BEAM PROFILES ANALYSIS FOR BEAM DIAGNOSTIC APPLICATIONS

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Abstract

Beam profile and its analysis play an important role in beam diagnostics of a particle accelerator system. Use of destructive screen monitor or non-destructive synchrotron radiation monitor for beam profile measurement is a simple way and has been widely used in synchrotron light source facility. Analyze beam profiles can obtain beam parameters including beam center, sigma, and tilt angle which has become a useful tool for beam diagnostic. In this report the comparison of fitting strategies affect the analysis results are studied. The computer simulated beam profiles with different background noise level and conditions are used to evaluate the computing time, and the estimated fitting errors.

INTRODUCTION

The two-dimensional (2D) images recorded by cameras are widely used in synchrotron light source facility due to the ability to provide extensive information on beam parameters, including beam center, sigma, and beam tilt. A beam profile monitor is a device which can converts the beam flux density as a function of position into a measurable signal. The instrumentation is equipped with a screen which placed in the beam's path and observation by using a charge-coupled device (CCD) camera. Thanks to inexpensive CCD cameras and availability of digital computer technology, the process of obtaining, storing, and analyzing of 2D images has become easier and faster. The images of beam as recorded by cameras are most conveniently represented by 2D circular or elliptical Gaussian distributions of light intensity. In this report the comparison of fitting strategies, including lsqcurvefit, polyfit, and moment, affect the analysis results are studied. The computer simulated Gaussian distribution beam profiles with different background noise level and conditions (zero-mean or nonzero-mean) are used to evaluate the computing time, and the estimated fitting errors.

ANALYSIS METHODS

The algorithms employed split into two categories: those that directly solve for the parameters, and those that employ iterative or non-iterative optimization to find their best results. Three of the methods compared here – the lsqcurvefit method, the polyfit method, and the moment method. The moment method fall into the first category which is a computationally simple method to directly calculation the image parameters. The polyfit is a non-iterative method which can be used to solve linear problem with faster and accurate analysis. The lsqcurvefit method is an iterative method which may require more computing time, but it can against high signal-to-noise (S/N) levels. The detail method described is shown as follows:

Least-Squares Curve Fitting Method

The least-squares curve fit, so called lsqcurvefit method [1] is one of the optimization toolbox in Matlab which can solve nonlinear curve-fitting problems in the least-squares sense that minimized the sum of squared differences between the measured and predicted data. Lsqcurvefit is an iterative method which returns results that minimizes the residuals when the tolerances supplied are satisfied. That is, given input data *xdata*, and the observed output *ydata*, find coefficients *x* that "best-fit" the equation *F(x,xdata):*

$$\min_{x} \frac{1}{2} \sum_{i} \left(F\left(x, xdata_i\right) - ydata_i \right)^2 \qquad (1)$$

,where *xdata* and *ydata* are vectors and *F(x,xdata)* is a vector valued function. This method can be used to deal with high signal-to-noise levels image, but it takes a long computing time due to it is an iterative method. Fortunately, the maximum number of function evaluations (MaxFunEvals, default 500) and iterations (MaxIter, default 400) allowed, and termination tolerance (TolX, default 1e-6) can be configured to satisfy operational requirements.

Polynomial Curve Fitting Method

The polynomial curve fit, so called polyfit method [2] in Matlab function which can find the coefficients, p, of a polynomial function, *P*(x), of degree, n, that fits the data. The results p is a row vector of length n+1 containing the polynomial coefficients in descending powers, as expressed by:

$$P\left(x\right) = p_1 x^n + p_2 x^{n-1} + \cdots + p_n x + p_{n+1} \qquad (2)$$

In the beam profile analysis case, data transformed into a form is necessary. Typically, the projected beam profile is a Gaussian distribution, by use of the natural log transformation to convert this non-linear relationship curve into a form that is amenable to polynomial curve fitting. Gaussian peak has the fundamental functional form $\exp(-x^2)$, into a parabola of the form $-x^2$, which can be fit with a second order polynomial (quadratic) function ($y = ax^2 + bx + c$). The equation for a one-dimensional beam Gaussian distribution profile is shown as following:

$$y = A e^{-\frac{(x-mu)^2}{2s^2}} \qquad (3)$$

,where A is the peak height, mu is the x-axis location of the peak maximum, and s is the sigma of the peak. All three parameters of the curve (A, mu, s) can be calculated from the three quadratic coefficients a, b, and c; the peak height (A), peak position (mu), and sigma (s) are given by

$$A = e^{\left(c-\frac{b^2}{4a}\right)}, \quad mu = -\frac{b}{2a}, \quad s = \sqrt{-\frac{1}{2a}} \qquad (4)$$

