MEASUREMENT OF COUPLING RESONANCE DRIVING TERMS IN THE LHC WITH AC DIPOLES*

R. Miyamoto†, R. Calaga, BNL, Upton, NY, USA
M. Aiba, PSI, Villigen, Switzerland
R. Tomás, G. Vanbavinckhove, CERN, Geneva, Switzerland

Abstract

Transverse betatron coupling in the LHC is measured from Fourier analysis of turn-by-turn beam oscillations excited by AC dipoles. The use of the AC dipole for optics measurements induces a small systematic error which can be corrected with an appropriate data interpretation. An algorithm to apply this correction to the measurement of the coupling resonance driving terms is developed and successfully applied to the LHC.

INTRODUCTION

An AC dipole excites a coherent beam motion in a ring like a kicker magnet for optics measurements [1]. It produces a resonance near the betatron frequency and drives the beam. Unlike kickers, the AC dipole can produce a sustained coherent motion with almost no emittance growth [1] and this made it the primary exciter of the LHC [2, 3]. Turn-by-turn position data of the excited motion from beam position monitors (BPMs) allows prompt measurements of optics parameters. Coupling resonance driving terms (CRDT) due to skew quadrupole fields can be also determined from corresponding spectral components of the turn-by-turn position [4].

As other transverse resonance sources, the AC dipole produces a pair (sum and difference) of resonances [5] and this may require a careful data interpretation to extract optics parameters from measurements using the AC dipole [6, 7, 8, 9]. This applies to not only 1D parameters but also coupling [6, 7, 8] and nonlinear terms [6, 10]. The first algorithm to remove this systematic effect for 1D parameters and coupling was proposed in [7] and successfully tested in the SPS for the measurement of the β-beating and modulus of the coupling coefficient [8]. It has been demonstrated that, for 1D case, the systematics effect of the AC dipole is equivalent to an additional quadrupole at its location and the motion can be well parametrized by introducing a new set of optics parameters [9]. This idea of using a new set of optics parameters is extended to the motion under the influence of skew quadrupole fields and AC dipoles and, based on this, a method to measure CRDTs without the systematic effect of the AC dipole is proposed [11]. Given the LHC has BPMs right next to each AC dipole, this method allows to measure the amplitudes and phases of CRDTs with a single excitation frequency in horizontal and vertical AC dipoles. This paper reviews the method and presents its successful applications to the LHC.

* This work partially supported by the US Department of Energy through the US LHC Accelerator Research Program (LARP).
† miyamoto@bnl.gov

THEORY

In the following, we use the normalized complex coordinates defined with β and α functions: $\tilde{z} \equiv z + i(\alpha^z z + \beta^z z')$ where z stands for either x or y. If positions at two adjacent BPMs, located at s_1 and s_2, are known, $\tilde{z}$ at s_1 is constructed as [12]

$$\tilde{z}_{s_1} = \frac{1}{i \sin \psi^z_{s_2,s_1}} \left(e^{i\psi^z_{s_2,s_1}} z_{s_1} - \sqrt{\frac{\beta^z_{s_1}}{\beta^z_{s_2}}} z_{s_2} \right), \quad (1)$$

where $\psi^z_{s_2,s_1}$ is phase advance from s_1 to s_2. Please note that this assumes the lattice between s_1 and s_2 contains only small errors.

1D Driven Motion

When driven by an AC dipole, turn-by-turn position on n-th turn at the location s can be expressed as [9, 11]

$$\tilde{x}_{s,n} = A^h \sqrt{\beta^h_s} e^{-2\pi i \nu^h n - i\psi^h_{s,s_h} - i\chi^h}, \quad (2)$$

where χ_h and s_h are the phase and location of the AC dipole, A^h, ν^h, β^h_s and ψ^h_{s,s_h} are the constant amplitude, tune, β-function, and phase advance of the driven motion. The β-functions and phase advances of the free and driven motions are related by

$$\beta^x_s = \frac{1 + (\lambda^h)^2 + 2\lambda^h \cos(2\Psi^h_{s,s_h})}{1 - (\lambda^h)^2} \beta^h_s \quad (3)$$

