LOW-BETA EMPIRICAL MODELS USED IN ONLINE MODELING AND HIGH LEVEL APPLICATIONS*

Y. Chao, G. Goh, TRIUMF
Vancouver, BC V6T 2A3 Canada

Abstract

Online physics model plays a central role in accelerator control system high level applications (HLA). A coherent and comprehensive development effort has taken place at SNS in the past few years leading to XAL [1], a framework fully integrated with the EPICS control system on which database driven accelerator models can be constructed and evaluated in real time as a platform for HLA development, execution and testing. The XAL physics model is a self-contained module undergoing constant feature enhancements. In this report work is described of establishing an infrastructure for online model fully compatible with XAL protocols, but aiming to avoid difficulties encountered by analytical accelerator modelling at low energies through an empirical approach.

OVERVIEW

TRIUMF is embarking on the Advanced Rare Isotope Laboratory (ARIEL) project, with an Injector complex including a thermionic electron gun, a 10 MeV injector cryomodule (ICM) and elements necessary for controlling and measuring beam properties, such as buncher cavity, solenoids, spectrometer and diagnostic/control devices. The ability to correctly and efficiently model the transport at low energy is critical to its success. Efficiency and correctness are however not always convergent objectives when it comes to low energy (low-β) modelling. One can emphasize correctness using stand-alone tools not easily integrated within the control system, or one can use faster analytic algorithms online to calculate effects at low β, risking mis-representing underlying physics. We attempt to bridge this gap with an online empirical model. The XAL platform, comprehensive and integrated with readily available building blocks, provides a natural infrastructure for developing HLA based on this empirical model. The same principle is applicable to other HLA platforms.

* Funding is received from the National Research Council of Canada.
#chao@triumf.ca

PROBLEMS WITH LOW-β MODELING BY ANALYTICAL MODELS

Analytical models in low-β beam transport fail to satisfactorily address the following effects with closed-form transfer functions over a wide range of phase space:

- Velocity changes within cavities
- Non-relativistic dynamics and arrival-time dependence
- Significant longitudinal-transverse momentum transfer
- Higher order aberrations in solenoids and in cavities
- Strong chromaticity due to large momentum spread
- Elements inside fringe fields of other elements
- Need to model space charge by dividing up elements

More issues not due to inadequate modelling ability, but logistics, also plague the transfer matrix approach to analytical models: They can become convoluted when all 6 phase space coordinates are needed to describe the element; Inadequate means to handle time-varying effect at off-design phase. It is also cumbersome to describe effects due to temporal distribution in a beam using transfer matrices defined between two spatial coordinates.

EMIPIRICAL MODEL

Problems cited above are no concern, of course, for a tracking program such as Astra, where raw EM fields are used to empirically track particles without abstracting them into analytical transfer functions, and the results are rigorous over a considerable area of the phase space. This suggests the possibility to capture pre-calculated tracking results in the form of interpolatable and polynomial-expandable data, to be used as an efficient online model, capable of giving results more rigorous than analytical models over a considerable range of working beam and hardware parameters. The goal is not to replace either tracking programs or analytical models (especially at high-β), but to bridge the gap with an efficient, easily integrated, and more accurate solution.

Table 1: Model Usage by High Level Applications

	Single Particle (Centroid) Diagnostics/Control	Phase Space Diagnostics/Control	Beam Propagation – without Space Charge	Beam Propagation – with Space Charge
Needed/Affected Elements	• Optical elements • Steering/BPM	• Optical elements • Phase space monitors	• Optical elements • Phase space /orbit monitors	• Optical elements • Phase space monitors
Model Method Used	• Transfer matrix concatenation	• Transfer matrix concatenation	• Propagate beam distribution	• Propagate beam distribution
Design Parameters Used as Reference	• Mostly	• Mostly	• No	• No
Segmented Units	• Needed for fringe fields	• Needed for fringe fields	• Inspection inside elements	• Compounding space charge
Dimensions	• Transverse or Longitudinal	• Transverse or Longitudinal	• Full 6D or any subspace	• Full 6D or any subspace
Optimization Methods	• Linear & analytic methods • Empirical methods	• Linear & analytic methods • Empirical methods	• Linear & analytic methods • Empirical methods	• Empirical optimization methods only
Examples	• Misalignment analysis • Orbit correction	• Phase space measurement • Betatron matching	• General simulation of beam propagation and experiments	• Full-effect simulation of experimental procedures

