\delta_{BS} = \frac{2 r_e \gamma^4 m_0 c^2}{3 E_0} \iint I_2 \rho_x(x) \rho_y(y) dx dy \quad (4)$$

$r_e$  is the classical electron radius, the mass of the electron and  $E_0$  its total energy before collision. A longitudinal distribution is not taken into account and all the particles reach the interaction point simultaneously, experiencing the field emitted by the centre of the opposite beam. It implies that the mean energy loss is overestimated, which is conservative regarding the evaluation of the beamstrahlung emission. Calculating  $\delta_{BS}$  supposing no vertical deviation is equivalent to the assumption of a small disruption parameter and leads to the same variation law as in reference [1] for  $\delta_{BS}/\delta_{BS0}$  [4].

The results obtained taking the vertical deviation into account are plotted in blue in Fig. 4, where  $\delta_{BS,n}$  designates the ILC nominal beamstrahlung parameter ( $\delta_{BS,n} = 2.1\%$ ).  $\delta_{BS}/\delta_{BS,n} = 1$  for head-on collision, so the classical approximation is still valid for ILC.

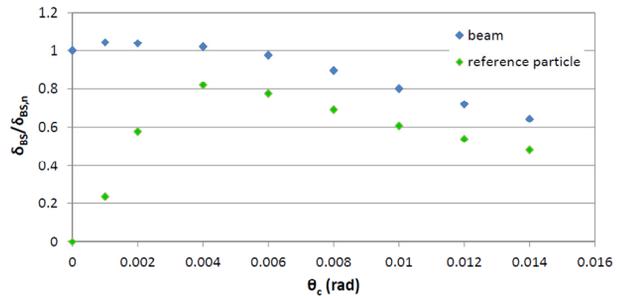


Figure 4: Beamstrahlung parameter as a function of  $\theta_c$  for ILC nominal beam parameters,  $\delta_{BS,n} = 2.1\%$ .

Results of Fig. 4 show that  $\delta_{BS}/\delta_{BS,n}$  first increases with  $\theta_c$  before getting less than one. This is due to a horizontal deviation of the reference particle in the presence of a crossing angle, generating additional radiation (see Fig. 5). The resulting energy loss for these central trajectories is plotted in green as a function of  $\theta_c$  in Fig. 4 (“reference particle”).

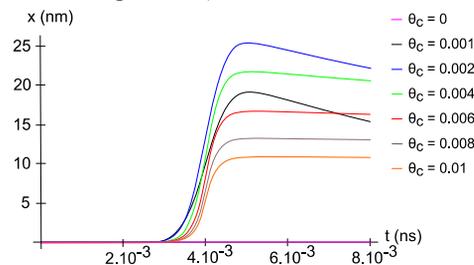


Figure 5: Horizontal deviation of the central trajectory with respect to the machine axis during collision for ILC nominal beam parameters.

## APPLICATION TO ILC AND CLIC

Introducing a crossing angle could be interesting to reduce the disruption in a linear collider. Reduction of beamstrahlung would be possible for very large crossing

angles only, since the energy loss resulting from the beam horizontal mean deviation has to be overcome. Nevertheless the energy spread with respect to the reference particle is smaller [4]. However, increasing the crossing angle significantly impacts on  $L$  (Eq. 2). To restore the nominal value, one can modify beam sizes at the interaction point and/or the number of particles per bunch  $N_0$ , the number of bunches per train  $n_b$  (and the repetition frequency  $f_{rep}$  of the machine). We have shown that calculation of the disruption parameter leads to same results as in reference [1]. This implies, assuming  $\phi > 1$ :

$$L \propto \frac{1}{\sigma_y \sigma_z \theta_c} \text{ and } D_y \propto \frac{\sigma_x}{\theta_c} L \quad (5)$$

Consequently for a given value of  $L$ ,  $\sigma_x$  should be minimized and  $\theta_c$  maximized to reduce the disruption. On

Table 1: Sets of Parameters for LPA Configuration.  $L_{0,n}$  is the Nominal Luminosity Without Hourglass.

	$\theta_c$ (mrad)	$\sigma_x$ (nm)	$\sigma_y$ (nm)	$\sigma_z$ ( $\mu\text{m}$ )	$\phi$	$\beta_y^*$ ( $\mu\text{m}$ )	$\gamma \epsilon_y$ (m.rad)	$N_0$ ( $10^{10}$ )	$D_y$	$\delta_{BS}$ (%)	$\sigma_\delta/\sigma_{\delta,n}$ (%)	$L/L_{0,n}$ (%)
ILCa	7	640	1.7	300	3.3	35	$4 \cdot 10^{-8}$	2	5.5	1.8	15	64
ILCb	7	640	1.7	200	2.2	45	$3 \cdot 10^{-8}$	1.9	5.5	2.8	30	94
ILCc	10	640	1.2	200	3.1	23	$3 \cdot 10^{-8}$	2	3.9	2.7	25	92
CLICa	10	45	1	44	9.8	7	$20 \cdot 10^{-8}$	0.37	0.07	5.8	-	10

$\sigma_\delta$  is the second order momentum of the energy distribution after collision with respect to the reference particle.  $\sigma_{\delta,n}$  corresponds to the nominal case. These parameters could not be evaluated for CLIC in the classical approximation.

