AMPLITUDE DEPENDENT TUNE SPREAD IN THE CR OPERATED AS AN ANTIPROTON COLLECTOR

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Abstract

In frame of the FAIR project [1] the Collector Ring is planned to be built for efficient cooling of antiprotons and rare isotopes beams. In order to accept hot antiproton beams coming from a separator [2] large aperture magnets are required. This paper examines the effects, which may influence on the beam dynamics because of large both an amplitude betatron oscillation (240 mm mrad) and large momentum spread (6%). Using analytic expressions the amplitude-dependent tune shifts driven by sextupole magnets, fringe field of quadrupole magnets and kinematic effects have been calculated. The obtained results are compared with numerical simulations. Tracking studies for the CR operated as an antiproton collector have been performed considering the real distribution of the magnetic field of the wide aperture quadrupole. We report on quantitative studies of the effects on the tune spread and its influence on the beam losses.

INTRODUCTION

In the Collector Ring [3] phase space reduction of an antiproton beam in the longitudinal phase space consists of two steps. Right after injection, the beam with a bunch length of 50 ns is rotated by a quarter of a synchrotron period within a mismatched bucket. Due to this the momentum spread is reduced from 6% to 2% on about 1 ms. Then the beam is adiabatically debunched to reduce further the momentum spread. Afterwards stochastic cooling is applied. The tune spread during bunch rotation can cause a crossing of “dangerous” resonances that leads to particle losses. In this paper we elaborate on the various sources of amplitude-dependent tune shift and list the contributions from each source to the antiproton beam loss.

Using analytic expressions the amplitude-dependent tune shifts driven by sextupole magnets, fringe field of quadrupole magnets and kinematics effects have been calculated. The obtained results are compared with numerical simulations.

To make an analytical estimation of the tune shift we use the following expressions for the amplitude tune dependence [4]

$$\begin{pmatrix}\Delta Q_x \\ \Delta Q_y\end{pmatrix} = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{xy} & C_{yy}\end{pmatrix}\begin{pmatrix}J_x \\ J_y\end{pmatrix} \qquad (1)$$

where J_x , and J_y are the horizontal and vertical action of the particle respectively. The anharmonic coefficient $C_{ii} = C^k + C^{sx} + C^{fr}$ includes the contribution of the kinematic C^k, sextupole C^{sx}, and fringe field C^{fr} effects The analytical expressions for C^k , C^{sex}, C^{fr} can be found in the literature [5,6]. The tune spread due to the field error perturbations in the main magnets is evaluated in a separate way using a numerical method and then compared with the analytical formulae.

SOURCES OF THE TUNE SHIFT

Sextupole Magnets

The first source of amplitude dependent tune shift are the 24 sextupole magnets located in the two CR arcs, where the dispersion function is not zero. 6 sextupole families are required to control the chromaticity and the dispersion function close to zero in the long straight sections, where the stochastic cooling systems and RF cavities are placed. Analytically the C^{sx} anharmonic coefficients can be calculated by the formulae given in ref. [5].

Fringe Field

The fringe field is another source of large amplitude contribution to the tune shifts. In the CR the normal-conducting quadrupoles require a 312-mm bore, which leads to strong, extended fringe fields proportional to the poletip field of the quadrupole (0.8 T, in this ring). With the need to minimize arc lengths in order to have good properties for the stochastic cooling, the poletip field and the fringe fields cannot be reduced by correspondingly lengthening the quadrupoles. Hence the impact of quadrupole end fields on the performance of the CR must be carefully evaluated. Analytically the fringe field anharmonic coefficients in the formula (1) can be calculated by the expressions [6]:

$$C^{fr}_{xx} = -\frac{1}{32\pi}\oint k_1''(s)\beta_x^2(s)ds; \qquad C^{fr}_{yy} = +\frac{1}{32\pi}\oint k_1''(s)\beta_y^2(s)ds; \qquad (2)$$

$$C^{fr}_{xy} = -\frac{1}{16\pi}\oint \beta_x(s)\left(k_1''(s)\beta_y(s) - 4k_1'\alpha_y(s)\right)ds,$$

$\beta_{x,y}$ and $\alpha_{x,y}$ Twiss (betatron and alfa) functions of the CR, k_1 – quadrupole strength (1/m). k' and k'' are the first and second derivatives of the k. To calculate k' and k'' each quadrupole magnet is represented by slicing the whole axial quadrupole field distribution in 100 thin segments of varying strength according to an Enge function with six parameters (a_i) of the form:

$$k(s) = \frac{k_0}{1-\exp(a_0 + a_2(s/d) + a_3(s/d)^2 + .. + a_5(s/d)^5)}, \qquad (3)$$

where s is the Cartesian distance to the field boundary. The quantity d is the full aperture of the quadrupole.