$$\tan \Psi^x_{s,s_h} = \frac{1 - \lambda^h}{1 + \lambda^h} \tan \Psi^h_{s,s_h}, \quad (4)$$

where $\delta^h = \nu^h - \nu^x$ and $\lambda^h = \sin(\pi\delta^h)/\sin[\pi(\nu^h + \nu^x)]$ are small parameters, typically $\delta^h \lesssim 0.01$ in the LHC, and $\Psi^x_{s_2,s_1}$ and $\Psi^h_{s_2,s_1}$ are shorthand notations of

$$\Psi^x_{s_2,s_1} = \psi^x_{s_2,s_1} - \pi\nu^x \quad (5)$$

$$\Psi^h_{s_2,s_1} = \psi^h_{s_2,s_1} - \pi\nu^h. \quad (6)$$

Equation (2) indicates that the β-function and phase advance of the driven motion are direct observables in this case [9]. The LHC has BPMs right next to each AC dipole and this allows us determine Ψ^h_{s,s_h} and hence β^x_s and ψ^x_{s,s_h} as well with Eqs (3) and (4), from a single excitation of the AC dipole. The above discussion also applies to the motion driven by a vertical AC dipole. We use a superscript v for parameters of the vertical driven motion.

Coupled Driven Motion

At the first order, turn-by-turn position of the motion driven by skew quadrupole fields is given by [4]

$$\begin{aligned}\tilde{x}^1_{s,n} &= 2iA^y f^-_s \sqrt{\beta^x_s}\, e^{-2\pi i\nu^y n - i\psi^y_{s,s_0} - i\phi^y} \\ &\quad + 2iA^y f^+_s \sqrt{\beta^x_s}\, e^{2\pi i\nu^y n + i\psi^y_{s,s_0} + i\phi^y} \qquad (7)\end{aligned}$$

$$\begin{aligned}\tilde{y}^1_{s,n} &= 2iA^x (f^-_s)^* \sqrt{\beta^y_s}\, e^{-2\pi i\nu^x n - i\psi^x_{s,s_0} - i\phi^x} \\ &\quad + 2iA^x f^+_s \sqrt{\beta^y_s}\, e^{2\pi i\nu^x n + i\psi^x_{s,s_0} + i\phi^x}\,, \qquad (8)\end{aligned}$$

where A^x, A^y, ϕ^x and ϕ^y are the constant amplitudes and constant phases of the main betatron motions, s_0 is the location of the reference BPM and f^-_s (f^+_s) is the CRDT of the difference (sum) resonance:

$$f^\mp_s = \frac{\sum_j \kappa_{s_j}\sqrt{\beta^x_{s_j}\beta^y_{s_j}}\, e^{-i(\Psi^x_{s,s_j} \mp \Psi^y_{s,s_j})}}{8i\sin[\pi(\nu^x \mp \nu^y)]}\,. \qquad (9)$$

In Eq (9), the summation runs over the skew quadrupole fields and κ_{s_j} and s_j are the effective strength and location of the j-th field. CRDTs, $f^\mp_s$, can be determined at each BPM location from the corresponding spectral components of the normalized complex position [4, 11, 13].

When the main motion is the driven motion excited with horizontal and vertical AC dipoles, Eqs (7) and (8) are modified to [11]

$$\begin{aligned}\tilde{x}^1_{s,n} &= 2iA^v f^{-,v,h}_s \sqrt{\beta^h_s}\, e^{-2\pi i\nu^v n - i\psi^v_{s,s_v} - i\chi^v} \\ &\quad + 2iA^v f^{+,v,h}_s \sqrt{\beta^h_s}\, e^{2\pi i\nu^v n + i\psi^v_{s,s_v} + i\chi^v} \qquad (10)\end{aligned}$$

