

# DESIGN OPTIMIZATION FOR A NON-PLANAR UNDULATOR FOR THE JETI-LASER WAKEFIELD ACCELERATOR IN JENA

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## Abstract

In a laser wakefield accelerator (LWFA), excited by a femtosecond laser pulse electrons are accelerated to several 100 MeV within a few centimetres. The energy spread of the electron beam, however is relatively large. In order to obtain monochromatic photons in an undulator despite the energy spread, the following idea was proposed. Two bending magnets and a drift space in between produce dispersion so that particles with different energies have different transverse positions. The beam enters a non-planar undulator, e.g. cylindrical pole geometry, where the K-value also varies with transverse position. If the two variations in the transverse direction (particle energy and K-value) compensate each other the generated light is more monochromatic than with a conventional planar undulator.

In this paper such a modified undulator design optimized for the JETI-LWFA in Jena is presented. An experiment to test this concept is in preparation.

## INTRODUCTION

The radiation produced in an undulator is very intense and concentrated in narrow energy bands in the spectrum, which consists of a fundamental frequency and higher harmonics. The photon wavelength of the n-th harmonic follows the undulator equation:

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} \right) \quad (1)$$

where  $\lambda_u$  is the period length of the undulator,  $\gamma$  is known as the relativistic Lorentz factor ( $E/0.511\text{MeV}$ ) and  $K$  is the undulator parameter given by:

$$K = \frac{B_0 e \lambda_u}{m_0 c 2\pi} \approx 93.36 B_0 [\text{T}] \lambda_u [\text{m}] \quad (2)$$

with  $B_0$  as the maximum flux density in the beam plane,  $c$  the speed of light,  $e$  the electric charge and  $m_0$  the rest mass of the electron.

A large energy spread  $\sigma_\gamma/\gamma \sim 1...10\%$  as generated by laser wakefield accelerators spoils the monochromaticity and brilliance of the undulator radiation. In order to compensate this effect and to generate monochromatic undulator radiation the idea was proposed to disperse different electron energies spatially  $\gamma \rightarrow \gamma(x)$  (see Fig. 1) and match the varying  $\gamma$  by a spatial variation of the magnetic flux

density  $B_0 \rightarrow B_y(x)$  [1]. For technical reasons  $\lambda_u$  was chosen to be constant.

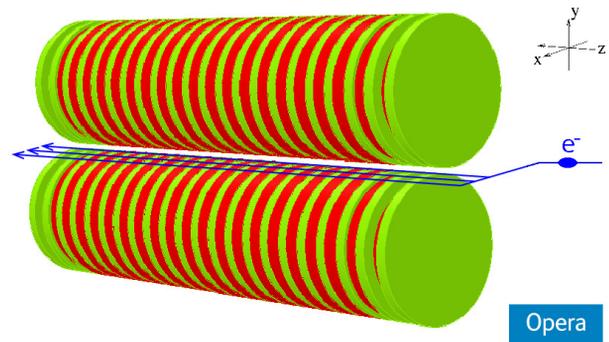


Figure 1: The lines represent trajectories for different electron energies.

Equating these two modifications in the equation (1) it is possible to generate radiation with a constant wavelength:

$$\lambda_n = \frac{\lambda_u}{2n\gamma(x)^2} \left( 1 + \frac{(93.36 B_0(x) [\text{T}] \lambda_u [\text{m}])^2}{2} \right) \quad (3)$$

## Energy Separation

The dispersion of the energy ( $d\Delta x$ ) can be achieved by a dogleg chicane. A complete design for this chicane satisfying all experimental requirements and for the boundary conditions discussed below is described in [2]. For the calculations in this paper a simplified model of the chicane consisting of two dipoles was used (see Fig. 2).

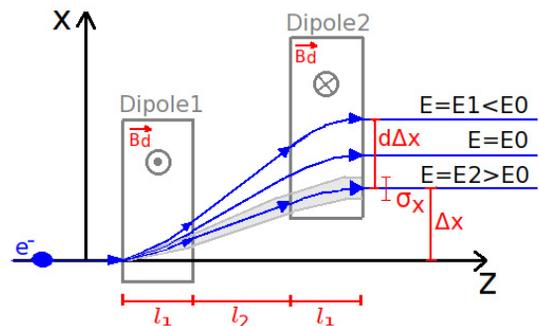


Figure 2: Schematic view of the chicane.

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The bending radius in the dipoles is proportional to  $\gamma$  and is given by:

$$r_L(x) = \frac{m_0 c \gamma(x)}{e B_d} \quad (4)$$

where  $B_d$  is the homogeneous flux density in the dipoles.

