PECULIARITIES OF THE EXCITATION OF AN OPTICAL RESONATOR BY AN ELECTRON BEAM*

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Abstract

The peculiarities of excitation an optical resonator installed in a storage ring by the electrons emitting undulator radiation wavelets (URWs) in an undulator or Self-Stimulated Undulator Klystron (SSUK) are investigated.

EXCITATION OF AN OPTICAL RESONATOR BY THE ELECTRON BEAM

In this paper we investigate the evolution of the electromagnetic undulator radiation (UR) fields excited by the electrons in an open resonator of a quasi-isochronous storage ring at the main and collateral synchronicity conditions [1].

The electric field strength of the circularly polarized Gaussian beam in an open resonator for the fundamental transverse TEM_{00} mode in paraxial approximation is given by [3], [4]

$$\vec{E}(x,y,z,t) = \vec{E}_{0,\lambda}\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w^2(z)}\right)\times$$
$$\times[\vec{e}_x\cos\psi + \vec{e}_y\sin\psi + \vec{e}_z\frac{r/k}{w_0 w(z)}\cdot\cos\psi] \qquad (1)$$

where $\psi = \omega_\lambda t - k_\lambda z - kr^2/R(z) + \varsigma(z)$, $r = x^2 + y^2$ is the radial distance measured from the central axis of the optical beam, z is the axial distance from the beam's waist (see Fig.1), $k_\lambda = \omega_\lambda / c = 2\pi/\lambda$ is the wave number, λ is the wavelength, $E_{0,\lambda} = E_\lambda |_{r=z=0}$, $w(z) = w_0\sqrt{1+z^2/Z_R^2}$ is the radius at which the field amplitude and intensity drops to $1/e$ and $1/e^2$ of theirs axial values, respectively, $w_0 = w(0)$ is the waist size, $Z_R = \pi w_0^2/\lambda$ is the Rayleigh length, $R(z) = z[1 + Z_R^2/z^2]$ is the radius of curvature of the wave-front (which coincides with the curvature of mirror at its location), $\varsigma(z) = arctg(z/Z_R)$. For our purposes the longitudinal component of field could be omitted, however.

The corresponding time-averaged intensity associated with the optical beam (1) is

$$I_\lambda(r,z) = I_{0,\lambda}\left(\frac{w_0}{w(z)}\right)^2\exp\left(-\frac{2r^2}{w^2(z)}\right), \qquad (2)$$

where $I_{0,\lambda} = I_\lambda(0,0)$ is an intensity at the center of beam at its waist, z=0, Fig.1.

First, let us consider excitation of TEM_{00} mode in the resonator by single electron periodically crossing an undulator or SSUK included in the optical resonator to find the electron beam energy and electric field strength in the URW after its first passage through the resonator. We accept that the URWs are emitted in a steady-state regime under the main synchronicity condition (when emitted URWs are overlapped in the optical resonator).

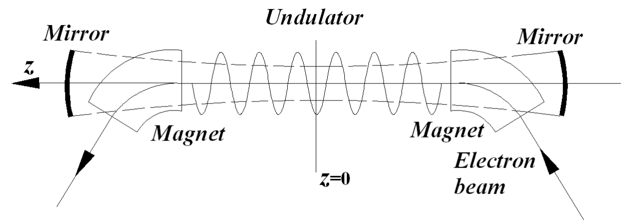

Figure 1. To the excitation of optical resonator.

Let the first harmonic is excited in a helical undulator. In this case the circularly polarized electric field strength of the emitted URW has the form (1) with the amplitude $E_0 = E|_{r=z=0}$, the stored energy of the URW is $\varepsilon_{URW} = \int_V (E^2/4\pi)dV = E_0^2 w_0^2 M\lambda_1/8$, where $M\lambda_1$ is the length of the URW, λ_1 is the wavelength of the emitted undulator radiation (UR) at the first harmonic, M is the number of periods in undulator.

