

COURANT-SNYDER INVARIANT DENSITY SCREENING METHOD FOR EMITTANCE ANALYSIS*

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Abstract

The emittance is an important characteristic of describing charged particle beams. In hadron accelerators, we often meet irregular beam distributions that are not appropriate to be described with a single rms emittance or 95% emittance or total emittance. In this paper, it is pointed out that in many cases beam halo should be described with very different Courant-Snyder parameters from the ones for beam core. A new method – Courant-Snyder invariant density screening method is introduced for analyzing emittance data clearly and accurately.

INTRODUCTION

The emittance and its shape of a beam are important physical parameters that express the beam quality and the matching to the beam line it will pass through. They are important in designing and commissioning accelerators and beam transport lines. In hadron accelerators where beam distributions are often irregular, one uses different definitions to better represent the beam distribution in different applications. In this paper, we introduce a new method to evaluate all kinds of beam distributions for both simulated and measured distribution data.

In decoupled linear beam transport systems, each particle follows tracks along ellipses in the phase planes, which can be expressed by the following formula (taking the horizontal phase plane as an example):

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = I \quad (1)$$

where α , β , γ are the Courant-Snyder parameters (hereafter referred to as C-S parameters), and there is a relation among them: $\beta\gamma - \alpha^2 = 1$; I is the Courant-Snyder invariant (hereafter referred to as C-S invariant) which is constant during beam transporting without acceleration. The emittance ε is defined as the area of the boundary ellipse, which is usually defined as the area divided by π and with a unit of $\pi\text{mm.mrad}$. For a simulated beam, in the horizontal phase plane, the rms emittance can be calculated by the following formulae:

$$\varepsilon = \sqrt{\mu_{20}\mu_{02} - \mu_{11}^2} \quad (2)$$

where $\mu_{20} = \overline{(x - \bar{x})^2} = \overline{x^2} - \bar{x}^2$,

$$\mu_{02} = \overline{(x' - \bar{x}')^2} = \overline{x'^2} - \bar{x}'^2,$$

$$\mu_{11} = \overline{(x - \bar{x})(x' - \bar{x}')} = \overline{xx'} - \bar{x}\bar{x}'$$

For a measured beam, the distribution is usually discrete. One obtains a density distribution $c(x_i, x'_j)$ for discrete phase point (x_i, x'_j) . Thus, the rms emittance is

defined as:

$$\varepsilon_{rms} = \sqrt{\langle x_i^2 \rangle \langle x_j'^2 \rangle - \langle x_i \cdot x_j' \rangle^2} \quad (3)$$

The C-S parameters for the rms ellipse can be calculated as:

$$\alpha = -\frac{\langle x_i \cdot x_j' \rangle}{\varepsilon_{rms}}, \quad \beta = \frac{\langle x_i^2 \rangle}{\varepsilon_{rms}}, \quad \gamma = \frac{\langle x_j'^2 \rangle}{\varepsilon_{rms}}$$

For high-intensity hadron accelerators, halo particles with large C-S invariants have great influence in calculating the rms emittance using the statistical method. Therefore, it is better to use a more general expression of the emittance that can give more detailed information for a beam distribution. A new method is introduced to give different C-S parameters and emittance for different beam fractions or boundaries.

METHODS OF EVALUATING BEAM EMITTANCE

There are usually different methods to evaluate the emittance for either a simulated beam or a measured beam. For high beam power hadron accelerators it is better to use total beam emittance as halo particles are also important in the operation [1].

Simulated Beams

For a simulated beam, the particle distribution in the phase space is given from simulation codes and the rms emittance can be calculated by Eq. (2) within or outside the codes. This statistical method of calculating the rms emittance is widely used in the accelerator simulation software. For different particle distributions, the emittance with different beam fractions can be calculated through the rms emittance by zooming in/out and keeping the same C-S parameters that is calculated together with the rms emittance.

Measured Beams

For a measured beam, the measurement noise and a typically small bias are contained in the measured signals. Thus, the contribution from the background must be minimized before calculating the rms emittance by Eq. (3). Here two influential methods to calculate the rms emittance of a measured beam distribution are described and commented.