After the second magnet the trajectories are displaced by:

$$\Delta x = 2r_L - 2\sqrt{r_L^2 - l_1^2} + \frac{l_1 l_2}{\sqrt{r_L^2 - l_1^2}} \quad (5)$$

with  $l_1$  the width of the dipole and  $l_2$  the length between the two dipoles.

### DESIGN OPTIMIZATION

Two possible non-planar undulators designs are considered for an experimental test of the concept at the JETI wakefield accelerator at the university of Jena, Germany, a cylindrical and a tilted planar design (see Fig. 3 and Fig. 4 respectively).

Both designs were optimized for an electron energy of 120MeV assuming an energy deviation of  $\pm 10\%$ . We further assume the undulators to be superconducting with a period length of  $\lambda_u = 10\text{mm}$  and a number of  $N_u = 100$  periods. The optimization goal is to achieve a monochromaticity of the undulator radiation in the order of the natural bandwidth  $\Delta\lambda/\lambda = 1/N_u = 1\%$ .

#### Analytic Field Models

In a cylindrical undulator (see Fig. 3) the field gradient along the x-axis is produced by the curvature of the undulator coils.

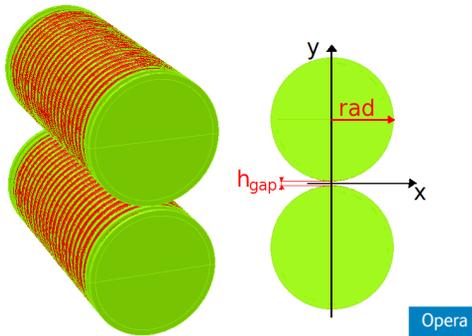


Figure 3: Schematic view of the cylindrical undulator.

In a tilted undulator (see Fig. 4) the two undulator coils are tilted relative to the middle plane.

Both models were optimized using idealized analytic expressions for the flux density [3] allowing for an efficient calculation of  $B_y(x)$ . Finite element (FE) simulations were employed to determine technically feasible parameter ranges. The dependence of  $\Delta\lambda/\lambda$  on all geometric parameters of the undulator models and the drift length  $l_2$  was investigated ( $l_1 = 50\text{mm}$  and  $B_d = 0.46\text{T}$  are predetermined by the already existing dipole magnets). Since the

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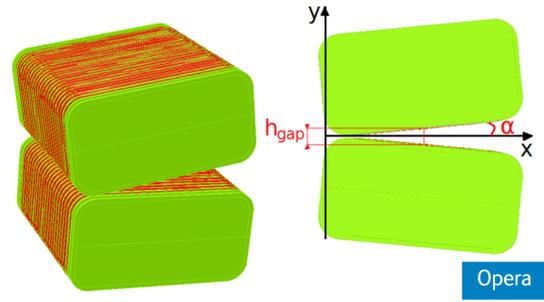


Figure 4: Schematic view of the tilted undulator.

geometric parameters turned out to be highly correlated, a technically reasonable choice was made for a subset of them, reducing the degrees of freedom to:

- $l_2, B_0$  in case of the cylindrical undulator.
- $l_2, \alpha$  in case of the tilted undulator.

The flux density and the wavelength for both undulators are plotted in Fig. 5.

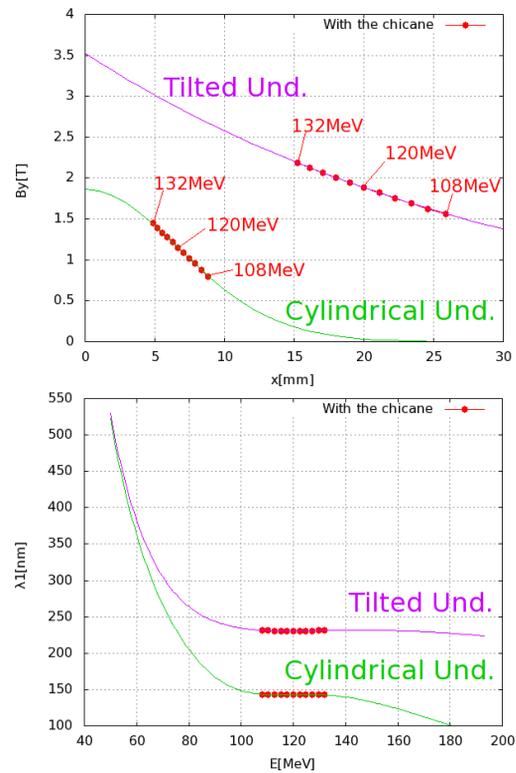


Figure 5: Comparison of calculated and required flux densities and resulting wavelengths for both geometries.

