

TRANSVERSE EMITTANCE REDUCTION WITH TAPERED FOIL*

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Abstract

The idea of reducing transverse emittance with tapered energy-loss foil is proposed by J.M. Peterson in 1980s and recently by B. Carlsten. In this paper, we present the physical model of tapered energy-loss carbon foil and analyze the emittance reduction using the concept of eigen emittance. The study shows that, to reduce transverse emittance, one should collimate at least 4% of particles which has either much low energy or large transverse divergence. The multiple coulomb scattering is significant, leading to a limited emittance reduction ratio.

INTRODUCTION

Small transverse emittances are of essential importance for the accelerator facilities generating free electron lasers, especially in hard X-ray region. The idea of reducing transverse emittance with tapered energy-loss foil is recently proposed by B. Carlsten [1], and can be traced back to J.M. Peterson's work in 1980s [2]. Peterson illustrated that a transverse energy gradient can be produced with a tapered energy-loss foil which in turn leads to transverse emittance reduction, and also analyzed the emittance growth from the associated multiple coulomb scattering. However, what Peterson proposed was rather a conceptual than a practical design.

In this paper, we build a more complete physical model of the tapered foil based on Ref. [2], including the analysis of the transverse emittance reduction using the concept of eigen emittance and confirming the results by various numerical simulations. The eigen emittance equals to the projected emittance when there is no cross correlation in beam's second order moments matrix [3]. To calculate the eigen emittances, it requires only to know the beam distribution at the foil exit. Thus, the analysis of emittance reduction and the optics design of the subsequent beam line section can be separated. In addition, we can combine the effects of multiple coulomb scattering and transverse energy gradient together in the beam matrix and analyze their net effect. We find that, when applying such idea to an electron linac or electron beam line, the energy spread increase and angular growth due to multiple scattering are not trivial; as a result, the transverse emittance can only be reduced with a limited ratio, e.g. down to about 65% the original value.

PHYSICAL MODEL

Let us consider the electron beam distribution passing through a tapered energy-loss foil, whose geometry is shown in Fig. 1. For convenience, we place the foil where the beam's phase space distributions in three planes are all up-right ellipses. Thus, the second order moments beam matrix (For convenience, only show x, z planes) is

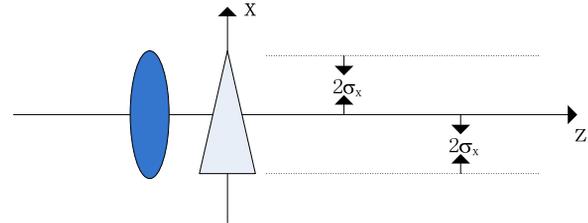


Figure 1: Geometry of the tapered energy-loss foil.

$$\Sigma_0 = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \quad (1)$$

where the 2-by-2 matrices A and B are

$$A = \begin{pmatrix} \sigma_{x0}^2 & 0 \\ 0 & \sigma_{x'0}^2 \end{pmatrix}, B = \begin{pmatrix} \sigma_{z0}^2 & 0 \\ 0 & \sigma_{d0}^2 \end{pmatrix}.$$

On exiting the foil, particles at and greater than $+2\sigma_x$ will not lose energy, while particles at $x=0$ lose energy $E_0 \times \Delta F$ and particles at $-2\sigma_x$ lose energy $2E_0 \times \Delta F$, with E_0 being the beam average energy at the entrance of the foil, ΔF the average energy loss factor of particles with $x=0$.

Therefore it produces a linear relationship between momentum and x position around the new mean momentum at the exit of the foil, as illustrated in Fig. 2. The correlation can be represented as

$$X_f = M_f X_0, \quad (2)$$

with $X = (x, x', z, \delta)^T$, and M_f is 4-by-4 matrix with diagonal elements to be 1 and $M_f(4, 1) = \Delta F / 2\sigma_x$. The slight downward net shift of energy distribution is ignored in Eq. (2).

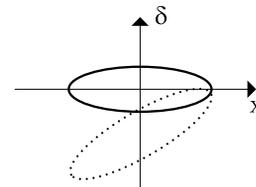


Figure 2: The beam distribution before (solid line) and after (dashed line) the foil.