Moment Method

For some applications such as image tracking, the computational efficiency is critical. The moment method in this kind of applications is common to use to analyze large data sets and computationally quicker but sacrifice precision. Image moments provide useful summaries of global image information. The moments involve sums over all pixels (M_{00}), and so are robust against small pixel value changes. If $I(x, y)$ is the image intensity at position x, y, then the image moments, up to second order, are shown as following:

$$\begin{aligned} M_{00} &= \sum_x\sum_y I(x,y), \quad M_{11} = \sum_x\sum_y xyI(x,y) \\ M_{10} &= \sum_x\sum_y xI(x,y), \quad M_{01} = \sum_x\sum_y yI(x,y) \\ M_{20} &= \sum_x\sum_y x^2I(x,y), \quad M_{02} = \sum_x\sum_y y^2I(x,y) \end{aligned} \qquad (5)$$

The position, x_c, y_c can be calculated as following:

$$x_C = \frac{M_{10}}{M_{00}}, \qquad y_C = \frac{M_{01}}{M_{00}} \qquad (6)$$

Define the intermediate variables a, b, and c, as following:

$$a = \frac{M_{20}}{M_{00}} - x_c^2, \quad b = 2\left(\frac{M_{11}}{M_{00}} - x_c y_c\right), \quad c = 2\left(\frac{M_{02}}{M_{00}} - y_c^2\right) \quad (7)$$

Then the orientation θ, and sigma, x_s, y_s can be calculated as following:

$$\begin{aligned} \theta &= \frac{\arctan\left(b,(a-c)\right)}{2} \\ x_s &= \sqrt{\frac{(a+c)+\sqrt{b^2+(a-c)^2}}{2}} \\ y_s &= \sqrt{\frac{(a+c)-\sqrt{b^2+(a-c)^2}}{2}} \end{aligned} \qquad (8)$$

However, moments become very noise-sensitive with increasing order. This method can yield perfect results only under ideal noise-free conditions. In the presence of noise it has a very low tolerance for non-zero mean noise [3].

SIMULATION MODEL

The algorithm was developed and tested in Matlab environment (R2008b) at a laptop computer. The specifications of computer in the experiment are as follows: Intel Core i5 450m 2.4GHz CPU, 4.0GB RAM. The simulated beam profile images model can be generated by a general two-dimensional elliptical Gaussian function, as expressed by

$$f(x,y) = Ae^{-\left(a(x-x_0)^2+2b(x-x_0)(y-y_0)+c(y-y_0)^2\right)} \qquad (9)$$

Here the coefficient A is the amplitude, x_0 ,y_0 are the center of x, y and a, b, c are defined as following:

$$\begin{aligned} a &= \frac{\cos^2\theta}{2s_x^2} + \frac{\sin^2\theta}{2s_y^2} \\ b &= -\frac{\sin 2\theta}{4s_x^2} + \frac{\sin 2\theta}{4s_y^2} \\ c &= \frac{\sin^2\theta}{2s_x^2} + \frac{\cos^2\theta}{2s_y^2} \end{aligned} \qquad (10)$$

,where theta (θ) is the rotate angle and s_x, s_y are the sigma of x and y. In the study, a circular Gaussian profile is generated (theta = 0). The parameters of the function are define as: A is 1; s_x, s_y are 30; and x_0,y_0 are 300.

In order to represent analysis results effect with background noise, a 2D Gaussian profile was generated with two kinds of noises, one is the zero-mean Gaussian background noise produced by normally distributed random function, and the other is the absolute value of above noise to produce non-zero mean background noise. The computation time is counting base on tic/toc command from Matlab, a tic command to start stopwatch timer and measure the time required by toc command for each analysis.

RESULTS AND DISCUSSION

The simulated beam profile parameters are circular Gaussian distribution with sigma 30 and center 300. Three fitting methods are used and compared its analyzed results including sigma, center and computing time. The results shown that all used methods can easily yield perfect results under ideal beam profile with noise-free conditions and with computing time 24.8 ms (lsqcurvefit), 1.8 ms (polyfit), and 14.9 ms (moment). The Table 1 compares the analysis results effect with difference zero-mean Gaussian background noise. It shown that difference noise levels for all three methods in center analysis is effect slightly. From the accurate point of view, the lsqcurvefit method is more robust, but with longer computing time. The polyfit method due to its non-iterative character, it can solve the problem faster than other methods within acceptable error. The moment method has larger error when noise level was increased.

