

# THE SPIN ABERRATION OF POLARIZED BEAM IN ELECTROSTATIC RINGS

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## Abstract

For a beam with nonzero transverse emittance and momentum spread passing through an electric field, for example an electric focusing lens or deflector, the orientation of a spin vector becomes a function of 6D initial phase coordinates that leads to spin aberrations. We investigate this process analytically and numerically.

## INTRODUCTION

At present, electrostatic storage rings are increasingly used not only in atomic physics, biology and chemistry [1], but also alleged experiments to search for the electric dipole moment [2]. Presumably the most successful experiment to search for the EDM is based on the measurement of the dependence of the spin precession on the strength of an external guiding field.

The spin  $\vec{S}$  is a quantum value, but in the classical physics representation the “spin” means an expectation value of a quantum mechanical spin operator:

$$\frac{d\vec{S}}{dt} = \mu\vec{S} \times (\vec{B} - c\beta \times \vec{E}) + d\vec{S} \times (\vec{E} + \vec{\beta}c \times \vec{B}) \quad (1)$$

where  $\mu$ ,  $d$  are the corresponding magnetic and electric dipole moments,  $c$  is the speed of light,  $\beta$  is the relative velocity and  $E$ ,  $B$  are the electric and the magnetic field vectors respectively. In an experiment to search for EDM it is desirable to minimize the first term of equation (1) described the contribution of the magnetic moment in spin motion.

In the rest frame of the particle the spin motion is determined by T-BMT equation (in SI units):

$$\begin{aligned} \frac{d\vec{S}}{dt} &= \vec{\omega}_G \times \vec{S} \\ \vec{\omega}_G &= -\frac{e}{m_0\gamma c} \left\{ Gc\vec{B} - \left( G - \frac{1}{\gamma^2 - 1} \right) (\vec{\beta} \times \vec{E}) \right\} \\ G &= \frac{g-2}{2}, \end{aligned} \quad (2)$$

where  $G$  is the anomalous magnetic moment,  $g$  is the gyromagnetic ratio and  $\omega_G$  is the spin precession frequency relatively of the momentum vector.

In a electrostatic storage ring with  $B = 0$  the spin precession frequency reads:

$$\vec{\omega}_G = -\frac{e}{m_0\gamma c} \left\{ \left( \frac{1}{\gamma^2 - 1} - G \right) (\vec{\beta} \times \vec{E}) \right\}. \quad (3)$$

The advantages of a purely electrostatic ring are especially evident in the so-called magic rings, when:

$$\frac{1}{\gamma_{mag}^2 - 1} - G = 0 \quad (4)$$

and the spin oriented in the longitudinal direction rotates in horizontal plane with the same frequency as the momentum vector, resulting in  $\omega_G = 0$  [3].

However, from (3) follows that this condition is only satisfied for particles with  $G > 0$  like protons and energy with magic value  $\gamma_{mag}$ . In addition, as we can also see, the frequency of precession depends on the field, and obviously it is different for particles with different trajectories.

Thus, the frequency of spin precession has a coherent component which is the same for all particles and a incoherent component, which determines the spin orientation smearing in time within an electric deflector. We denote the spin incoherent component as spin aberration. It depends on momentum spread in the beam  $d\omega_G/dp$  and the emittance  $d\omega_G/d\varepsilon_{x,y}$ .

In this paper we consider various causes leading to aberrations of spin motion, we estimate their values using simple analytic techniques and compare them with numerical results obtained with the simulation program COSY Infinity [4]. Based on these results, we consider different methods to reduce spin aberrations. Some of the methods have been proposed previously. Nevertheless, we tested them using our analytical and numerical models for our electrostatic test lattice.