For a single electron, $\vec{J}(\vec{r},t) = e\vec{v}_e(t)\delta[\vec{r} - \vec{r}_e(t)]$, where $\vec{r}_e(t)$, $\vec{v}_e$ (t) are the electron radius-vector and its velocity respectively. In this case the energy emitted by the electron in the URW per single pass is $\Delta\varepsilon_e = \int_0^T\int_V \vec{J}(\vec{r},t)\cdot$ $\vec{E}_{URW}(\vec{r},t)dVdt = e\int_0^T \vec{v}_e(\vec{r}_e,t)\vec{E}_{URW}(\vec{r}_e,t)dt = e\beta_{e,\perp}E_0 M\lambda_u$.

The damping of the URW energy for a single revolution is

$\Delta\varepsilon_{URW} = \varepsilon_{URW}[1 - \exp(-\delta T_{URW})] \simeq \pi w_0^2 M\lambda_1 E_0^2/4F_\lambda$,

where δ is a damping decrement of the URW, $1 - \exp(-\delta T_{URW}) \simeq \delta T_{URW} \simeq 2\pi/F_\lambda$, F_λ is the resonator finesse related to the mode frequency ω_λ, T_{URW} is the period of the URW circulation in the optical resonator.

In the steady-state regime the energy loss by the URW per single turn is equal to the energy gain of the electron in the fields of undulator and the URW.

From the energy balance $\Delta\varepsilon_e = \Delta\varepsilon_{URW}$, the accepted condition $Z_R = M\lambda_u/2$ or $w_0^2 = M\lambda_u\lambda_1/2\pi$ and in a case of high finesse $F_\lambda/2\pi \gg 1$ the electric field strength $E_0 = 8e\beta_\perp F_\lambda/M\lambda_1^2$ and the energy of the URW $\varepsilon_{URW} = 8e^2K_\perp^2F_\lambda^2/\pi\lambda_1(1+K^2)$. From here it follows that the amplitude of the URW electric field strength, the

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energy of the URW and the number of photons emitted in the unexcited resonator by the electron in the resonator mode for a single pass are the following

$$\Delta E_0 = 2\pi E_0 / F_\lambda = 16\pi e\beta_\perp / M\lambda_1^2 ,$$
$$\Delta\varepsilon_{URW,1} = 32\pi e^2 K_\perp^2 / \lambda_1 (1+K^2)$$
$$N_\gamma = 16\alpha K^2 / (1+K^2) , \qquad (3)$$

where $\alpha = e^2/\hbar c = 1/137$.

Non-synchronous condition of excitation of the resonator $\Delta\, l \neq 0$ can be investigated by analogy with excitation of resonators by the periodic electron bunches in the parametric FELs [2]. In this case behavior of the energy variation of the URW in the optical resonator is similar to the energy dependence of an oscillator excited by an external force.

Note that in many applications the beam intensity is determined by the rms beam size $\sigma(z)$ with the form

$$I_\lambda(r,z) = I_{0,\lambda}\left(\frac{\sigma_0}{\sigma(z)}\right)^2 \exp\left(-\frac{r^2}{2\sigma^2(z)}\right). \qquad (4)$$

In this case, according to (2) and (4), $w(z) = 4\sigma(z)$ and $Z_R = 4\pi\sigma^2(z)/\lambda_1$.

The electric field strengths of an URWs emitted by a particle *j* in a storage ring during its pass through the undulator can be descried by the expression $E_j(t) = E_0\sin[\omega_1(t-t_j)]$ in the time interval $0 < t - t_j < MT_1$, and $E_j(t) = 0$ outside of the interval, where the moment $t_j = (j-1)T_e$ corresponds to the arrival of the j-*th* URW to the observation point, $T_1 = \lambda_1 / c$ is the period of the UR wave, T_e is the period of the electron revolution.