$$\begin{aligned}\tilde{y}^1_{s,n} &= 2iA^h (f^{-,h,v}_s)^* \sqrt{\beta^v_s}\, e^{-2\pi i\nu^h n - i\psi^h_{s,s_h} - i\chi^h} \\ &\quad + 2iA^h f^{+,h,v}_s \sqrt{\beta^v_s}\, e^{2\pi i\nu^h n + i\psi^h_{s,s_h} + i\chi^h} \qquad (11)\end{aligned}$$

and the direct observables become $f^{\mp,h,v}_s$ and $f^{\mp,v,h}_s$. A relation between $f^\mp_s$ and $f^{\mp,h,v}_s$ is given by

$$\begin{aligned}f^\mp_s = \frac{1}{\sqrt{1-(\lambda^h)^2}}\frac{\sin[\pi(\nu^h \mp \nu^y)]}{\sin[\pi(\nu^x \mp \nu^y)]}\Big[& e^{i(\psi^h_{s,s_h} - \psi^x_{s,s_h})} f^{\mp,h}_s \\ & - \lambda^h e^{-i(\psi^h_{s,s_h} + \psi^x_{s,s_h})}(\lambda^{c,h})^{\mp 1}(f^{\pm,h}_s)^* \\ & - 2\pi i\delta^h e^{i(\Psi^h_{s,s_h} - \Psi^x_{s,s_h})}\hat{f}^{\mp,h}_{s,s_h} \\ & - 2\pi i\delta^h e^{-i(\Psi^h_{s,s_h} + \Psi^x_{s,s_h})}(\lambda^{c,h})^{\mp 1}(\hat{f}^{\pm,h}_{s,s_h})^*\Big]\,, \qquad (12)\end{aligned}$$

where $\lambda^{c,h} = \sin[\pi(\nu^h - \nu^y)]/\sin[\pi(\nu^h + \nu^y)]$ is another small parameter,

$$\begin{aligned}f^{\mp,h}_s = \frac{1}{\sqrt{1-(\lambda^h)^2}}\Big[& e^{\mp i(\Psi^v_{s,s_v} - \Psi^y_{s,s_v})} f^{\mp,h,v}_s \\ & + \lambda^v e^{\pm i(\Psi^v_{s,s_v} + \Psi^y_{s,s_v})} f^{\pm,h,v}_s\Big]\,, \qquad (13)\end{aligned}$$

and

$$\begin{aligned}\hat{f}^{\mp,h}_{s,s_h} &= \frac{e^{\pi i(\nu^h \mp \nu^y)\mathrm{sign}(s-s_h)}}{2i\sin[\pi(\nu^h \mp \nu^y)]} \\ &\quad \times \Big[f^{\mp,h}_s - e^{-i(\psi^h_{s,s_h} \mp \psi^y_{s,s_h})} f^{\mp,h}_{s_h}\Big]\,. \qquad (14)\end{aligned}$$

We note $\hat{f}^{\mp,h}_{s,s_h}$ is the CRDTs which only includes the effects from skew quadrupole fields between s_h and s [6, 7, 8, 11]. Relations similar to Eqs (12)-(14) hold between $f^\mp_s$ and $f^{\mp,v,h}_s$ as well. As the case of 1D parameters, having BPMs next to the AC dipoles makes all the parameters on right-hand-sides of Eqs (12)-(14) measurable and allows to determine $f^\mp_s$ from $f^{\mp,h,v}_s$ or $f^{\mp,v,h}_s$, measured with a single excitation frequency in horizontal and vertical AC dipoles. We normally take the average of $f^\mp_s$ from $f^{\mp,h,v}_s$ and that from $f^{\mp,v,h}_s$ and this makes the result less sensitive to the BPM calibration error [14].

As seen in Eq (12), $f^\mp_s$ and $f^{\mp,h,v}_s$ are different by a global factor $\{\sin[\pi(\nu^h \mp \nu^y)]/\sin[\pi(\nu^x \mp \nu^y)]\}$ and this is because resonance strengths depend on the tunes and the AC dipoles effectively change them [6]. Because the second to forth terms of Eq 12 are proportional to the small parameters δ^h or λ^h, taking into account this global factor and simply scaling $f^{\mp,h,v}_s$ and/or $f^{\mp,v,h}_s$ provides a good approximation of $f^\mp_s$ in many cases. However, this simple scaling does not always work properly and sometimes we must fully use Eqs (12)-(14), for instance when the magnitudes of f^-_s and f^+_s are comparable. The next section presents applications of the presented theory to measurement and simulation data of the LHC.