Table 1: Analysis Results Effect with Zero-Mean Gaussian Background Noise

S/N	Lsqcurvefit	Polyfit	Moment
Sigma (Error %)			
20:1	29.90 (-0.33)	30.98 (3.27)	30.78 (2.60)
10:1	29.82 (-0.60)	33.72 (12.4)	39.28 (30.9)
Center (Error %)			
20:1	299.9 (-0.03)	299.8 (-0.07)	298.8 (-0.40)
10:1	299.7 (-0.10)	298.7 (-0.43)	302.7 (0.90)
Computing Time (ms)			
20:1	26.1	1.6	14.7
10:1	27.0	1.7	13.3

However, zero-mean noise models are often unrealistic, especially when dealing with non-negative image functions. Thus, the analysis results effects with nonzero-mean background noise are studied and results as shown in Table 2. The moment method has shown high noise-sensitive, the error of sigma is significant. The others two method (lsqcurvefit and polyfit) still have good results in this kind of background noise condition.

Table 2: Analysis Results Effect with Nonzero-Mean Background Noise

S/N	Lsqcurvefit	Polyfit [a]	Moment
Sigma (Error %)			
20:1	30 (0)	30.3 (1.0)	Failure [b]
10:1	30 (0)	30.9 (3.0)	Failure [b]
Center (Error %)			
20:1	300 (0)	300 (0)	326.3 (8.77)
10:1	300 (0)	299.8 (-0.07)	327.9 (9.30)
Computing Time (ms)			
20:1	24.6	1.75	14.0
10:1	28.5	2.05	13.1

[a] Due to polyfit method can't deal with the curve that mixed with non-linear (Gaussian) and linear (Offset) components, thus, it is necessary to subtract the baseline from the curve before processing.
[b] Error is more than 200% from the actual value.

BACKGROUND NOISE

The important factor when using statistical methods such as moment analysis (the results shown above) is the amount of background noise. The simple method is presented here that can be used to deal with this problem. This method is to cut off any signal below a certain threshold, the cutting level must be determined manually. While being very efficient to implement, however this approach is of limited use if the noise level varies too much. For the signal-to-noise ratio of 10:1, by cutting the 30% of amplitude can reduce the sigma and center error in a certain degree, as shown in Fig. 3. If cutting level is too much, the analysis results will be affected, especially the sigma of the curve will be reduced significantly.

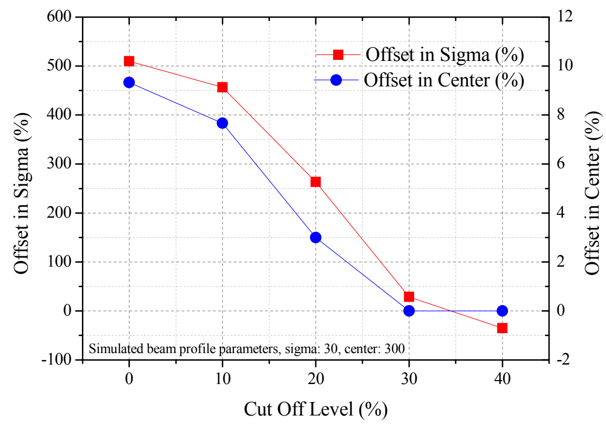

Figure 1: Cutting Level Affects under Nonzero-Mean Background Noise (10%) for Moment Method.

SUMMARY

The three methods compared different features of lsqcurvefit, polyfit, and moment methods. The iterative method, lsqcurvefit, is a stable analytic method which will yield the results that are accurate to within its convergence tolerances. In this report, although, the polyfit method has faster computing time, it can't deal with the Gaussian distributions with an offset condition, due to the mixed system with non-linear and linear components. Moment analysis gives a large calculation error in the present of noise, but after making simple cutting process, it can achieve better results in permissible error range.

REFERENCES

[1] http://www.mathworks.com/help/toolbox/optim/ug/lsqcurvefit.html
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[3] Matthias Gruber and Ken-Yuh Hsu, Moment-Based Image Normalization With High Noise-Tolerance, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 19, no. 2, 1997, pp 136-138.