The beam matrix after the foil is then calculated by

$$\Sigma = M_f \Sigma_0 M_f^T, \quad (3)$$

Meanwhile, multiple coulomb scattering induces additional energy spread $\Delta\sigma_\delta^2$ and transverse divergence $\Delta\sigma_{x'}^2$. Although there is weak correlation between x' and δ for 100% particles due to coulomb scattering, for the central (~96%) particles the correlations is negligible. Therefore these two effects are taken into account by directly adding $\Delta\sigma_\delta^2$ and $\Delta\sigma_{x'}^2$ to the beam matrix Σ ,

$$\Sigma_f = \begin{pmatrix} A_f & C_f \\ C_f & B_f \end{pmatrix}, \quad (4)$$

where the 2-by-2 matrices A_f, B_f, C_f are

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$$A_f = \begin{pmatrix} \sigma_{x0}^2 & 0 \\ 0 & \sigma_{x0'}^2 + \Delta\sigma_{x'}^2 \end{pmatrix},$$

$$B_f = \begin{pmatrix} \sigma_{z0}^2 & 0 \\ 0 & \sigma_{z0}^2 + \Delta F^2/4 + \Delta\sigma_\delta^2 \end{pmatrix},$$

$$C_f = \begin{pmatrix} 0 & \Delta F\sigma_{x0}/2 \\ 0 & 0 \end{pmatrix}.$$

The eigen emittance can be obtained by calculating the eigen values of the $J_4\Sigma_f$ [3], J_4 is the four dimensional unit symplectic matrix,

$$J_4 = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}, \quad (5)$$

where the 2-by-2 matrix J is

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (6)$$

The horizontal eigen emittance is in the form

$$\varepsilon_{x,eig} = \frac{1}{2\sqrt{2}} \sqrt{\mathcal{A} - \sqrt{\mathcal{A}^2 - \mathcal{B}^2}}$$

$$= \frac{1}{2\sqrt{2}} \sqrt{\mathcal{A}} \sqrt{1 - \sqrt{1 - (\mathcal{B}/\mathcal{A})^2}}, \quad (7)$$

with

$$\mathcal{A} = 4(\varepsilon_{x0}^2 + \varepsilon_{z0}^2) + 4(P + Q) + G,$$

$$\mathcal{B} = 8\sqrt{(\varepsilon_{x0}^2 + P)(\varepsilon_{z0}^2 + Q)},$$

$$G = \sigma_{z0}^2 \Delta F^2 \gamma^2, P = \sigma_{x0}^2 \Delta\sigma_{x'}^2 \gamma^2, Q = \sigma_{z0}^2 \sigma_\delta^2 \gamma^2.$$

where G , P and Q are related to the transverse energy gradient, angular growth and energy spread increase, respectively; γ is the Lorenz factor; $\varepsilon_{x0} = \gamma\sigma_{x0}\sigma_{x0'}$, $\varepsilon_{z0} = \gamma\sigma_{z0}\sigma_{z0}$ are the initial normalized x and z emittances.

Note that G , P and Q are all positive, therefore on the RHS of Eq. (7), $\frac{1}{2\sqrt{2}}\sqrt{\mathcal{A}} \geq \sqrt{(\varepsilon_{x0}^2 + \varepsilon_{z0}^2)/2} \geq \min(\varepsilon_{x0}, \varepsilon_{z0})$. Both ε_{x0} and ε_{z0} are assumed to be larger than the target x eigen emittance. In other cases, like, ε_{z0} is smaller than the target x eigen emittance, one can make emittance exchange between x and z planes to realize the transverse emittance reduction, which is not the interest of the present study.

To achieve small emittance, it requires

$$\left(\frac{\mathcal{B}}{\mathcal{A}}\right)^2 = \frac{64(\varepsilon_{x0}^2 + P)(\varepsilon_{z0}^2 + Q)}{(4(\varepsilon_{x0}^2 + \varepsilon_{z0}^2) + 4(P + Q) + G)^2} \ll 1. \quad (8)$$

so that

$$\varepsilon_{x,eig} \approx \frac{1}{4} \frac{\mathcal{B}}{\sqrt{\mathcal{A}}} < \min(\varepsilon_{x0}, \varepsilon_{z0}). \quad (9)$$

Due to coulomb multiple scattering, a few percent (~2%) of particles have large transverse divergence and away from the beam core in (x, x') phase space, and similarly in (z, δ) phase space. In practice, these particles (totally about 4% percent) should be collimated.

