USE OF A GRID WAVEGUIDE FOR PARTICLE ENERGY DETERMINATION*

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Abstract

We consider prospects of use of a grid waveguide for determination of energy of charge particles in bunches. The method under consideration is based on measurement of a waveguide mode frequencies. Earlier we developed two variants of this method. One of them is based on use of a thin dielectric layer in a waveguide. Other variant is based on use of a waveguide loading with a system of wires coated with a dielectric material. In this paper we offer new version of this method. It consists in application of a circular waveguide having a grid wall. Analytical solution of the problem is obtained by means of the averaged boundary conditions. It is shown that propagating mode can be excited under certain conditions, and this mode is single. The main advantage of the method consists in enough strong dependency of particle energy on the mode frequency in wide range of energy. Note as well that this structure can be used for generation of a monochromatic radiation possessing frequency depending on the bunch velocity.

INTRODUCTION

Cherenkov radiation is widely used for detection of charged particles [1]. A new method of determination of the charged particles energy was developed in papers [2–9]. It is based on the measurements of frequencies of modes generated by a bunch moving in the waveguide. For this method, it is important to provide a strong dependence of modes frequencies ω_m on Lorenz-factor $\gamma = \left(1-\beta^2\right)^{-1/2}$ of the charged particles.

For this goal the ray of structure was offered. For example, it can be usual dielectric layer in the circular waveguide [2]. However the dependency of the mode frequencies on γ is enough weak, especially in the ultra-relativistic case $\gamma >> 1$. Obviously, this peculiarity is typical for all detectors based on Cherenkov effect. However it is possible to overcome partly this imperfection.

Earlier we offer two ways to achieve this goal. One of them is based on use of a thin dielectric layer in a waveguide [8]. Decrease of the layer thickness results in increase of dependency of ω_m on γ and in some extension of the Lorenz-factor range which is suitable for measurement. However, it should be noted that this effect is not very large.

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Other variant is based on use of a waveguide loading with some metamaterial [2–9]. One of them is a system of wires coated with a dielectric or magnetic layer [9]. It was shown that, in the waveguide with such filling, the bunch generates both the ordinary modes and additional (“anomalous”) modes for some narrow range of γ. Frequencies of these modes strongly decrease with increase of γ. The strong dependence $\omega_m(\gamma)$ takes place for relatively low frequencies, and the mode amplitudes of radiation of typical bunch are enough large for measurement. Thus, these modes give essential advantages for measurement of particles energy in comparison with modes in waveguide with ordinary material. However, this effect takes place for enough small values of γ (as a rule $\gamma \sim 1 \div 10$).

Now we offer a new structure that allows measurement of γ in the ultra relativistic case. This structure is easy for manufacture and can be embedded in the accelerator channel without big difficulty.

ANALYTICAL RESULTS

Let us consider a circular waveguide which has a grid wall with rectangular cells (the z axis coincides with a waveguide axis). The grid has period d_z for “z-wires” parallel to the z axis and period d_ϕ for “ϕ-wires” orthogonal to the z axis. Here we consider the case of bunch that moves along the waveguide axis with velocity $\vec{V} = c\vec{\beta}$. The thickness of the bunch is assumed to be negligible.

The grid wires are perfect, and the following conditions are assumed to be fulfilled:

$$r_0 << d_{z,\phi} << \Lambda \equiv \min(a, c/\omega) . \quad (1)$$

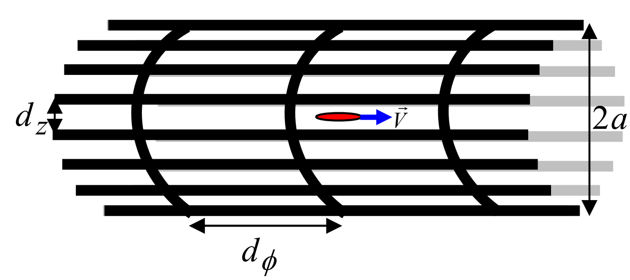

Figure1: Geometry of the problem.