Table 2: Coordinate Convention of Empirical Model

Index	1	2	3	4	5	6
Coord.	δX (m)	δP_X (MeV/C)	δY (m)	δP_Y (MeV/C)	T_{SLIP} (ns)	P_Z (MeV/C)

Table 3: Components of Empirical Model and Implementation inside XAL Framework

Properties of:	Element transport	Design machine state	Actual machine state	Specific input beam
Data	Empirical interpolation table	• Hardware (B_{DES}, Φ_{DES}) • Beam (ΔT_{DES}, P_{DES} …)	Hardware setting (amplitude, phase….)	• Instantaneous (α, β, P) • Cumulative (T_{SLIP})
Implementation	Element attribute	Element attribute	EPICS Process Variable	Probe

Application in Different Schemes

It is useful to define the scope of HLA usage of the empirical model. Table 1 lists all schemes in which an online model can be used by HLA's. One criterion of particular interest is whether the HLA is based only on transfer matrix (including high order coefficients), or requires detail in beam distribution (with or without space charge). The former (deterministic) approach uses single-particle, usually on-design transport to derive analytical relations between measured/predicted responses and actuator effects, and obtains answer by analytical optimization techniques. The latter (dynamic) approach relies on empirical construction of the same relations, including model effects not easily reducible via algebraic methods, and effects due to ad hoc distribution of beam coordinates or deviation from design, and obtains answer by empirical, heuristic optimization techniques. The above distinction is critical in determining the type of empirical model appropriate for a particular HLA. In principle one can adopt the empirical approach in all cases, whereas if suitable, the deterministic approach is more efficient, tractable and offers more insight.

Time-Based Formalism

The scheme needs to account for effects introduced by time varying elements, inevitably demanding a system to track elapsed time as the beam travels from one point to the next. Examination of available options led to the adoption of a time-based coordinate system that is versatile and conceptually straightforward. Table 2 lists the coordinates used. The first 4 are conventional coordinates relative to the design particle, whereas absolute momentum makes up the 6^{th}. The 5^{th} coordinate, superseding the conventional one, is the cumulative elapsed time relative to design at any given point. This fulfils the need to keep track of elapsed time for time-varying elements. At each such element this is translated into a quantity pertinent to the element, such as relative phase lag.

Space charge simulation in XAL is done by interspersing sub-divided transport with space charge matrices based on equivalent uniform distribution [2]. Conversion algorithm was implemented to preserve this formulation in the time based representation.

Implementation inside XAL Framework

Use of empirical model in a time based formalism marks departure from standard XAL protocols and, in order to take advantage of infrastructure provided by the latter, modification and augmentation are needed. Table 3 shows how and where important ingredients of the empirical model are implemented. Machine description in XAL is accomplished through database definition or explicitly constructed XML files with machine layout and hardware attributes. Hardware design parameters are defined at this level. In the empirical model, both design-independent interpolation tables and design phase Φ_{DES} and transit time (ΔT_{DES}) are part of hardware attributes in XML files, which can thus have unique correspondence to particular machine design or tuning states. The all-important 7^{th} coordinate of cumulative elapsed time is most naturally carried inside the beam probe.