APPLICATIONS

On July 2nd, 2011, the injection optics of the LHC Beam2 was measured with both kickers and AC dipoles. Figure 1 compares the amplitudes and real parts (effectively showing the phases) of the CRDTs measured with the kickers and AC dipoles. In this case, the simple scaling and the full calculation using Eqs (12)-(14) have almost no difference. Figure 2 shows the other measurement of the CRDTs of the LHC Beam2 for the injection optics, performed on February 20th, 2011. In this case, the full calculation provides a better agreement with the result from the kickers. The difference between the two cases is $|f^-_s|$ is smaller and comparable to $|f^+_s|$ for Fig 2.

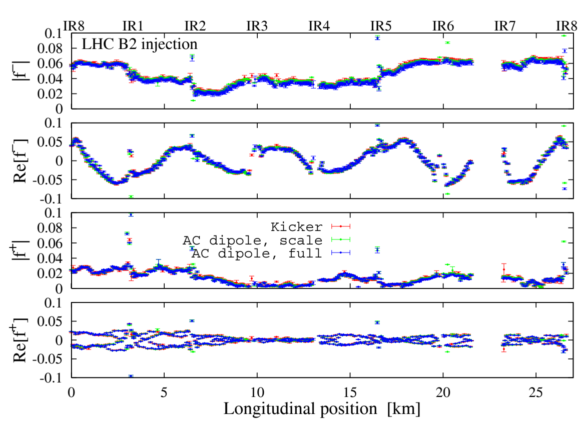

Figure 1: CRDTs of the LHC Beam2 measured with the kickers and AC dipoles. The simple scaling and full calculation provide almost identical result in this case.

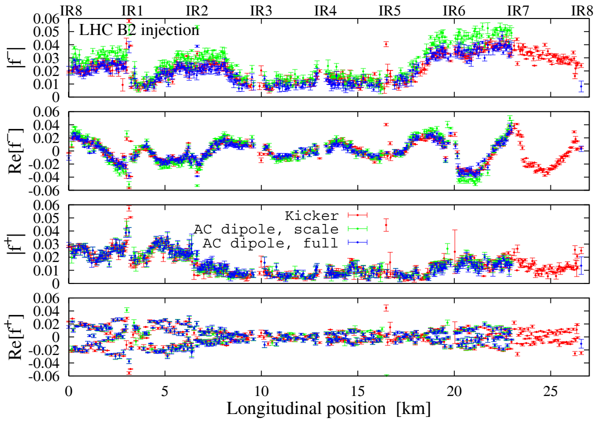

Figure 2: Another example of CRDTs measured with the kickers and AC dipoles. The full calculation gives a result closer to that from the kickers in this case.

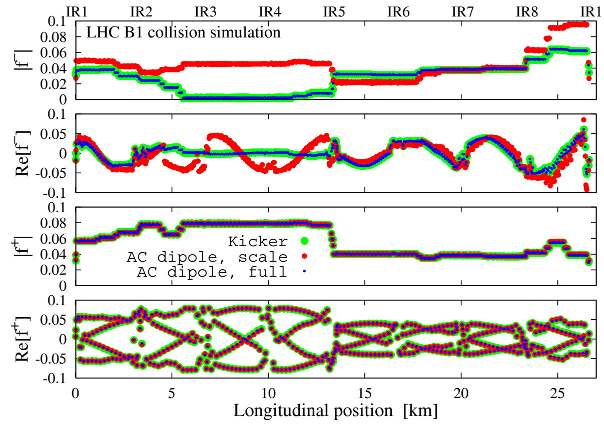

Figure 4: CRDTs in a simulated lattice reconstructed from the kick and AC dipole excitations. Skew quadrupole errors are based on the measurement in Fig 3.

