

# EMITTANCE VARIATION DEPENDENCE ON RESONANCE EXTRACTION PARAMETERS AT ELSA\*

S. Zander, O. Preisner, F. Frommberger, W. Hillert, ELSA, Bonn, Germany

## Abstract

The Electron Stretcher Facility ELSA (see Fig. 1) consists of several accelerator stages, the last one being a stretcher ring providing a beam of polarized electrons with an energy of up to 3.5 GeV. In order to guarantee a high duty cycle, a slow extraction via a third integer resonance is applied to the stretcher ring. The emittance of the extracted beam as well as the efficiency of the extraction process depend on different parameters as the sextupole strength being necessary for the excitation of the third integer resonance or the adjusted tune. In order to optimize the quality of the extracted beam, an accurate comprehension of the influence of these parameters is indispensable. Beam profiles are detected using dedicated synchrotron light monitors optimized for low intensities. The emittance was investigated by the method of quadrupole scan. The experimental studies are accompanied by numerical simulation studies. The results of the change of the emittance depending on different resonance extraction setups obtained by the experimental as well as by the theoretical studies will be presented.

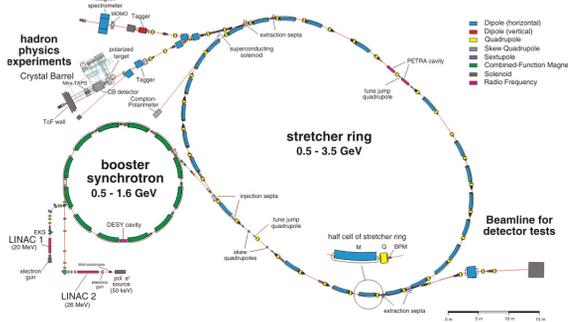


Figure 1: Electron Stretcher Facility ELSA

## INTRODUCTION

At ELSA, polarized electrons are extracted via a third integer resonance in order to provide a high duty cycle. The usual elliptical shape of the particle motion of the horizontal phase space changes to a triangle shape due to the excitation of the third integer resonance by sextupole magnets. In the case of a third integer resonance, the phase space is divided into an area with stable particle motion and one with unstable motion (cf. Fig. 2). The stable area of the phase space is defined by the so called unstable fixed points. The triangle between these three points form the stable area of the phase space. The size  $a_{\text{fix}}$  of this area de-

pends on the horizontal tune  $Q$  and the sextupole strength  $g$ :

$$a_{\text{fix}} \propto \frac{Q - Q_{\text{res}}}{g}. \quad (1)$$

In order to transfer the electrons from the stable to the unstable area of the phase space during the extraction time in a controlled way, the tune is shifted towards the third integer resonance. Thereby, the stable area of the phase space is reduced. Outside of the stable area, the electrons move along the separatrix branches and, with every turn, diverge further from the center of the triangle. In the case of a third integer resonance, an electron reaches the same separatrix branch after three turns. The rise of the displacement over these three turns is called pitch. The pitch increases with larger distances from the unstable fixed points.

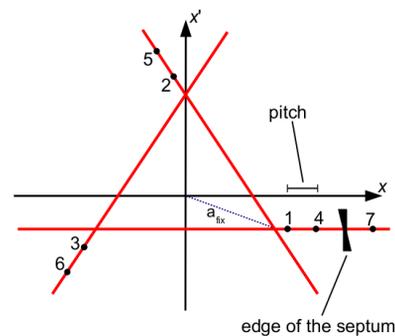


Figure 2: Phase space at a third integer resonance

At ELSA, a septum magnet is used for deflecting electrons into the extraction beamline. The beam properties in the extraction beamline depend on the position of the edge of the septum magnet and on the pitch at the edge. The pitch can be varied by shifting the tune over the extraction time and by the strength of the sextupoles magnets. In the following, the influences of these parameters are investigated.

## NUMERICAL SIMULATION STUDIES

Using the accelerator optics simulation program package MAD-X [1], dedicated tracking studies were arranged for the examination of the influence of different parameters on the emittance of the extracted beam. The extraction process—beginning with different, in phase space Gaussian shaped particle distributions—was simulated with a steep linear tune ramp. Because of the true-to-life considerations made for the synchrotron radiation the simulations are very time-consuming, but this is indispensable for

\* Work supported by the DFG within the SFB / TR 16

the calculation of the emittance in order to get a scattering around the separatrix branches as shown in Fig. 3.

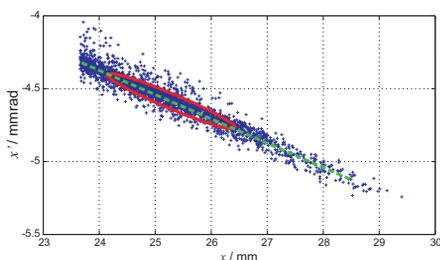


Figure 3: Coordinates in phase space after particle deflection into the extraction channel

In the extraction beamline, the beam is sharply cut by the septum edge, resulting in a non-Gaussian shape. Nevertheless the usual algorithm

$$\varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \quad (2)$$

is used for the calculation of the emittance of the extracted beam as a good approximation to the real value.