Interpolation Table

Interpolation tables lie at the heart of the empirical model. They are constructed by extracting transport properties calculated by a proven tracking program covering extensive areas of phase space over a wide range of hardware parameters. This information can predict the transport of beam centroids to a very high degree of accuracy regardless of the presence of space charge, and can approximate incoherent space charge effects by interleaving phenomenological point-like space charge transfer matrices with subdivided empirical transports. The table contains complete information on transport properties across well defined end points as functions of multi-dimensional variables. The independent variables include the following: phase Φ (e.g., RF), other settable parameters of the element such as B field, and incoming momentum P. Dependence on transverse coordinates is expressed in polynomial expansion to arbitrary order, including 0^{th}, as individual terms in the interpolation table. The different treatment of dependencies on longitudinal and transverse coordinates stems from the observation that there is no advantage in expanding the former into polynomials for lack of usable symmetry. Direct interpolation often better captures their ad hoc combined behavior. Extracted transport properties include the following: total transit time ΔT, total exit momentum P, and relative transverse coordinates (δX, δP_X, δY, δP_Y), all to arbitrary order of the transverse coordinates. It should be noted that the time coordinate has been made interchangeable with phase, as this is the only place where time information of a particle is relevant. A particle carries with it the cumulative time lag relative to design, and at each element entrance this is converted to phase as the interpolation variable. The cumulative time lag is updated at the exit through interpolating ΔT as function of relevant variables. Table 4 shows a concept interpolation table for a cavity, with a 3D basis of interpolation spanned by columns of phase Φ, input momentum P and amplitude E. The rows consist of transport coefficients represented by shorthand notations in the left column, followed by their respective physical meanings.

Interpolation Table for Different Applications

Depending on the type of application, different interpolation tables are used. Of those in Table 1, single particle, linear transfer matrix based HLA's benefit more from tables derived from small <u>transverse</u> amplitude, near design transport, while phase space propagation requires higher order, large amplitude scans of both beam and machine parameters. Different tables are created for these applications. An additional subtlety arises regarding the phase reference for subdivided cavities. In single particle (deterministic) applications the phase through each subsegment is completely determined from the initial parameters, thus initial phase alone can serve as interpolation variable for all subsequent segments. In (dynamic) applications where spontaneous effects can arise between segments, such as due to space charge, the interpolation must be computed dynamically based on the non-deterministic phase entering each segment separately.

VALIDATION OF MODEL

A test of the empirical model constructed by the above principles is shown in Figure 1, where a realistic 1500 particle distribution is run through a 3.5 m, 300keV-10 MeV beam line described by the model with 2 solenoids and 2 cavities. The results compare well against Astra, and much better than an analytic model can conceivably do over this energy range. The empirical model with elements embedded in fringe fields was used to build a demo application for simulated solenoid alignment (Figure 2), which proved to perform correctly at 300 keV.

Table 4: Interpolation Table Structure for a Cavity

Phase	Φ_1	Φ_1	Φ_1	Φ_1	Φ_2	Φ_2	Φ_2	Φ_2
P	P_1	P_1	P_2	P_2	P_1	P_1	P_2	P_2
Amp.	E_1	E_2	E_1	E_2	E_1	E_2	E_1	E_2
7/0000	Constant total transit time ΔT							
6/0000	Constant exit momentum P							
7/1100	$\partial^2(\Delta T)/\partial(\delta X)\partial(\delta P_X)$: ΔT dependence on X, P_X							
6/0011	$\partial^2(P)/\partial(\delta Y)\partial(\delta P_Y)$: P dependence on Y, P_Y							
1/1000	$\partial(\delta X)/\partial(\delta X)$: First order transverse transfer coefficient							
2/1110	$\partial^3(\delta P_X)/\partial(\delta X)\partial(\delta P_X)\partial(\delta Y)$: 3rd order transverse coefficient							
4/2210	$\partial^5(\delta P_Y)/\partial^2(\delta X)\partial^2(\delta P_X)\partial(\delta Y)$: 5th order transverse coefficient							

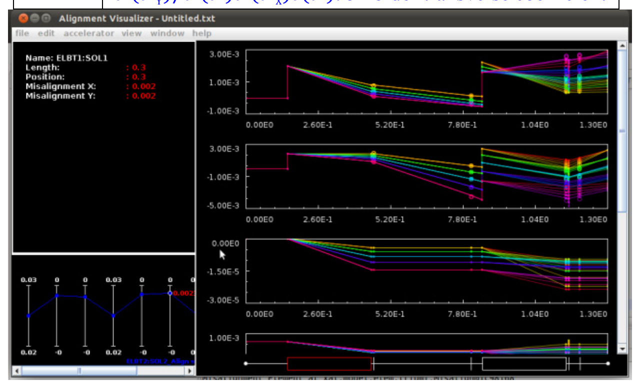

Figure 2: Empirical model based solenoid alignment HLA

ACKNOWLEDGMENTS

We appreciate efforts by Paul Chu and Jane Richards in helping bring XAL online at TRIUMF, and generous knowledge sharing and consulting from Paul Chu.

