ENERGY LOSS AND LONGITUDINAL WAKE FIELD OF RELATIVISTIC SHORT ION BUNCHES IN ELECTRON CLOUDS *

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Abstract

The aim of our study is the numerical computation of the wake field and energy loss per unit length for a relativistic, short (< 10 ns) ion bunch penetrating an electron cloud residing in the beam pipe. We use two different self-consistent PIC codes: A standard 2-D electrostatic PIC code and a higher order 3-D PIC code which is based on the full-wave approach for the Maxwell equations in the time domain. The parameter scope of the codes refers to the CERN LHC and SPS accelerators.

INTRODUCTION

The interaction between an ion beam and an electron cloud may lead to coherent instabilities and beam loss. In this context, wake fields induced by a short, relativistic ion bunch in an electron cloud have already been obtained numerically in Ref. [1, 2]. Furthermore, analytic expressions for wake fields and impedances in the framework of the dielectric response theory were reported in Ref. [3]. In the present work we focus on the energy loss of bunches, as a result of the longitudinal wake field induced in the electron cloud in field free and dipole sections.

ENERGY LOSS AND RF PHASE SHIFT

This study has been motivated by observations in SPS and LHC. In both machines an increase in the shift of the synchronous phase $\Delta\phi_s$ with decreasing bunch spacing has been observed [4]. During beam storage the phase shift in a rf bucket is

$$\sin(\Delta\phi_s) = \frac{\Delta W_p}{qV_{rf}} \quad (1)$$

where q is the ion charge, V_{rf} is the rf amplitude and ΔW_p is the energy loss per particle and per turn. The measurement of $\Delta\phi_s$ can be used to gain information on the longitudinal impedance spectrum [5]. In LHC the observed dependence of the phase shift on the bunch spacing indicates that electron clouds can be the source of the energy loss. In general the stopping power S (energy loss of the total bunch per length unit) can be written as

$$\frac{dW}{ds} = -\int \rho_i(\vec{r})E_z(\vec{r})d^3r \approx -q\int \lambda(z)E_z(z)dz \quad (2)$$

where ρ_i is the bunch charge density, $E_z(z)$ is the longitudinal electric field induced by the bunch, $\lambda(z)$ is the line density of the bunch and $q = Ze$ is the ion charge. The energy loss per ion and per turn is

$$\Delta W_p = \frac{L}{N_i}\frac{dW}{ds} \quad (3)$$

where L is the ring circumference and N_i the number of ions in the bunch.

ELECTRON EQUATION OF MOTION

The rigid bunch of velocity $v_0 \approx c$ interacts with the electrons via its transverse electric field $E_r^i(r,z)$. Here we ignore the beam's magnetic field as well as any magnetic field induced by the electrons. For a transverse Gaussian beam profile one obtains for the electric field

$$E_r^i(r,z) = \frac{q\lambda(z,t)}{2\pi\epsilon_0 r}\left[1-\exp\left(-\frac{r^2}{2\sigma_\perp^2}\right)\right] \quad (4)$$

For the sake of simplicity we assume a round beam of radius $a = 2\sigma_\perp$. The line density of the bunch is assumed to be Gaussian with

$$\lambda(z) = \frac{N_i}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad (5)$$

where $z = z_0 - ct$, N_i is the number of ions in the bunch, σ_z is the rms bunch length. The resulting electron equation of motion is

$$r'' + \kappa^2(r,z)r = \frac{eE_r^e(r,z)}{m_e c^2} \quad (6)$$

where $\kappa(r,z)$ represents the focusing force due to the beam's transverse electric field and E_r^e is the electric field of the electron cloud. For $r < a$ the focusing gradient is

$$\kappa(z) = \frac{\sqrt{2\lambda(z)r_e}}{a} \quad (7)$$

The electron space charge field $E_r^e(r,z)$ induced by the bunch in the cloud has to be obtained from Gauss law with the electron charge density $\rho_e(r,z)$.