For the variation of the distance of the septum to the design orbit, a new feature was implemented into the THINTRACK module of MAD-X: Using the command `APER_OFFSET={Δx, Δy}`, the given aperture can be shifted around the design orbit. It follows from theory that the larger the distance of the septum to the design orbit of the circulating beam, the larger the increment of the pitch of the particle at the edge and thus the width of the extracted beam becomes. Figure 4 shows the predicted expansion of the emittance when shifting the edge of the septum away from the design orbit.

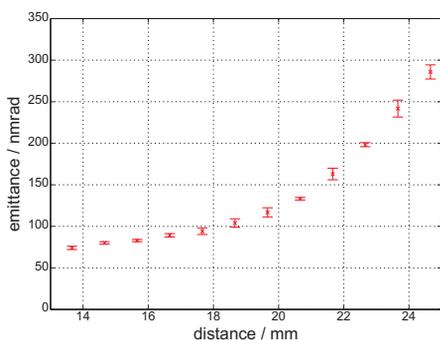


Figure 4: Emittance dependence on the distance of the septum to the design orbit

For comparison, the emittance of the circulating beam at the standard adjustments and the standard beam energy of 2.35 GeV is 473 nmrاد for all simulations presented here.

According to Eq. 1, a variation of the strength of the extraction sextupole magnets is equivalent to a variation of the distance of the fixed points to the origin of the phase space. The effect on the emittance is the same observed at the variation of the distance of the septum edge to the design orbit, what is shown in Fig. 5.

## 06 Beam Instrumentation and Feedback

### T03 Beam Diagnostics and Instrumentation

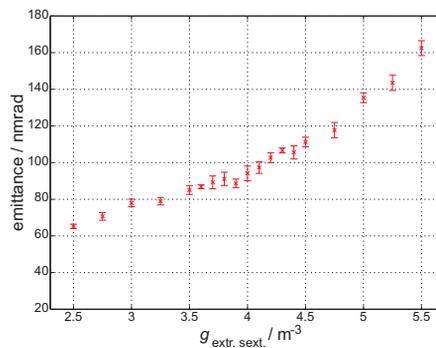


Figure 5: Emittance dependence on the strength of the extraction sextupole magnets

Applying an acceleration voltage to the RF cavities, MAD-X also considers synchrotron oscillations and thus energy oscillations during the tracking calculations. Due to

$$Q_x = Q_{x,0} + \xi_x \cdot \frac{\Delta p}{p_0}, \quad (3)$$

the energy oscillations of the particles lead to tune oscillations so that the tune of a single particle depends directly on its energy deviation. The consequence is an increase and a decrease of the individual stable phase space areas of the particles and thus the position of the separatrix branches. This results in a larger scattering around every branch, enlarging the emittance of the extracted beam as shown in Fig. 6.

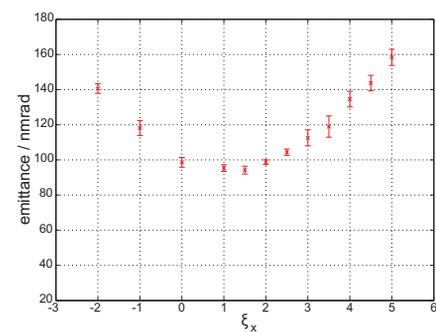


Figure 6: Emittance dependence on the horizontal chromaticity  $\xi_x$

## EMITTANCE MEASUREMENTS AT THE EXTERNAL BEAMLINES

At the external beamline leading to the hadron physics experimental setups, a synchrotron light monitor for each of the experimental areas is provided at the dipoles MB2 and MB3 (see Fig. 7). With these synchrotron light monitors, the emittance was determined via the quadrupole scan method. The width of the beam profile measured by synchrotron light monitors is not only determined by the emittance  $\varepsilon$  but also by the dispersion  $D(s)$  at the longitudinal

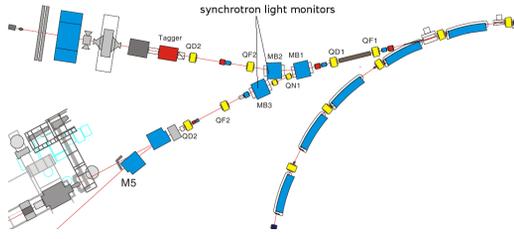


Figure 7: Synchrotron light monitors at the external beamline

position  $s$ :

$$\sigma_x(s) = \sqrt{\varepsilon_x \cdot \beta_x(s) + \left( D_x(s) \cdot \frac{\Delta p}{p_0} \right)^2}. \quad (4)$$

The dispersion function is determined by measuring the shift of the center of the beam profile depending on the variation of the RF frequency and different adjustments of the external beamline. So the dispersion function

$$D_x(s) = m_{11}(s)D_0 + m_{12}(s)D'_0 + m_{16}(s) \quad (5)$$

is evaluated. Hereby,  $D_0$  is the Dispersion and  $D'_0$  the change of the Dispersion at the beginning of the external beamline.  $m_{11}$  and  $m_{12}$  are the matrix elements of the transfer matrix.