## SPIN OSCILLATION OF NON-MAGIC PARTICLE

For a particle with an energy different from the “magic” value  $\gamma \neq \gamma_{mag}$  the condition (4) is violated for  $G - \frac{1}{\gamma^2 - 1} \neq 0$ . Expanding  $G - (1/(\gamma^2 - 1))$  in Taylor series in the vicinity of the corresponding magic momentum  $p = p_m$  we get:

$$\begin{aligned} \left( \frac{1}{\gamma^2 - 1} - G \right)_{p=p_m+\Delta p} &= 0|_{p=p_m} - 2G\frac{\Delta p}{p} + \\ &+ \frac{1+3\gamma^2}{\gamma^2} G \left( \frac{\Delta p}{p} \right)^2 + \dots \end{aligned} \quad (5)$$

### Precession of longitudinal spin component

First, we consider the spin precession in linear approach versus momentum:

$$\vec{\omega}_G = -\frac{e}{m_0\gamma c} \left\{ -2G\frac{\Delta p}{p} (\vec{\beta} \times \vec{E}) \right\}. \quad (6)$$

Furthermore the spin components will be described in the following way:  $z$  is orientated along the momentum vector,  $x$  and  $y$  are horizontal and vertical directions respectively. Taking into account that the vertical electric field component is expected to be rather small and moreover the velocity in the vertical direction  $\beta_x, \beta_y \ll \beta_z$ , equation (3) can be further simplified to understand the qualitative behavior of spin motion:

$$\begin{aligned} \frac{dS_x}{c\beta_z dt} &= + \frac{e}{m_0\gamma c^2} 2G \frac{\Delta p}{p} E_x S_z, \\ \frac{dS_z}{c\beta_z dt} &= - \frac{e}{m_0\gamma c^2} 2G \frac{\Delta p}{p} E_x S_x. \end{aligned} \quad (7)$$

In normalized coordinates  $d\varphi = 2\pi dn = 2\pi \frac{c\beta_z}{L_{cir}} dt$ , where  $L_{cir}$  is orbit circumference, we get:

$$\frac{d^2 S_z}{d\varphi^2} + \left( \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} 2G \frac{\Delta p}{p} \right)^2 S_z = 0. \quad (8)$$

Thus, the spin oscillates in the horizontal plane with tune  $\nu_{sz}$  satisfy:

$$S_z = S_{z0} \cos 2\pi\nu_{sz}n, \quad \nu_{sz} = \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} G \frac{\Delta p}{p}, \quad (9)$$

where  $\bar{E}_x$  is the average value of the deflecting electric field. Thus we find that the spin of each particle oscillates with its own frequency, which clearly lead to spin aberrations and to a complete decoherence of the beam for a certain period called the spin coherence time. Using a common definition we can say that the spin coherence time (SCT) has the finite value. For example, if we assume that the maximum deviation of the momentum equal to  $10^{-4}$ , we have identified from (9)  $\nu_{sz} = 1.588 \cdot 10^{-4}$ , or SCT = 6300 turns, which is approximately 1 msec. At the same time we have made 3D tracking with COSY Infinity and have found  $\nu_{sz} = 1.636 \cdot 10^{-4}$  that coincides with good accuracy with the analytical estimation.

### RF cavity as a method to increase SCT

The idea of using an RF cavity to reduce the precession of the spin for a particle having an energy difference from the magic values was already expressed by other authors some time ago, for instance [5] or in Richard Talman's notes.

Obviously, the particle oscillating around the magic momentum also changes the behavior of the spin motion. It is easy to see from the equation (8) with  $(\Delta p/p) = (\Delta p/p)_m \cos(\nu_z \varphi)$ , where  $\nu_z$  — longitudinal tune due to RF field, that:

$$\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} 2G \left( \frac{\Delta p}{p} \right)_m \cos(\nu_z \varphi) \right\}^2 S_z = 0. \quad (10)$$

This equation describes the pendulum motion in a rapidly oscillating field. Now, instead of the oscillations with a frequency  $\nu_{sz}$ , the spin vibrates within a very narrow angle  $\Phi_{\max}$  with  $\nu_z$  tune  $\Phi \sim \Phi_{\max} \sin(\nu_z \varphi)$ .

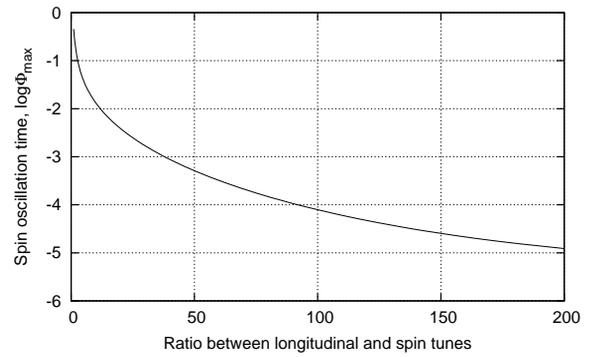


Figure 1: Spin oscillation angle versus longitudinal and spin tunes ratio.