REFERENCES

[1] http://www.ornl.gov/~t6p/Main/XAL.html
[2] C. Allen and N. Pattengale, Theory and Technique of Beam Envelope Simulation, LANL LA-UR-02-4979.

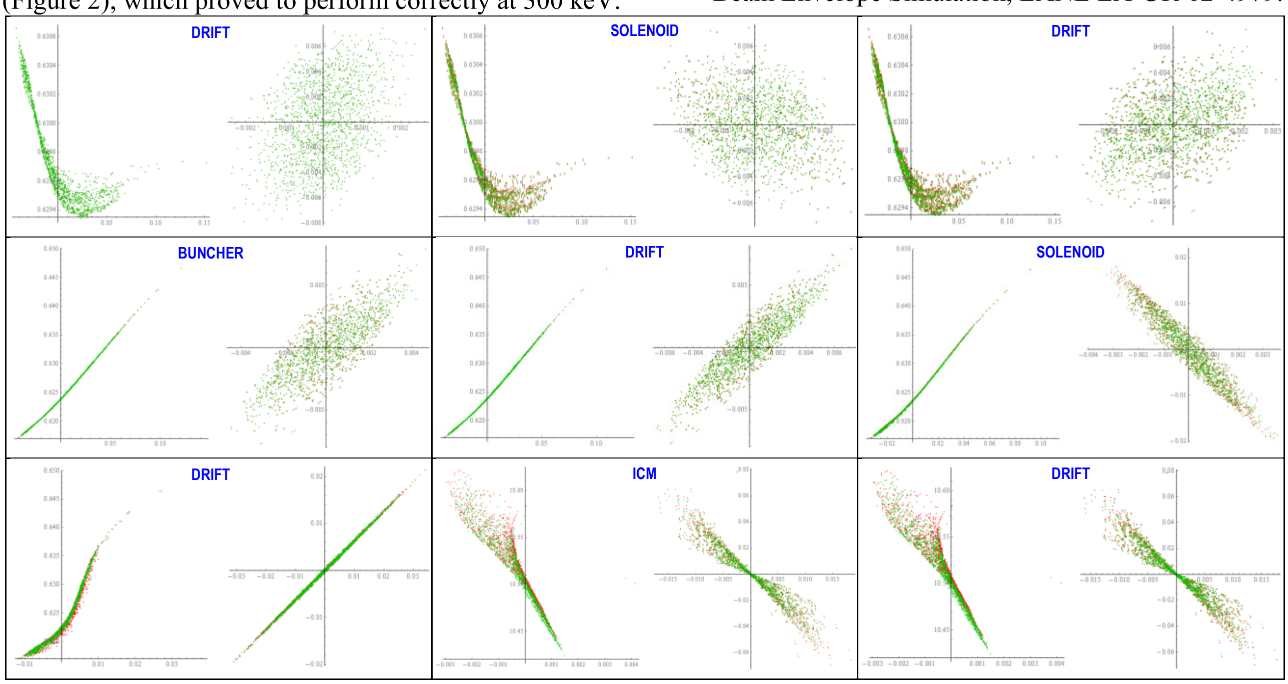

Figure 1: Beam propagation by Empirical Model (green) & Astra (red) from 300 keV to 10 MeV through two solenoids, one buncher and one ICM cavity. All plots: <u>Left</u>: P_Z (MeV/C) vs ΔT (ns); <u>Right</u>: $\delta P_{X/Y}$ (MeV/C) vs $\delta X/Y$ (m).