For the central 98% particles around the mean momentum, the average energy loss factor (denotes as ΔF_{98}) and energy spread increase depend on the relative foil thickness $df = L_f/L_0$ (L_0 is the radiation length), beam energy, and weakly on the foil material. For Carbon foil, the relationship is empirically found based on plenty of simulations with G4beamline code [4],

$$\Delta F_{98} = \langle \frac{\Delta\gamma}{\gamma} \rangle (98\%) \approx \frac{L_f}{L_0} \frac{83}{\gamma} = \frac{83}{\gamma} df. \quad (10)$$

$$\Delta\sigma_\delta^2 (98\%) \approx \left(\frac{35.5}{\gamma} \frac{L_f}{L_0}\right)^2 = \left(\frac{35.5}{\gamma} df\right)^2. \quad (11)$$

These two equations are valid for df of 10^{-5} to 10^{-3} and beam energy of 100 MeV to 12 GeV. In the cases of $df > 10^{-3}$, the dependences of ΔF_{98} and $\Delta\sigma_\delta^2 (98\%)$ on the $1/\gamma$ are no longer linear. Although ΔF_{98} will increase, the $\Delta\sigma_\delta^2 (98\%)$ will increase more quickly than ΔF_{98} with the increasing df , leading to emittance growth in three planes.

For the central 98% particles, the angular growth using Gaussian approximation [5] is

$$\langle \theta^2 \rangle (98\%) = \left(\frac{35.5}{\gamma}\right)^2 (1 + 0.038 \ln(df))^2 df. \quad (12)$$

We notice that the value of $(1 + 0.038 \ln(df))^2$ varies little with df , therefore replace this term with a constant value 4/9 (it is accurate to 10% for $10^{-6} < df < 10^{-3}$). Then the angular growth approximately linearly depends on df ,

$$\Delta\sigma_{x'}^2 = \frac{1}{2} \langle \theta^2 \rangle \approx \frac{157.4}{\gamma^2} df. \quad (13)$$

Here we make an approximation that $\Delta\sigma_\delta^2 (98\%)$ and $\Delta\sigma_{x'}^2$ do not depend on x, which greatly simplifies the analysis while does not lead to large disparity between the physical model and real circumstance (see below).

Submit Eqs. (10), (11) and (13) into Eq. (7), we have

$$G \approx 6890\sigma_{z0}^2 df^2,$$

$$P \approx 157.4\sigma_{x0}^2 df, \quad (14)$$

$$Q \approx 1260\sigma_{z0}^2 df^2 \approx \frac{G}{5.5}.$$

From Eq. (8), to achieve small x eigen emittance, the most effective way is to increase G and minimize P and Q . However, Q changes monotonically with G , therefore increasing G leads to larger Q meanwhile, which in turn limits the available minimal $\varepsilon_{x,eig}$ to be about 65% of ε_{x0} .

Let us consider a case that the beam's initial normalized emittances are 0.7/0.7/1.4 μm , $\beta_x = 1$ cm at the foil and rms bunch length $\sigma_z = 0.5$ mm. With these parameters, the x eigen emittance with respect to the relative foil thickness df and beam energy is shown in Fig. 3. To achieve $\varepsilon_{x,eig} = 0.5$ μm , it requires energy greater than 1.845 GeV and relative foil thickness df about 2×10^{-4} . It is really hard to reduce $\varepsilon_{x,eig}$ close to the theoretical minimal $\varepsilon_{x,eig}$, unless we collimate more particles, in that case, Eqs. (10-13) should be modified, too.

NUMERICAL EXPERIMENT

Here we show a numerical experiment. The main parameters of the beam and the foil are listed in Table 1. We collimate 4% particles with large transverse divergence and energy loss using collimators after the foil. Fig. 4 presents the beam distribution before and a distance after the foil. The x eigen emittance calculated from the beam distribution at the exit of the foil is $0.543 \mu\text{m}$, which agrees fairly well with that calculated from the physical model, $0.498 \mu\text{m}$.