The value  $\Phi_{\max} \sim (\nu_{sz}/\nu_z)^2$  depends on the frequency ratio.

Practically the same results were obtained by numerical simulation with COSY Infinity. We can see that for our ratio  $\nu_z/\nu_{sz} \approx 150$  the spin aberration is determined by  $\Phi_{\max} \sim 10^{-4}$ , which is an extremely small quantity and does not play a major role.

### Second order approach of spin tune versus $\Delta p/p$

From the expression (5) one can easily find the frequency of the oscillations of the spin in the second approach versus momentum:

$$\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \left[ -2G \frac{\Delta p}{p} + \frac{1+3\gamma^2}{\gamma^2} G \left( \frac{\Delta p}{p} \right)^2 \right] \right\}^2 S_z = 0 \quad (11)$$

and with RF  $(\Delta p/p) = (\Delta p/p)_m \cos(\nu_z \varphi)$  we can see that the spin tune has non-zero average value:

$$\begin{aligned} \nu_{sz} &= \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \left\langle -2G \left( \frac{\Delta p}{p} \right)_m \cos(\nu_z \varphi) + \frac{1+3\gamma^2}{\gamma^2} G \left( \frac{\Delta p}{p} \right)_m^2 \cdot \right. \\ &\quad \left. \cos^2(\nu_z \varphi) \right\rangle = \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \frac{1+3\gamma^2}{\gamma^2} \frac{G}{2} \left( \frac{\Delta p}{p} \right)_m^2 \quad (12) \end{aligned}$$

Taking the second approach into account, spin motion performs fast oscillations with a frequency of RF relative to the average position, which in turn oscillates with a very low frequency  $\sim (\Delta p/p)^2$  determined by the average value from equation (12).

But even for spin tune values we expect for  $(\Delta p/p)_{\max} = 10^{-4}$  the number of turns for the spin decoherence is  $\approx 6 \cdot 10^7$ , corresponding to SCT  $\approx 180$  sec. We tested this behaviour with COSY Infinity and have found almost complete agreement with our analytical estimation.

## OFF-AXIAL PARTICLES

In [6] was shown that particles flying into the deflector with different deviation from the axis, have different angular momentum, and therefore perform oscillations with respect to different energy levels.

Figure 2 shows the COSY Infinity tracking results for two particles with different initial horizontal displacements

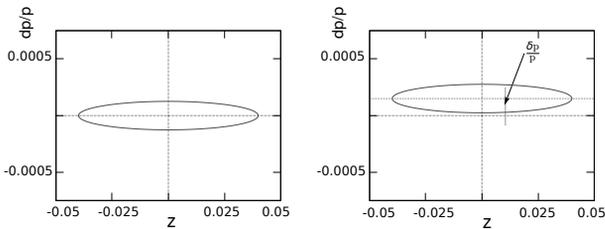


Figure 2: Phase trajectory in longitudinal plane for initial coordinates  $x = 0, y = 0$  (a) and  $x = 3\text{mm}, y = 0$  (b).

in an electrostatic ring. As can be seen, the equilibrium energy level for off-axis particle shifts up  $\delta p/p$  from the magic momentum. Besides this, the effect has incoherent character. This means that the RF cavity will not be able to reduce the aberration of the spin. Even in the first approach SCT is very short for off-axis particles due to spin oscillation with tunes:

$$\nu_{sz} = \frac{e\bar{E}_x L_{cir}}{\pi m_0 c^2 \gamma} G \frac{\delta p}{p}. \quad (13)$$

### Equilibrium energy level modulation

Since the main reason for spin rotation of particle is its displacement of the equilibrium energy momentum, which in turn depends on the initial particle displacement in the horizontal plane. The only solution to increase SCT is the modulation of the energy level itself relative of the magic level. This situation can be realized under the condition when the longitudinal and transverse horizontal frequencies become closer each to other. For this purpose, we have reduced the tunes to the values  $\nu_x = 1.31$ ,  $\nu_y = 0.64$ . Figure 3 shows the COSY Infinity tracking results. Trajectory of particle compared to the previous case (see fig. 2b) is modified.