Owing to these inequalities we can use method of the averaged boundary conditions (ABC) [10]. In the case under consideration (where only TM-field is generated), the ABC has the form

$$E_{\omega z}\Big|_{r=\pm a} = -\frac{i\omega d_z}{2\pi c}\ln\left(\frac{d_z}{2\pi r_0}\right)\times$$
$$\times\left(1+\frac{c^2}{\omega^2\delta}\frac{\partial^2}{\partial z^2}\right)\left(H_{\omega\phi}\Big|_{r=a+0} - H_{\omega\phi}\Big|_{r=a-0}\right), \quad (2)$$

where $\delta = (1 + d_\phi / d_z + \kappa)/(d_\phi / d_z + \kappa)$. Parameter κ is charged with contact between conductors in points of their intersections. In case of perfect contact $\kappa = 0$, and $\delta = (d_z + d_\phi)/d_\phi$, i.e. $\delta = 1$ for grid from z-wires only ($d_\phi = \infty$), $\delta = 2$ for grid with square cells ($d_z = d_\phi$), and $\delta \approx d_z / d_\phi >> 1$ in the case where $d_\phi << d_z$.

We give the analytical solution for wave field (so-called "wakefield") in the case of Gaussian bunch moving along the waveguide axis. The transverse dimension of the bunch is negligible, and longitudinal distribution of the charge is determined by the function $\exp\left(-\zeta^2/(2\sigma^2)\right)$, where $\zeta = z - Vt$ and σ is much less than the typical wavelength. The non-zero components of wave field (existing only behind the charge) are

$$E_r^W = \frac{4q\gamma}{a^2}\frac{k_0 a}{W(k_0 a)}\exp\left(-\frac{\omega^2\sigma^2}{2V^2}\right)\sin\left(\frac{\omega_0\zeta}{V}\right)\times$$
$$\times\begin{Bmatrix} K_0^2(k_0 a) I_1(k_0 r) \text{ for } r < a \\ -K_0(k_0 a) I_0(k_0 a) K_1(k_0 r) \text{ for } r > a \end{Bmatrix},$$
$$E_z^W = \frac{4q}{a^2}\frac{k_0 a}{W(k_0 a)}\exp\left(-\frac{\omega^2\sigma^2}{2V^2}\right)\cos\left(\frac{\omega_0\zeta}{V}\right)\times \quad (3)$$
$$\times\begin{Bmatrix} K_0^2(k_0 a) I_0(k_0 r) \text{ for } r < a \\ K_0(k_0 a) I_0(k_0 a) K_0(k_0 r) \text{ for } r > a \end{Bmatrix},$$
$$B_\phi^W = \beta E_r^W,$$

where

$$W(x) = I_1(x)K_0(x) - I_0(x)K_1(x),$$

$k_0 = \omega_0 V^{-1}\gamma^{-1}$, $\gamma = (1-\beta^2)^{-1/2}$, $\beta = V/c$, $\zeta = z - Vt$.

Here k_0 is a real positive solution of the following dispersion equation:

$$I_0(ka)K_0(ka) = \chi, \quad (4)$$

where $\chi = \dfrac{\delta\beta^2 - 1}{\delta}\dfrac{d_z}{2\pi a}\ln\left(\dfrac{d_z}{2\pi r_0}\right)$.

This equation can have only a single real root $k = k_0$. This root exists under condition $\chi > 0$, that is $\beta > \delta^{-1/2}$. Thus, propagating mode can be generated only in the case $\delta > 1$. It means that generation of radiation (wakefield) is possible only in the case of grid possessing both z-wires and ϕ-wires. This fact can be seemed wonderful because in the problem under consideration only current in z-wires are excited. However the distribution of the charge on the grid depends on the orthogonal ϕ-wires too. This circumstance is reflected in the ABC (2) owing to presence of parameter $\delta \neq 1$.