Kinematic Effect

Kinematic perturbations appear even for ideal magnets and alignment. General, such terms become significant for small circular accelerators or wherever beams are deflected in high fields and the beam sizes are large. The kinematic non-linearity arises from high order terms proportional to the transverse momenta p_x, p_y into the expansion of the standard square-root relativistic Hamiltonian. The first correction to the tune shift comes from octupole like terms: p_x^4, $p_x^2 p_y^2$ and p_y^4. A transformation to action variables results in the kinematic anharmonic coefficients [6]:

$$C_{xx}^k = \frac{3}{16\pi}\oint \gamma_x^2(s)ds; \quad C_{yy}^k = \frac{3}{16\pi}\oint \gamma_y^2(s)ds, \quad C_{xy}^k = \frac{1}{8\pi}\oint \gamma_x\gamma_y(s)ds, \tag{4}$$

the $\gamma_{x,y}$ are the usual Twiss gamma functions.

Field Errors of Magnets

The nonlinear fields in the magnets can be represented by

$$B_y + iB_x = B_0\left(1 + \sum_{n=0}(b_n + ia_n)(x+iy)^n\right) \tag{5}$$

where B_0 is the main field, and b_n and a_n are the normal and skew multipole components. x and y are horizontal and vertical displacements of the charged particles. The first order tune shifts can be calculated by formula [7]:

$$\Delta Q_{x,y} = \oint \frac{ds}{2\pi\rho_0}\beta_{x,y}\sum_{n=0}^{9}\left(C_n^{x,y,b}b_n + C_n^{x,y,a}a_n\right) \cdot \tag{6}$$

Here the C_n coefficients are proportional to the action $J_{x,y}$ and depend on the beam orbit.

NUMERICAL APPROXIMATION

To prove the analytical calculations of all anharmonic coefficients numerical simulations have been performed. 15000 particles were tracked each with a different starting position *(x,x',y,y',dp/p)* within the ring acceptance of 240 mm·mrad in both planes for 1024 turns. A FFT is performed on the horizontal and vertical turn by turn position data and particles fractional horizontal and vertical tunes are calculated.

In the numerical simulations each magnet is represented by slicing the whole axial field distribution in 100 thin segments. Each segment is represented by a linear transfer matrix. The quadrupole field strength of each segment is varied according to formula (3). The dipole magnet is modelled as a hard edge fringe field. Between slices thin lenses with zero length are introduced. These lenses produce the kick angle change, which is calculated depending on the nonlinear source. In case of field errors the kick angle change is

$$\Delta x' + i\Delta y' = \alpha\sum_{n=0}(b_n + ia_n)(x+iy)^n, \tag{7}$$

where $\alpha = K_Q ds$ for the quadrupole magnet and $\alpha = d\theta$ for the dipole magnet; ds, $d\theta$ are the length and angle of one thin segment. For the CR magnets the calculated field harmonics are given in table 1 and used in our simulations.

In the case of the quadrupole fringe field the kick angle is calculated by [8]:

$$\Delta x' = K_Q''\left[x^3 + 3xy^2\right]/12, \quad \Delta y' = K_Q''\left(y^3 + 3yx^2\right)/12. \tag{8}$$

Table 1: Field Harmonics b_n for Dipole (*D*) and Quadrupole (*Q*) Magnets (unit 10^{-4})

n	2	3	4	5	6	7	8	9
D	6.3	0.03	1.7	≈0	≈0	-0.02	≈0	-0.01
Q	0.0	≈0	0.01	-3.0	0.01	≈0	0.01	10.