In a non-planar undulator a finite width of the electron beam and therefore of the beamlets for each energy obviously also affects the bandwidth of the radiation. Therefore after the optimization the maximum beamlet width consistent with the optimization goal ( $\sigma_x$ ) was determined.

The assumed parameters, the optimized parameters calculated with Simplex-Method [4] and a summary of the results are listed in the Table 1.

Table 1: Summary of Parameters and Results

<b>Electron beam parameters</b>		
Electron energy $E_0$ [MeV]	120	
Energy spread $\Delta E/E_0$ [%]	$\pm 10$	
<b>Fixed geometric parameters</b>		
Cyl.Und.	Tilted.Und.	
Period length $\lambda_u$ [mm]	10	10
Radius $rad$ [mm]	30	-
Gap height $h_{gap}$ [mm]	1.5	2
<b>Optimized parameters for E0</b>		
Cyl.Und.	Tilted.Und.	
Max. flux density $B_0$ [T]	1.87	-
Angle $\alpha$ [degrees]	-	2.86
Length in Chicane $l_2$ [mm]	290	864
<b>Results for E0</b>		
Cyl.Und.	Tilted.Und.	
Dispersion $d\Delta x$ [mm]	3.96	10.66
Wavelength $\lambda_1$ [nm]	143	230
$\frac{\Delta\lambda}{\lambda}$ [%]	0.5714	0.4009
$\sigma_x$ [mm]	0.094	0.234

### Comparison with FE-Simulations

The optimization results for both geometries were checked with FE-Simulations. For this purpose a model of both undulators was designed with the software Opera-3D and the magnetic flux density was analyzed in a central period background. The models are shown in Fig. 6.

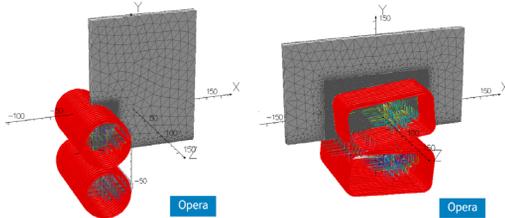


Figure 6: FE-Simulations in Opera-3D.

The comparison of the x-dependence of the flux density amplitude between the analytic and the FE-models are plotted in the Fig. 7. It can be seen that the agreement is very good within the error margin of the numerical method for the cylindrical model. The agreement for large values of x for the tilted undulator is also good. The apparent deviation at small x values comes from the fact, that the analytic model was infinite in x with the pole planes crossing in the origin, while the FE model was finite, with realistic coils that terminate before touching.

### CONCLUSION

In this paper two designs of non-planar undulators optimized for the JETI-LWFA were investigated. The designs

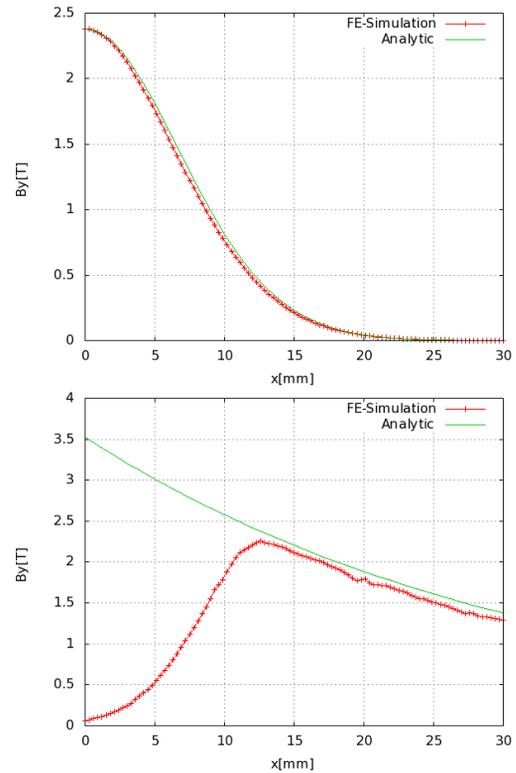


Figure 7: Comparison of  $B_y(x)$  for FE-simulations and analytic field models.

were compared analytically and by using FE-calculations (Opera-3D). Both designs decrease the energy spread of the emitted radiation. But with the tilted undulator geometry and technically realistic parameters, the required dispersion of the beam becomes too large. However, with the cylindrical geometry all design criteria are met or even exceeded. Therefore the cylindrical geometry is chosen for the actual experiment. A detailed design is now underway and fabrication is planned for next year.

### ACKNOWLEDGMENTS

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