It can be obtained some simple approximations for the propagating mode frequency in two particular case:

$$\omega_0 \approx V\gamma/(2a\chi) \text{ for } \chi << 1 \text{ (not very large } \gamma), \quad (5)$$

$$\omega_0 \approx \frac{2c}{a}\sqrt{\gamma^2 - 1}\, e^{-C-\chi} \text{ for } \chi > 1 \ (\gamma >> 1), \quad (6)$$

where $C = 0.577...$ is Euler constant. According to (6) the dependency of ω_0 on γ for large values of γ can be enough strong due to factor $\exp(-\chi)$ strongly decreasing

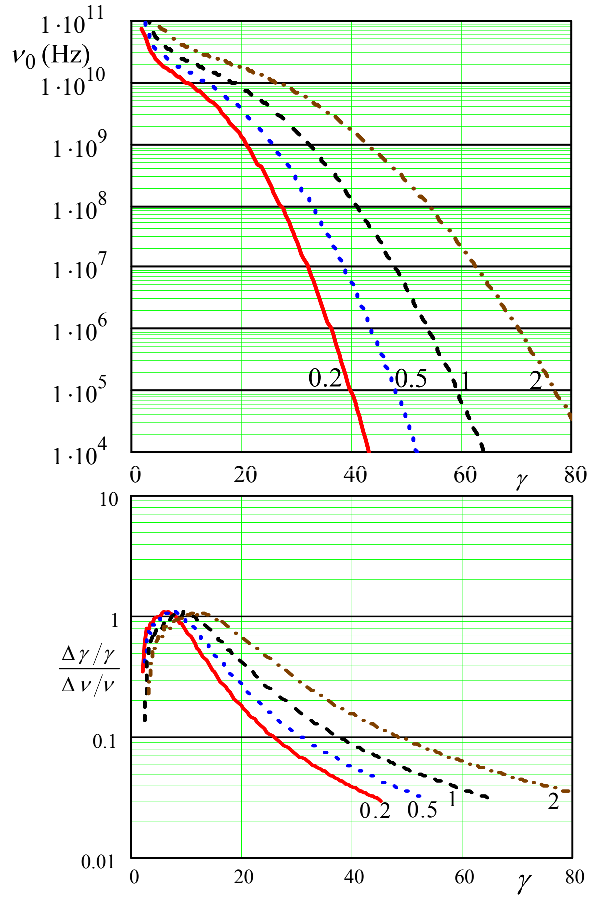

Figure 2: The mode frequency (top) and the ratio between relative accuracies of γ and frequency (bottom) depending on γ; $a = 3\,\mathrm{cm}$, $r_0 = 0.5\,\mathrm{mm}$, $d_z = 2\pi a / 38 \approx 5\,\mathrm{mm}$; magnitudes of d_ϕ (cm) are given close to the curves.

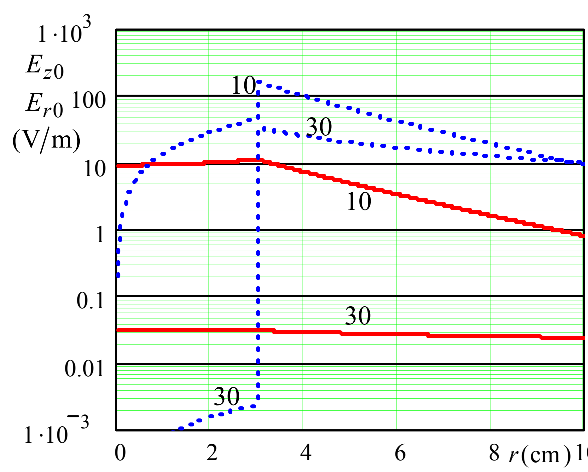

Figure 3: Dependencies of amplitudes of components

E_z^W (solid red) and E_r^W (dotted blue) on distance from waveguide axis; $q = 1\text{pC}$, $\sigma = 3\text{mm}$, $a = 3\,\text{cm}$, $d_z = d_\phi = 2\pi a / 38 \approx 5 = \text{mm}$, $r_0 = 0.5\,\text{mm}$, magnitudes of γ are indicated near the curves.

with increase in γ. This circumstance is especially attractive for determination of particle energy.