The total electric field strength of the sum of URW emitted by a particle on its *n*-th pass through the undulator and URWs emitted by the particle on its previous passages through the undulator and reflected by mirrors of the optical resonator in the time interval $0 < t - t_n < MT_1 - n\Delta t$ at $n < n_M$ has the form

$$E(t, n<n_M) = \sum_{j=1}^{n<n_M} E_j(t) \text{ or}$$

$$E(t, n<n_M) = E_0 \sum_{i=1}^{n<n_M} \sin[\omega_1(t-(i-1)\Delta t)]\, e^{-(n-i)\beta} , \quad (5)$$

where we took into account the damping of the URWs with the decrement $\beta = \delta T_{URW} = 2\pi/F_\lambda$ determined by the mirror reflectivity $r = 1 - 2\pi/F_\lambda$, $\Delta t = T_e - T_{URW}$ is the slip time between the URWs emitted by the electron in the undulator and circulating in the optical resonator, $n_M = MT_1/\Delta t$ is the number of revolutions corresponding to the partial overlapping of the emitted URWs (in general case n_M is not whole number).

The expression (5) at $0 < t - t_n < MT_1 - n\Delta t$, $n < n_M$ is brought to the geometric progression

$$E(t, n<n_M) = E_0 \,\mathrm{Im}\{e^{i\omega_1 t}\sum_{j=1}^{n} e^{-(i\omega_1\Delta t+\beta)(n-1)}\} =$$
$$= E_0\,\mathrm{Im}\{e^{i\omega_1 t}\frac{1-e^{-(i\omega_1\Delta t+\beta)n}}{1-e^{-(i\omega_1\Delta t+\beta)}}\} ,$$

which can be presented in the form

$$E(t, n<n_M) = E_0 a_n \sin(\omega_1 t) + E_0 b_n \cos(\omega_1 t)$$

or

$$E(t, n<n_M) = E_n \sin(\omega_1 t + \xi_n) , \qquad (6)$$

where $E_n = E_0 A_n$, $A_n = \sqrt{a_n^2 + b_n^2}$,

$$a_n = \frac{1 - e^{-\beta}\cos\alpha - e^{-\beta n}\cos(\alpha n) + e^{-\beta(n+1)}\cos[\alpha(n-1)]}{1 - 2e^{-\beta}\cos\alpha + e^{-2\beta}},$$

$$b_n = \frac{e^{-\beta}\sin\alpha - e^{-\beta n}\sin(\alpha n) + e^{-\beta(n+1)}\sin[\alpha(n-1)]}{1 - 2e^{-\beta}\cos\alpha + e^{-2\beta}},$$

$\xi_n = \arccos a_n / A_n$, $\alpha = \omega_1\Delta t$.

According to (6) the value $A_1 = 1$.

If $\alpha = 0$ then

$b_n = \xi_n = 0$, $A_n = a_n = (1 - e^{-\beta n})/(1 - e^{-\beta})$,

$A_n|_{\beta\to 0} \to n$, $A_n|_{\beta n >> 1} = 1/(1-e^{-\beta})|_{\beta\to 0} = 1/\beta$.

If $\alpha \neq 0$, $\beta << 1$, $n < n_M$ then the amplitude of the URW E_n is the quasi-periodic function of the revolution number *n* with the frequency α (period $n_T = T_1/\Delta t$) and with negative-going amplitude for the damping time $\tau \simeq T_1/\beta$.

At $n < n_c >> n_\tau$, $n_\tau = \tau/T_1 = 1/\beta$ the amplitude of URW E_n tends to the steady state regime:

$$a_n = \frac{1 - e^{-\beta}\cos\alpha}{1 - 2e^{-\beta}\cos a + e^{-2\beta}}, \quad b_n = \frac{e^{-\beta}\sin\alpha}{1 - 2e^{-\beta}\cos a + e^{-2\beta}},$$

$$A(n) = \frac{1}{\sqrt{1 - 2e^{-\beta}\cos\alpha + e^{-2\beta}}}, \qquad \xi_n =$$
$$\arccos\frac{1 - e^{-\beta}\cos\alpha}{\sqrt{1 - 2e^{-\beta}\cos a + e^{-2\beta}}} .$$

In this specific case at $\alpha \ll 1$, $\beta \ll 1$ the amplitude

$$A(n) \simeq \frac{1}{\sqrt{\alpha^2 + \beta^2}} , \ \xi_n \simeq \arccos\frac{\beta}{\sqrt{\alpha^2+\beta^2}} \qquad (7)$$

A transient behavior of the amplitude A_n and the relative power ($P_n = A_n^2$) for the time interval τ is presented on the Fig. 2. At this interval the amplitude and the emitted power can be much higher then theirs steady state averages. That is why it will be useful to extract periodically the energy of the optical beam in the optical resonator with period $T_{extr} \simeq \tau$ or to change the phase of the stored radiation in the URWs to π for one revolution of the beam in the ring (overload conditions).