STOPPING POWER FOR SHORT BUNCHES

First we will ignore the effect of electron space charge. Furthermore we will assume that the bunch length σ_z is short relative to the electron oscillation length in the bunch center $\kappa^{-1}(0) = \kappa_0^{-1}$

$$\kappa_0\sigma_z \lesssim 1 \quad (8)$$

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and that the electrons are homogeneously distributed inside the beam pipe of radius $R_p \gg a$. In this case the majority of the electrons will simply receive a transverse impulse kick $\Delta p_\perp(b)$ from the passing bunch (see also Ref. [6]). b is the impact parameter or transverse distance between the electron and the beam axis. The total energy gain of the electrons per unit length is

$$\frac{dW_e}{ds} = \frac{1}{2} m_e n_e \int_0^{R_p} 2\pi \Delta p_\perp^2(b) b db \qquad (9)$$

and the stopping power

$$S = -\frac{dW_e}{ds} \approx 4\pi Q_i^2 n_e r_e \ln\left(\frac{R_p}{a}\right) \qquad (10)$$

The corresponding rf phase shift per unit length is (assuming $\Delta\phi_s \ll 1$)

$$\frac{d\Delta\phi_s}{ds} \approx \frac{4\pi Q_i n_e r_e}{V_{rf}} \ln\left(\frac{R_p}{a}\right) \qquad (11)$$

In a strong dipole field the electrons can only respond along the magnetic field lines and the stopping power given by Eq. 10 is reduced by the factor $1/2$. With electron space we use Mulser's 'oscillator model' [7] in order to calculate the stopping power. The energy loss of the bunch is obtained from the energy transferred into plasma waves

$$S = \frac{Q_i^2 \kappa_e^2}{4\pi\varepsilon_0} \ln\left(\frac{R_p}{a}\right) \exp(-\kappa_e^2 \sigma_z^2) \qquad (12)$$

which is exactly Eq. 10 multiplied by an exponential factor. $\kappa_e = \omega_{pe}/c$ is the inverse 'dynamical Debye length' and ω_{pe} the electron plasma frequency. For electron cloud densities exceeding $\kappa_e\sigma_z \approx 1$ the stopping power is reduced by the plasma shielding effect of the cloud.

SIMULATION MODEL

In order to go beyond the impulse kick approximation and to study the effect of the self-consistent electron space charge field a two-dimensional (2D), electrostatic Particle-In-Cell (PIC) code was employed. For selected parameters the results were compared to a full-wave, three-dimensional (3D) PIC code [8]. In the 2D model the electrons evolve in a (x, y) plane perpendicular to the bunch direction of motion. Poisson's equation is solved in 2D for the electrostatic potential each time step using the electron charge density $\rho_e(x, y, t)$ and the known bunch density $\rho_i(x, y, z = z_0 - ct)$. At the beam pipe electrons are either elastically reflected, absorbed or multiplied according to the SEY given in Ref. [9]. In our single bunch simulations we observe that the detailed choice of the SEY does not affect the stopping power. Only the electron density further behind the bunch can be affected. Finally, the stopping power in the 2D simulation model is obtained from Eq. 2 with the longitudinal electric field

$$E_z(z) = -\frac{1}{\pi a^2} \int_0^a \frac{\partial\phi}{\partial z} dxdy \qquad (13)$$