- Threshold analysis method

Applying a threshold commonly means that all values of a distribution above the threshold remain unchanged while all values below the threshold are set to zero [2]. Various methods have been developed to select the most

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appropriate threshold.

- SCUBEE analysis method

For the self-consistent unbiased elliptical exclusion (SCUBEE) [3] method, the C-S parameters are calculated firstly from the data after thresholding them at a high percentage of the peak signal (20% for instance) to exclude all background signals. These C-S parameters define the shape of the exclusion ellipse, and then zoom in the exclusion ellipse starting at zero. The average of the signals measured outside the exclusion ellipse is taken as the bias and needs to be subtracted from the raw data. The rms emittance can be evaluated from the data within the ellipse after subtracting the average current signal found outside the ellipse.

C-S Invariant Density Screening Method

The methods discussed above are good to evaluate the rms emittance of regular beam distributions that are symmetric and have the same contour shape for the inner and outer parts. In these cases, when calculating the emittance of different beam fractions, one can zoom in/out the rms ellipse. However, for some beams with irregular distributions, the simple rms emittance calculation methods are not appropriate. For example, for a beam with a large halo that may be non-symmetric or incomplete due to the acceptance limit, errors can arise in calculating the rms emittance, because the halo particles with large C-S invariants have very important weights to the calculations. In this case, it is better to use different C-S parameters to express the emittance shapes of the beam core and the beam halo. In addition, to better evaluate the importance of a macro-particle or a signal by minimizing statistical errors or noise, one can use its neighboring particles or signals. This can be done by accounting the particles or signals in a small phase ellipse defined by a given C-S invariant, namely, the C-S invariant density is used to mark the particle or signal. The emittance of different beam fractions can be calculated by including particles or signals with different density thresholds. This is called the C-S invariant density screening method. More details about the method are described below.

Usually a beam distribution has a denser core and sparser halo, and this is the case to be dealt with by the new method. Certainly it is also often to see beams with irregular distributions of no good symmetry, e.g. those having several islands, and they can not be dealt with by the method very effectively. In the following descriptions, only a simulated beam with macro-particles is used, but it is quite similar for a measured beam.

Before the C-S parameters have been calculated, an initial set of C-S parameters that are quite arbitrary are given. In order to determine the C-S invariant density for each particle, one can count the number of macro-particles within a small ellipse that is centered on the particle with a pre-set ellipse size and uses the initial C-S parameters. Then after, all the particles are labeled by their C-S invariant densities. There are two ways to use the density distribution: 1) One is to calculate the rms emittance by ruling out the outermost halo particles. One

just needs to set a threshold on density to rule out sparse the irregular halo particles, and takes the usual statistical method to obtain the rms emittance and the C-S parameters. This part is similar to the two methods discussed above. 2) The other is to calculate the C-S invariant boundary and the C-S parameters for any given beam fraction. Or one calculates the beam fraction for a given C-S invariant boundary. For the latter, by setting a density and a narrow window, one can calculate the fit ellipse for the selected particles and the beam fraction with all the particles within the ellipse. In this way, one can obtain the beam fraction and the C-S parameters for the beam core, the beam halo or any given C-S invariant boundary. One needs to repeat the above procedures again by replacing the initially-set C-S parameters with the calculated ones so that the orientation of the small ellipse for calculating the C-S invariant density can be better presented.

With the C-S invariant density screening method one can describe the emittance and the C-S parameters for different beam fractions more accurately, and it can be applied to both simulated and measured data. Figure 1 shows the procedures in applying the C-S invariant density screening method.

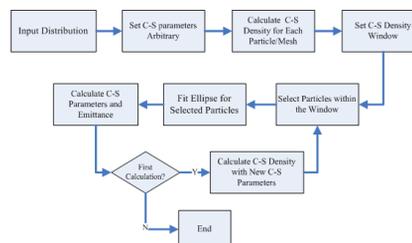


Figure 1: Procedures of the C-S invariant density screening method.