Reducing  $\sigma_y$  to 1.7 nm for  $\theta_c = 7$  mrad enables to restore the luminosity neglecting the hourglass effect. In that case  $D_y$  is divided by 3.4. But the luminosity loss due to hourglass effect is  $\sim 35\%$  (ILCa). Decreasing  $\sigma_x$  could help to reduce it since  $\sigma_{zeq} \approx \sigma_x/\theta_c$ , but beamstrahlung becomes stronger due to the horizontal deviation of the beam's central trajectory ( $\delta_{BS} \approx 10\%$ ) [4]. Restoring nominal  $L/L_{0,n}$  is done by decreasing the vertical emittance ( $\epsilon_y$ ) by 25% and  $\sigma_z$  from 300  $\mu\text{m}$  to 200  $\mu\text{m}$  (to stay in the parameter plane [2]). With nominal  $\sigma_x$ , and with  $\sigma_y$  less than 2 nm, we get  $L/L_0 > 0.9\%$  for both values of  $\theta_c$ , and  $D_y = 5.5$  (ILCb, ILCc). For ILCb,  $N_0$  can be slightly relaxed to reach the desired value of  $L/L_{0,n}$ . The beamstrahlung parameter is similar to nominal for ILCb and ILCc, but the energy spread after collision is much smaller [4]. It has to be mentioned that the reduction of  $\beta_y^*$  is strong compared to the nominal design ( $\beta_{y,0}^* = 400 \mu\text{m}$ ) and optics study has to be done to check its feasibility.

For CLIC the nominal vertical beam size  $\sigma_y = 1$  nm is very small, and this parameter cannot be further decreased. This implies that we cannot preserve the nominal luminosity. But  $\beta_y^*$  could still be divided by 10 thanks to the crossing angle pushing the hourglass limit. Consequently the constraint on vertical emittance could be relaxed from  $2 \cdot 10^{-8}$  m.rad to  $20 \cdot 10^{-8}$  m.rad (Table 1, CLICa). The disruption parameter becomes very small in this case. In such a small disruption regime, the beamstrahlung parameter can be calculated with the

the other hand,  $\sigma_y$  should be smaller to restore the nominal luminosity. This could be possible considering the new limit for the hourglass effect in the presence of crossing angle:  $\beta_y^* \geq \sigma_{zeq} \sim \sigma_x/\theta_c$ . In the following discussion,  $L$  is the geometrical luminosity including the hourglass effect, and  $L_0$  is the nominal luminosity neglecting hourglass effect defined as  $L_{0,n} = \frac{N_0^2 n_b f_{rep}}{4\pi \sigma_x \sigma_y}$ , with zero crossing angle and nominal beam sizes. For ILC and CLIC nominal parameters, the luminosity reduction due to the hourglass effect is 10%, and  $L/L_{0,n} = 0.9\%$ . This is the ratio to be restored with the LPA configuration. Two values of  $\theta_c$  are considered for ILC: 7 mrad and 10 mrad, and  $\theta_c = 10$  mrad for CLIC. Calculations of  $D_y$ ,  $\delta_{BS}$ ,  $\sigma_\delta/\sigma_{\delta,n}$  and  $L/L_{0,n}$  are given in Table 1 for selected cases.

expression of reference [1], and it is divided by 5 compared to nominal  $\delta_{BS,n} = 29\%$ . Now the resulting background should be evaluated, and the required luminosity re-calculated. Then the possibility to modify  $n_b$  or  $N_0$  taking advantage of the larger emittance has to be studied, since 10 mrad crossing angle implies 90% luminosity loss with the nominal parameters.

## CONCLUSION

Disruption can be significantly reduced thanks to the LPA configuration in a linear collider. Concerning beamstrahlung, the horizontal deviation of the reference particle in the presence of a crossing angle prevents  $\delta_{BS}$  to decrease considerably, but the energy spread after collision is smaller. A new set of parameters preserving nominal luminosity has been suggested for ILC for  $\theta_c = 7$  mrad or 10 mrad, but optics studies have to be performed to verify the possibility to reduce  $\beta_y^*$  using the existing final focus design. Background production has to be simulated in these new schemes.

## REFERENCES

- [1] K. Yokoya, P. Chen, "Beam-beam phenomena in linear colliders", Springer Berlin/Heidelberg (1992).
- [2] ILC Reference Design Report, Accelerator (2007).
- [3] CLIC Conceptual Design Report, (2010).
- [4] R. Versteegen, "Conception et optimisation de la région d'interaction d'un collisionneur linéaire e-e-", PhD thesis, Université Paris XI (2011).
- [5] P. Raimondi, *et al.*, arXiv:physics/0702033 (2007).
- [6] Wolfram Research Inc., *Mathematica 8.0* (2010).