K'' is the second derivation of the quadrupole strength K_Q. When the kinematic effect is calculated, each thing lens is considered as a drift space, where particle trajectories are transformed proportional to terms in $p_{x,y}$

$$\Delta(x,y) = \frac{p_{x,y}(p_x^2 + p_y^2)ds}{2(1+\delta)^3}, \tag{9}$$

The sextupole magnets for the case of a thin lens approximation change the particles angles by

$$\Delta x' = m\left[(x + D_x\delta)^2 - y^2\right]\cdot 0.5, \qquad \Delta y' = -m(x + D_x\delta)y. \tag{10}$$

$m=(L/BR)d^2B/dx^2$ is the integrated strength of the sextupole. $\delta=\Delta p/p$. D_x is the dispersion function.

TUNE CALCULATIONS

The magnitudes of the anharmonic coefficients, which are obtained from various sources, are listed in table 2. The first characteristic of the CR with respect to the sextupole and kinematic effects is that the anharmonic coefficients are relatively small for $\Delta p/p$=0. Such coefficients are also observed in our numerical simulations. The quadrupole fringe field has a dominant effect on the tune shift compare with sextupole and kinematic effects.

Table 2: Calculated Anharmonic Coefficients for the CR from Various Sources. δ=0

	C_{xx}	C_{yy}	C_{xy}
Sextupole			
analytical	-2.58	-1.82	0.89
numerical	-3.55	-3.33	-1.45
Quadr. fringe			
analytical	15.58	25.81	19.98
numerical	31.91	38.55	-0.16
Kinematics			
analytical	1.34	4.15	1.34
numerical	1.34	4.02	1.32

Figures 1 and 2 illustrate the calculated horizontal and vertical tunes obtained by a FFT for particles with a different transverse staring position within the transverse acceptance of 240 mm·mrad and a momentum spread of 6% for 1024 turns. In Fig.1 we present the tune spread due to the individual effect independently from each other. These simulations allow us to identify which effect is dominating if there is no compensation. One can see that the horizontal and vertical tune spreads (ΔQ_h=0.031, ΔQ_v=0.036) arise from field errors of the main magnets. The fringe field of the quadrupole magnets has a dominant effect only in the horizontal plane and gives a tune spread of ΔQ_h=0.025, ΔQ_v=0.005. In Fig.2 the tune spread calculated by action of all sources before bunch rotation is shown for three cases (without correction and with sextupole correction for two sextupole settings). If the sextupole correction is not applied then the chromatic effect of quadrupole magnets is strong and produces the tune spread of ΔQ_h=0.3, ΔQ_v=0.6.

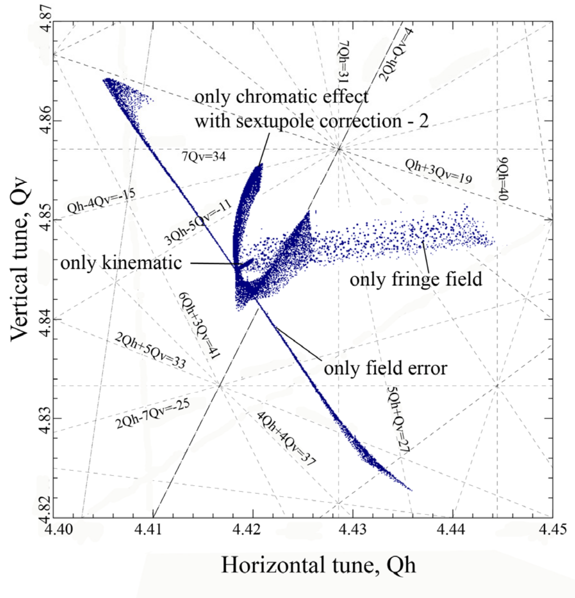

Figure 1: The tune spreads due to each individual effect independently from each other (δ=6%).