The emittance of the extracted beam is determined by the so called quadrupole scan method. The measurement of the beam width  $\sigma$ , depending on the quadrupole strength  $k$ , enables a fit depending on the parameters  $(\varepsilon\beta_0)$ ,  $(\varepsilon\alpha_0)$ ,  $(\varepsilon\gamma_0)$ :

$$\sigma(k)^2 = m_{11}(s, k)^2(\varepsilon\beta_0) - 2m_{11}(s, k)m_{12}(s, k)(\varepsilon\alpha_0) + m_{12}(s, k)^2(\varepsilon\gamma_0). \quad (6)$$

Here, the matrix elements  $m_{11}$  and  $m_{12}$  depend on the optics between the monitor and the varied quadrupole magnet. The unambiguous determination of the emittance  $\varepsilon$  is only possible if the minimum of the beam width is crossed while the quadrupole strength  $k$  is varied.

By a general transformation of the Twiss parameters  $(\alpha, \beta, \gamma)$ , one obtains:

$$\gamma(s) = m_{21}(s)^2\beta_0 - 2m_{21}(s)m_{22}(s)\alpha_0 + m_{22}(s)^2\gamma_0. \quad (7)$$

For the beam waist, the condition  $\gamma_t = \frac{1}{\beta_t}$  holds.

Now, the beta function can be expressed by the width of the beam waist  $\sigma_t$  which leads to the emittance  $\varepsilon$ :

$$\varepsilon^2 = \sigma_t^2 [m_{21}(s)^2(\varepsilon\beta_0) - 2m_{21}(s)m_{22}(s)(\varepsilon\alpha_0) + m_{22}(s)^2(\varepsilon\gamma_0)]. \quad (8)$$

Quadrupole scans for different adjustments of the sextupole strength and the horizontal tune were conducted to investigate the influence on the emittance at the external beamline. As shown in Fig. 8 the emittance for a small sextupole strength is also small. This result is in good agreement with Eq. 1: The smaller sextupole strength  $g$  leads to

a larger distance between the center of the triangle of the stable area of the phase space and the unstable fixed points  $a_{\text{fix}}$ . Therefore, the unstable fixed points are close to the edge of the septum, which leads to a smaller pitch. Due to the smaller pitch, the emittance at the external beamline is also smaller.

The emittance for a horizontal tune far away from the third integer resonance (at ELSA  $Q_{\text{Res}} = 4\frac{2}{3}$ ) is smaller than near the resonance. This result is consistent with expectations, because the difference between the horizontal tune and the resonance is proportional to the distance  $a_{\text{fix}}$ . So the increasing of the difference between the horizontal tune and the resonance also leads to a smaller emittance.

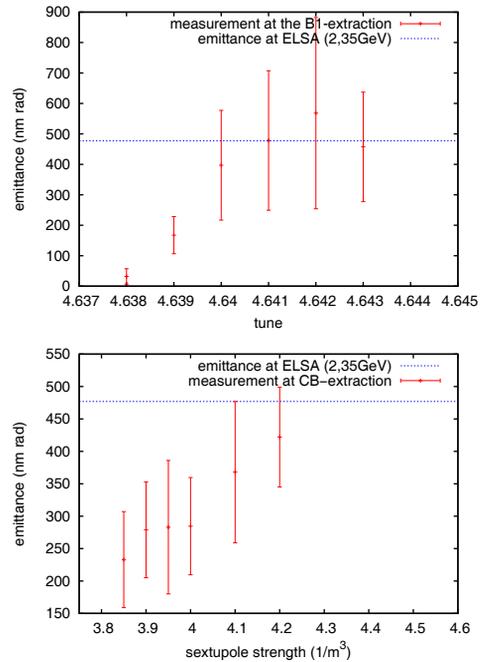


Figure 8: Dependence of emittance on extraction parameters. The blue line shows the equilibrium emittance at the stretcher ring.

## CONCLUSION

The particle distribution of the phase space depending on the resonance extraction was simulated with MAD-X due to the examination of the beam properties at the external beamline. The emittance was simulated and measured for different adjustments of the extraction parameters. For a certain set of values for sextupole strength and tune, a reduction of the emittance at the external beamline can be achieved in relation to the emittance at ELSA, if a reduction of the extraction efficiency is acceptable.

## REFERENCES

- [1] H. Grote and F. Schmidt, CERN-AB-2003-024 ABP.