The total electric field strength of URWs emitted by a particle on its $n > n_M$ pass of the undulator and its previous passages is equal to $E(t, n > n_M) = E(t,n) -$

$E(t - MT_1, n)$, where $E(t)$ is determined by (5) but under conditions of n is the arbitrary number (without the restriction $n < n_M$). In this case the summation of the URWs is carried out in the limits from $n - n_M$ to n. The steady state regime occurs at $n > n_M$.

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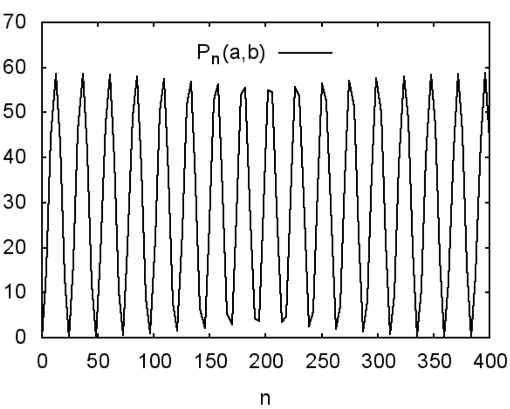

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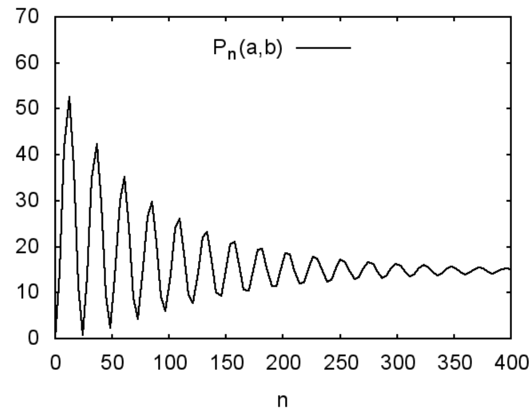

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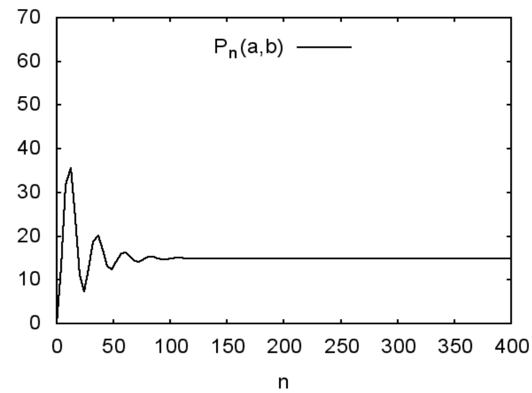

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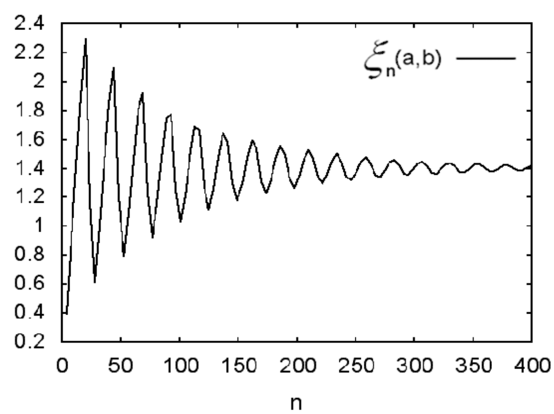

(d)

Figure 2: Time dependence of the SSUR power emitted by a particle at $\alpha = \pi/12$, $\beta = 0$ (a); $\alpha = \pi/12$, $\beta = 0.01$ (b); $\alpha = \pi/12$, $\beta = 0.05$ (c), $\alpha = \pi/12$, $\beta = 0.01$ (d), $n_c >> n_\tau$.