NUMERICAL RESULTS AND DISCUSSION

Figures 2–5 show some results of computation of wave field. One can see that dependency of the mode frequency on Lorentz factor is essential for enough large range of magnitudes of γ (Fig.2). For waveguide with radius of the order of several centimeters this range can be several decades. Relative accuracy of determination of γ (i.e. $\Delta\gamma/\gamma$), as a rule, is less than relative accuracy of measurement of frequency $\Delta\nu/\nu$, but this values has the same order in some range of γ (see Fig.2).

The radial component of electric field on the outward surface of waveguide is the most convenient for measurement because it is much more than longitudinal component (Fig.3). Dependency of the radial component on γ is shown in Fig.4. One can see that its values for bunch with the charge of 1 pC is enough large for measurements.

Figure 5 shows typical wakefield. Note that this computation was performed with account of finite conductivity of wires. Consequently, exponentially decrease of wakefield with increase of $|\zeta|$ occurs.

Note that the structure under consideration can be examined as generator of monochromatic microwave radiation too. It is important that radiation has single propagating mode in our model where the averaged boundary conditions are used. Really, radiation has some spectrum of shorter waves with wavelengths $\le d_\phi$. However these waves can be filtered out, and we will obtain almost monochromatic radiation with frequency ω_0 that can be varied with variation of the bunch velocity.

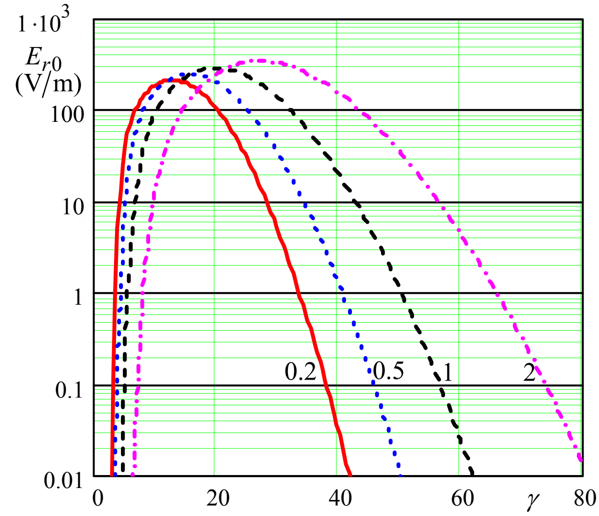

Figure 4: Amplitude of the component E_r^W on the outward surface of waveguide ($r = a + 0$) depending on γ; magnitudes of d_ϕ (cm) are indicated near the curves, other parameters are the same as in Fig 3.

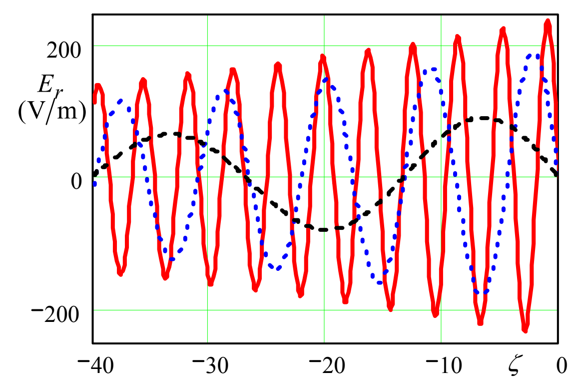

Figure 5: Component E_r^W on the outward surface of waveguide depending on the distance $\zeta = z - Vt$ for $\gamma = 15$ (solid red), $\gamma = 20$ (dotted blue), $\gamma = 25$ (dashed black). Conductivity of wires is $5 \cdot 10^7 (\text{om}\,\text{m})^{-1}$ (copper), other parameters are the same as in Fig 3.

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