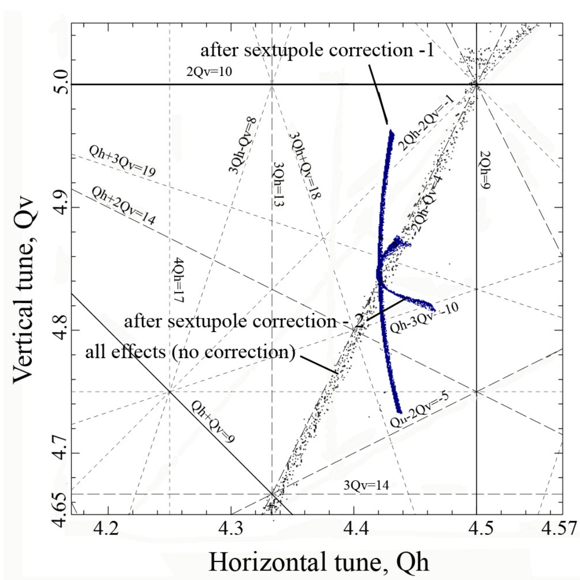

Figure 2: The tune spreads calculated with all sources acting without and with sextupole correction (δ=6%).

Six families of sextupole magnets are used to correct the chromaticity. In Fig.1 one can see that the sextupole correction is able to reduce the tune spreads down to ΔQ_x=0.008 and ΔQ_y=0.012. If we consider the action of all nonlinear sources the sextupole correction has to be optimised with respect to the minimal beam loss. As a first step we investigate the beam loss depending on each individual effect. Fig.3 shows the beam survival during bunch rotation over 1000 turns, when the momentum spread is reduced from 6% to 2%. If there is no correction beam loss of 23% is observed due to the action of all sources. One can see that the chromatic effect has the largest contribution to the beam loss before stochastic cooling stats. Other effects all together cause beam loss of less then 2%..

The sextupole correction allows us to reduce the beam loss to 4 - 7% depending on the sextupole setting. We consider two sextupole settings. First, the sextupole magnets can be tuned to the setting that gives the small horizontal tune spread (ΔQ_h=0.01), while the vertical tune spread is 0.2. At this setting the beam loss is 4% after the first 1000 turns. But for the long term stability the dynamic aperture is reduced because the tune spread crosses the structure resonances Q_h-$3Q_v$=-10 and Q_h+$2Q_v$=14 (fig.2, sextupole correction-1). Second, the sextupole magnets can be adjusted to a setting at which the tune spread is about 0.02 in both planes. At such a setting the tune spread does not cross the structure resonances as shown in fig.2. But, in this case, the beam loss is 7% because the ring beta-functions are distorted for large momentum deviation. This leads to an emittance dilution, which cause particle loss.

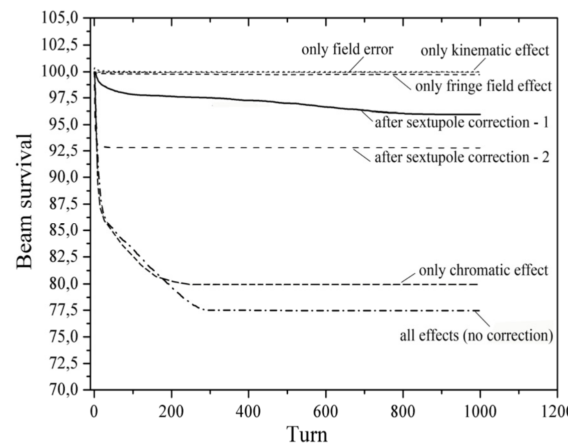

Figure 3: The calculated beam loss in the CR due to different nonlinear sources.

REFERENCES

[1] An International Accelerator Facility for Beams of Ions and Antiprotons, Baseline Technical Design Report, GSI, 2006. www.gsi.de/fair/reports.

[2] A.Dolinskii, K.Knie et al. NIM A629 (2011) 16.

[3] M. Steck, "Advanced design of the FAIR ring complex", PAC’09, Vancouver , FR1GRI03.

[4] Z. Parsa et.al. Second-order perturbation theory for accelerators, Particle accelerators 1987, Vol.22. pp.205-230.

[5] "7 GeV Advanced Photon Source – Conceptual Design Report", Argonne National Laboratory Report ANL-87-15, April 1987.

[6] E.B.Levichev, P.A. Piminov, “Analytic estimation of the non-linear tune shift due to quadrupole magnetic fringe field. Preprint 2000-014, BINP, Novosibirsk.

[7] R. Ostojic, T. M.Taylor, EPAC’96, Sitges, MOP018G.

[8] P. Krejik. Nonlinear quadrupole end-field effect in the CERN antiproton accumulator. IEEE CH2387-9/87/0000/-1278