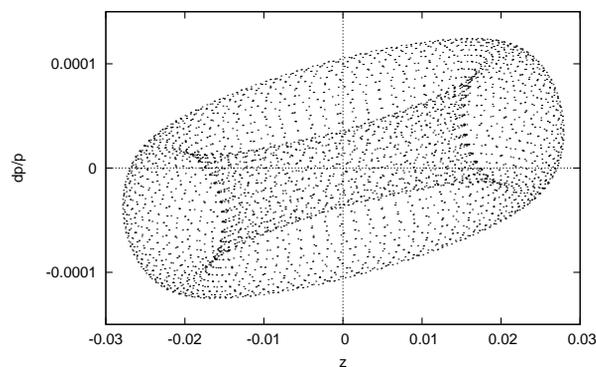


Figure 3: Phase trajectory modification in longitudinal plane for initial coordinates  $x = 3\text{mm}, y = 0\text{mm}$  at  $\nu_{x,y} = 1.31, 0.64$ .

Now, the particles carrying the Lissajous curve are painted over the phase plane with respect to the magic level of energy, and aberration of the spin vector is determined only by the second order momentum deviation  $\left(\frac{\delta p}{p}\right)^2$ . In particular, at  $\frac{\delta p}{p} = 10^{-4}$  and the beam emittance  $2\text{mm} \cdot \text{mrad}$ , SCT will be  $\sim 500$  sec.

## 05 Beam Dynamics and Electromagnetic Fields

### D02 Non-linear Dynamics - Resonances, Tracking, Higher Order

## FRINGE FIELD INFLUENCE

As we mentioned already the spin precession frequency depends on the electric field, and obviously it is different for particles with different trajectories. Just as it was done in [7], we have identified spin aberrations of the spin due to the difference of particle trajectories in electric quadrupoles and deflectors, with COSY Infinity taking into account the fringe fields.

Since the fringe field can lead to a coherent spin rotation, we should note that we are interested in the incoherent component of the spin oscillations.

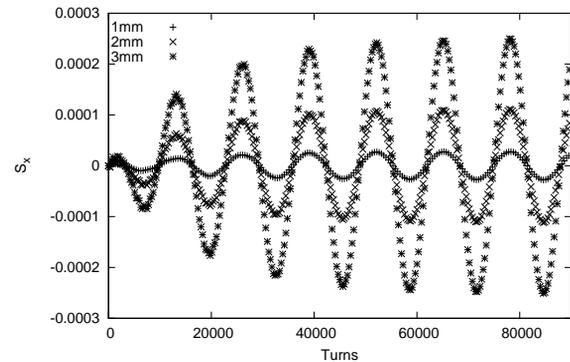


Figure 4: Incoherent spin oscillation for initial horizontal deviation  $x = 1, 2, 3\text{mm}$  in the lattice with fringe fields in all elements.

Figure 4 shows that the first spin aberrations increase, but then stabilized at a very low level acceptable for EDM experiments.

## CONCLUSION

In this paper we examined aberrations of spin emerging from various causes. We are convinced that the use of RF cavities is extremely important. However, the use of RF cavities does not permit to reach a required SCT, while keeping the beam in the ring for 3000 seconds. For this purpose it is necessary to paint the beam in the longitudinal plane by coupling between the longitudinal and transverse planes, which allows to increase SCT up to 500 sec.

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## REFERENCES

- [1] S. P. Møller, NIM, A 394, pp. 281–286 (1997).
- [2] Y. K. Semertzidis (Muon g-2 Collaboration), Nucl. Phys. B, Proc. Suppl. 117, p. 373 (2003).
- [3] F. J. M. Farley and Y. K. Semertzidis, Prog. Part Nucl. Phys. 52, 1 (2004).
- [4] M. Berz, Computational Aspects of Design and Simulation: COSY INFINITY NIM A298, p. 473 (1990).
- [5] A. P. Lysenko et al., Part. Accel. 18, p. 215 (1986).
- [6] Yu. Senichev and S. P. Møller, EPAC'00, Vienna, June 2000, pp. 794–796.
- [7] N. Golubeva and Yu. Senichev, Proc. of Linac Conf., 1988, CEBAF-report-89-001, p.663.