Figure 3 compares the CRDTs of the collision optics of the LHC Beam1, where β^* is 1.5, 10, 1.5, and 3 m for the interaction points 1, 2, 5, and 8, measured with the AC dipoles on February 22nd, 2011, based on the simple scaling and full calculation. This has been the case where we observed the largest discrepancy between the scaling and full calculation in the LHC, so far. The measurement is done before any coupling correction is applied and $|f_s^+|$ is comparable to or even larger than $|f_s^-|$ in some regions, making the result from the simple scaling inaccurate.

To confirm that the result from the simple scaling can be inaccurate on the level seen in Fig 3, a simulation is performed with MADX [15]. Skew quadrupole magnets in the model lattice are fitted to the measurement in Fig 3 to produce a similar situation. Then, a single particle is tracked through this model lattice with an initial transverse displacement, which simulates kicks, or with AC dipole fields. Turn-by-turn position of the particle is recorded at each BPM location and the same analyses as real measurements are applied to the recorded data. Figure 4 compares CRDTs in the simulated lattice reconstructed from the simulated kick and AC dipole excitations. As clearly seen, the result of the AC dipole from the full calculation agrees well with the result of the kick, whereas the result from the simple scaling has a large discrepancy in some regions with respect to that of the kick. In this way, both experiments and simulations verify our algorithm and also show that the simple scaling does not always work properly.

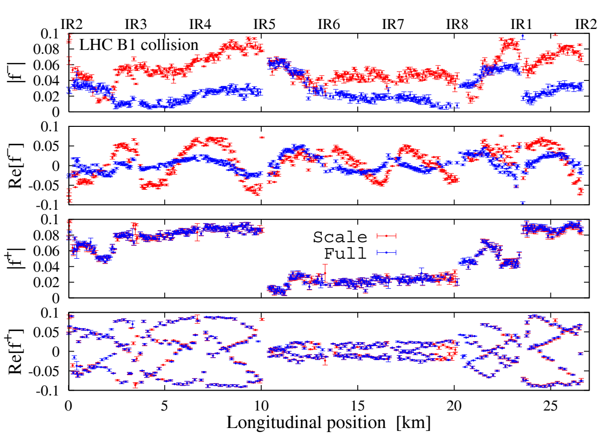

Figure 3: CRDTs of the collision optics of the LHC Beam1. A case where the largest discrepancy between the simple scaling and the full calculation is observed.

CONCLUSIONS

A method to measure the CRDTs using the AC dipoles without the systematic effect of the AC dipole is presented. It is tested in experiments and simulations of the LHC. The method allowed to measure the amplitudes and phases of the CRDTs without the systematic effect of the AC dipole for the first time in a real machine.

ACKNOWLEDGMENT

Authors would like to thank to S. Fartoukh and M. Giovannozzi for useful discussions.

REFERENCES

[1] M. Bai *et al.*, Phys. Rev. E **56**, p. 6002 (1997).
[2] J. Serrano *et al.*, CERN BE Note 2010-14-CO, 2010.
[3] R. Tomás, PRST-AB **13**, 121004 (2010).
[4] R. Bartolini *et al.*, Part. Accel. **59**, p. 93 (1998).
[5] S. Peggs, PAC'99, p. 1572.
[6] R. Tomás, PRST-AB **5**, 054001 (2002).
[7] S. Fartoukh, CERN-SL Report 2002-059-AP, 2002.
[8] N. Catalán Lasheras *et al.*, PAC'03, p. 2225.
[9] R. Miyamoto *et al.*, PRST-AB **11**, 084002 (2008).
[10] R. Miyamoto *et al.*, PAC'09, p. 3073.
[11] R. Miyamoto, BNL C-A/AP Note 410.
[12] A. Franchi, Ph.D thesis, Frankfurt University (2006).
[13] A. Franchi *et al.*, CERN BE Note 2010-016, 2010.
[14] M. Hayes *et al.*, EPAC'02, p. 1290.
[15] http://www.cern.ch/mad.