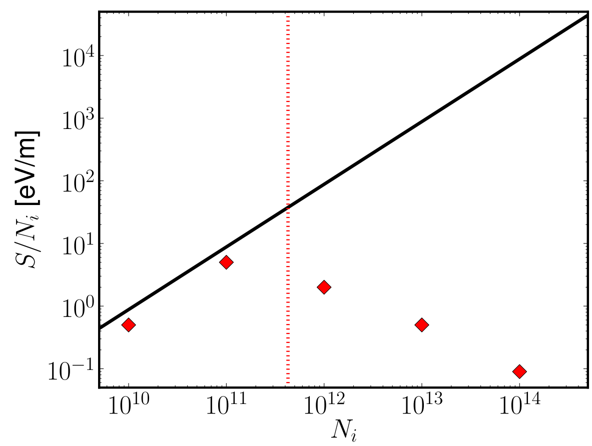

Figure 1: Stopping power as a function of the ion number in the bunch. The analytic results obtained from Eq. 10 is represented by the solid curve. The symbols represent the results obtained from the simulation. The red dashed line corresponds to $\kappa_0\sigma_z = 10$.

where the potential $\phi(x, y, z)$ is obtained numerically from the 3D Poisson equation at the end of a simulation run using the stored 2D densities $\rho_e(x, y, t)$ for each time step.

SIMULATION RESULTS

For a bunch length $\sigma_z = 0.1$ m, beam radius $a = 0.002$ m and pipe radius $R_p = 0.01$ m, which are close to the LHC parameters at injection energy, the stopping power divided by the number of bunch particles N_i is shown in Fig. 1 as a function of N_i. The electron density is chosen as $n_e = 10^{12}$ m^{-3}. The analytic results from Eq. 10 is compared to the stopping power obtained by solving Eq. 6 numerically (see previous Section). Electron space charge is included in the simulations, but it does not affect the stopping power and the wake field for $n_e = 10^{12}$ m^{-3}. The comparison of the analytical and numerical results shown in Fig. 1 indicates that the kick approximation is valid for $\kappa_0\sigma_z \lesssim 10$. Using Eq. 10 for an LHC bunch with $N_i = 10^{11}$ we arrive at $S \approx 6$ eV/m for the energy loss per ion and unit length, which corresponds to $\Delta\phi_s \approx 10^{-4}$ deg/m, for $V_{rf} = 3.5$ MV. For $N_i = 10^{11}$, bunch length $\sigma_z = 0.25$ m, beam radius $a = 0.004$ m and pipe radius $R_p = 0.02$ m, which are close to the SPS parameters at extraction energy, the stopping power obtained from Eq. 12 as a function of the electron cloud density is shown in Fig. 2. The symbols represent the simulation results and the vertical line indicates that the dynamical Debye length κ_e^{-1} is equal to the rms bunch length σ_z. One can observes that the stopping power from the simulations starts to drops at $\kappa_e\sigma_z \approx 1$. However, Eq. 12 underestimates the simulation results for electron densities above this value. It is worth noting that Eq. 12 with the substitution $\sigma_z \approx \sigma_z/2$

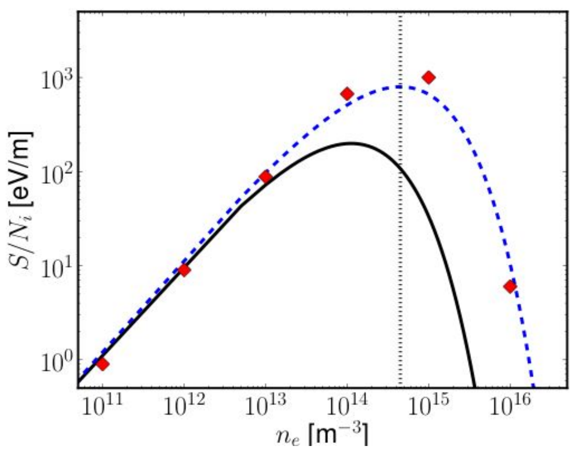

Figure 2: Stopping power as a function of the electron density. The analytic result obtained from Eq. 12 for $\sigma_z = 0.25$ m is represented by the solid curve. The symbols represent the results obtained from the simulations. The dashed blue curve is Eq. 12 with an adapted bunch length. The dotted line corresponds to $\kappa_e \sigma_z = 1$.