PRACTICAL APPLICATIONS OF THE C-S INVARIANT DENSITY SCREENING METHOD

For a Simulated Beam Distribution

A simulated beam with a normal beam core and a halo part is used for the study of the effectiveness of different emittance evaluation methods. It comes from the study of the scattering effect of the proton beam crossing through a PBW (Proton Beam Window) in the RTBT beam transport line at CSNS. Figure 2 shows the calculated 95% emittance ellipse of the simulated beam by using the rms ellipse zooming method and the C-S invariant density screening method. The parameters of the ellipses from the two methods are compared in Table 1. One can find that there is a large difference between the two methods in evaluating the C-S parameters when the distribution is irregular. With the C-S invariant density method, one can define the phase ellipse shape much more accurately than the rms zooming method.

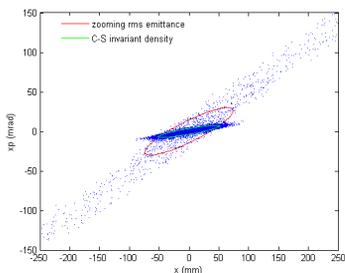


Figure 2: Comparison of the 95% emittance ellipses between the two method.

Table 1: Parameters of the two 95% emittance ellipse

Methods	Emittance ($\pi \cdot \text{mm} \cdot \text{mrad}$)	Alpha	Beta (m)
RMS ellipse zooming	1075.4	-1.82	5.10
C-S density screening	191.5	-1.86	14.15

For a Measured Beam Distribution

A set of measured emittance data from ISIS, Rutherford Appleton Lab (RAL) is also used to evaluate the C-S invariant density method. These data was obtained by applying a double-slit emittance measurement system to a 65 keV H- beam extracted from a PIG H- ion source. Figure 3 (left) shows the contours of the raw data in the vertical phase plane. Figure 3 (right) shows the C-S invariant density contours for the same data. One can find that the two contours are different, as the C-S invariant density contour shows more hiberarchy. This is because the density at each mesh point is calculated with the intensities in the neighboring meshes defined by a given C-S invariant ellipse of having the C-S parameters fit to the distribution.

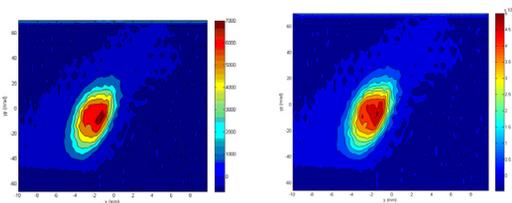


Figure 3: (left) Contour plot in the vertical phase plane of the measured emittance data for a beam extracted from PIG H- ion source in ISIS, RAL; (right) C-S invariant density contour plot for the same data as shown in Figure 3(left).

By applying the threshold method, the SCUBEE method and the C-S invariant density screening method to the same data, one can make the comparison among them. Figure 4 shows the emittance as a function of beam fraction by applying the three methods. The dashed and dotted lines are obtained by zooming the rms emittance ellipse, and the solid line is obtained by the data fitting with different C-S invariant densities. One can find that

the three methods give almost the same result for the beam core and quite different ones for the halo part that is with larger beam fraction than 0.85.

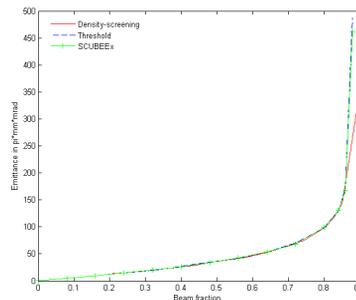


Figure 4: Emittance as a function of beam fraction with the three methods.

Figure 5 shows the C-S parameters calculated by the C-S invariant density screening method. It shows clearly that the C-S parameters change from the beam core to the beam halo, while they are supposed to be constant in the other two methods.

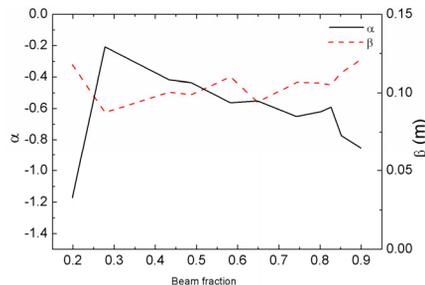


Figure 5: C-S parameters evaluated by applying C-S invariant density screening method.

CONCLUSIONS

A new emittance analysis method “C-S invariant density screening” has been developed. The method can be used for both simulated data and measured data. The comparisons show that it has significant advantages over usually used emittance analysis methods based the rms emittance calculation, especially when irregular distributions are concerned.

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