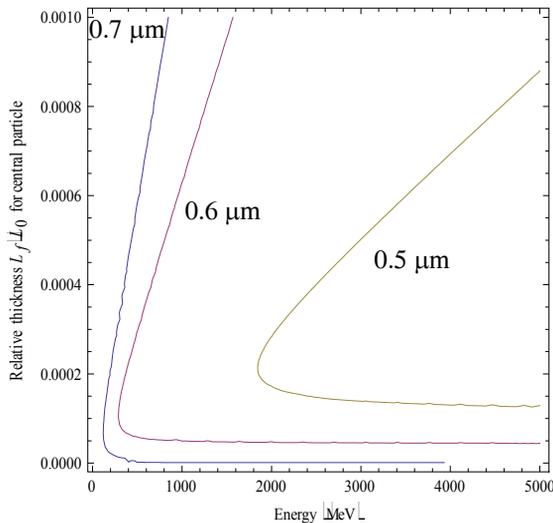


Figure 3: x eigen emittance with respect to energy and relative foil thickness $df = L_f/L_0$.

Table 1: Parameter Setting for Numerical Experiment

Parameters	Value	Unit
Energy	2	GeV
$\epsilon_{x0}/\epsilon_{y0}/\epsilon_{z0}$	0.7/0.7/1.4	μm
β_x/β_y at foil entrance	1/1	cm
Bunch length σ_z	0.5	mm
Relative thickness L_f/L_0	2×10^{-4}	
Foil material	Carbon	

We also simulate the case of a partially tapered foil, whose thickness varies linearly with x in the range of $[-\sigma_x, \sigma_x]$ and constant out of this range. Consider 96% central particles, the calculated x eigen emittance from the beam distribution at the exit of the foil is $0.47 \mu\text{m}$, 14% smaller than that produced by a purely tapered foil. We design a beam line section to remove the x, δ correlations. Because the beam is partially x, δ correlated, the correlation can not be completely removed. Fig. 5 shows the x slice emittances at the exit of this beam line section, which accord with the x eigen emittance calculation result quite well.

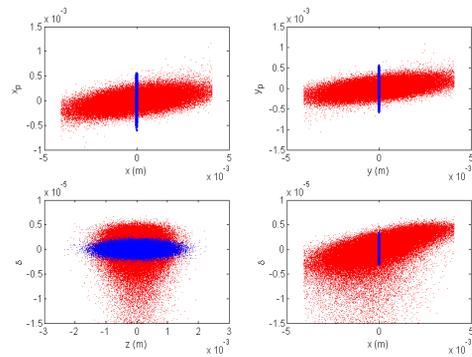


Figure 4: Beam distribution before (blue plots) and a distance after (red plots) the tapered carbon foil.

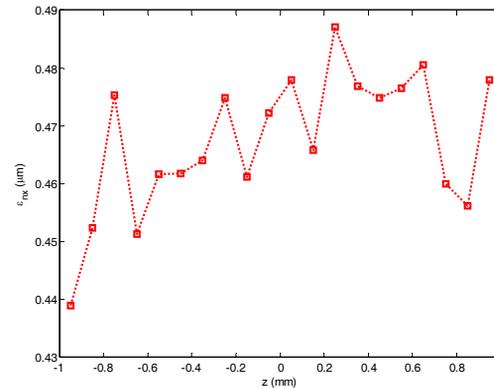


Figure 5: Beam distribution after the correlation removing beam line section.

CONCLUSION

The idea of reducing transverse eigen emittance with tapered energy-loss carbon foil is analyzed in this paper. The multiple coulomb scattering will cause about 4% of particles losing too much energy or having too large transverse divergence. For the remaining particles, the average energy loss factor, angular growth and energy spread growth mainly depend on the beam energy and relative foil thickness, and weakly on foil material. We build a physical model of the tapered foil, and find that the energy spread growth due to multiple coulomb scattering is not trivial, but comparable to the transverse energy gradient; as a result, the available minimal x eigen emittance is only about 65% of ϵ_{x0} .

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