reproduces the simulation results very well, also in the case of a shorter LHC bunch. For electron cloud densities above $\kappa_e \sigma_z \approx 1$ the bunch excites undamped plasma waves in the electron cloud. This can clearly be seen in Fig. 4. In Fig. 3 we compare the longitudinal electric field obtained from the 2D model with the result from 3D EM simulations for $n_e = 10^{12}$ m^{-3} and SPS bunch parameters. For $n_e = 10^{12}$ m^{-3} and $\kappa_e \sigma_z \ll 1$ electron space charge plays only a very minor role. The shape of the two wake fields shown in Fig. 3 is very similar, we only observe some slight discrepancies in the wake field amplitudes. For $n_e = 10^{15}$ m^{-3} ($\kappa_e \sigma_z \approx 5$) the wake fields are shown in Fig. 4. Both simulations show the excitation of a long range plasma wave behind the bunch. In the 3D simulation the wave decays faster, which can be a result of longitudinal effects, that are absent in the 2D model.

CONCLUSIONS

The energy loss and wake field of a short, relativistic ion bunch in an initially homogeneous electron cloud has been studied within a 2D electrostatic simulation model. The results were compared to a 3D full EM simulation. We found that for sufficiently short bunches or $\kappa_0 \sigma_z \lesssim 10$ the energy loss can be described very well by an analytic formula. For electron densities well above the typical $10^{11} - 10^{12}$ m^{-3} or $\kappa_e \sigma_z \gtrsim 1$ the space charge field of the electrons reduces the energy loss. This 'plasma-shielding' effect is more important for longer bunches. For $n_e = 10^{12}$ m^{-3} and LHC bunch parameters we estimate a rf phase shift per unit length of $\Delta\phi_s \approx 10^{-4}$ deg/m.

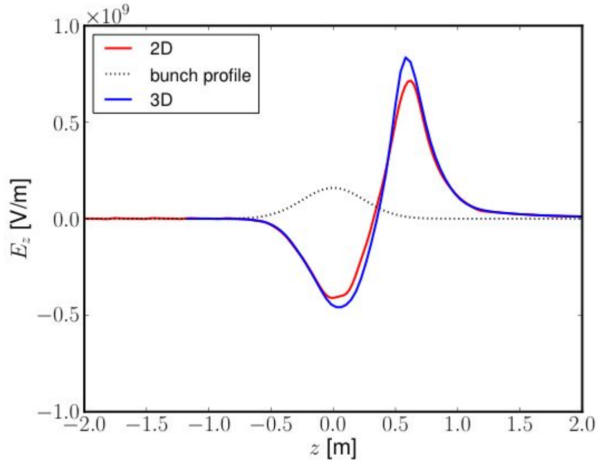

Figure 3: Longitudinal electric field obtained from the 2D ES and the 3D EM simulations for $n_e = 10^{12}$ m^{-3}.

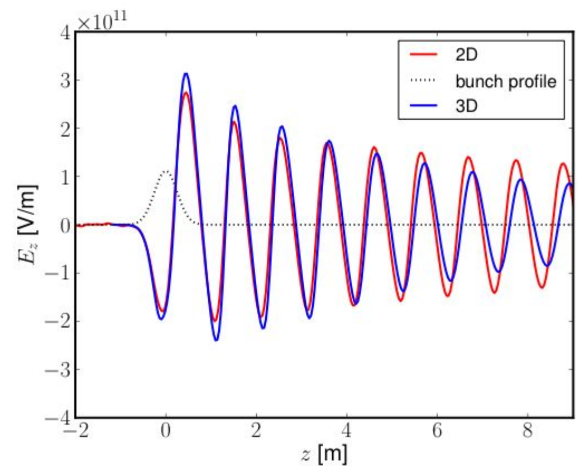

Figure 4: Longitudinal electric field obtained from the 2D ES and the 3D EM simulations for $n_e = 10^{15}$ m^{-3}.

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