AUTOMATIC POLE AND Q-VALUE EXTRACTION FOR RF STRUCTURES*

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Abstract

In this contribution a procedure for automatic pole fitting and external Q-value estimation for complex spectral data is presented. This approach works on scattering parameters either measured or resulting from numerical simulations. We present the algorithm and an application example.

INTRODUCTION

The experimental characterization of RF structures like accelerating cavities often demands for measuring resonant frequencies of eigenmodes and corresponding (loaded) Q-values over a wide spectral range. A common procedure to determine the Q-values is the -3dB method, which works well for isolated poles, but may not be applicable directly in case of overlapping multiple poles residing in close proximity (e.g. for adjacent transverse modes differing by polarization). Although alternative methods may be used in such cases, this often comes at the expense of inherent systematic errors. We have developed an automation algorithm, which not only speeds up the measurement time significantly, but is also able to extract eigenfrequencies and Q-values both for well isolated and overlapping poles. At the same time the measurement accuracy may be improved as a major benefit. To utilize this procedure merely complex scattering parameters have to be recorded for the spectral range of interest. In this paper we present the proposed algorithm applied to experimental data recorded for superconducting higher order mode damped multicell cavities as an application of high relevance.

THEORY

Modes and Poles in RF-Structures

An arbitrary electromagnetic field inside an rf-structure can be decomposed into an infinite set of specific eigenmodes, which are a solution to the Helmholtz-eigenproblem with appropriate boundary conditions. Here, we will focus on the electric field, but the following argument holds for the magnetic field analogously. The electric field inside an rf-structure can be written as:

$$\mathbf{E}(\mathbf{r},t) = \sum_{i=1}^{\infty} \mathbf{E}_i(\mathbf{r})\, e^{-\alpha_i t} \sin(\omega_i t + \phi_i). \qquad (1)$$

with $\mathbf{E}_i(\mathbf{r})$, α_i, ω_i and ϕ_i denoting the spatial distribution of the electric field, an attenuation constant, the resonance frequency of the eigenmode and the phase offset of the i-th mode to a certain reference phase, respectively. In frequency domain equation (1) reads as

$$\underline{\mathbf{E}}(\mathbf{r},\omega) = \sum_{i=1}^{\infty} \frac{\underline{\mathbf{E}}_i(\mathbf{r})}{\omega - \underline{p}_i}. \qquad (2)$$

The complex quantities are given by:

$$\begin{aligned} \underline{\mathbf{E}}_i(\mathbf{r}) &= \mathbf{E}_i(\mathbf{r}) e^{j\phi_i} \\ \underline{p}_i &= \omega_i + j\alpha_i \end{aligned} \qquad (3)$$

with $j^2 = -1$ denoting the imaginary unit.

As a consequence of equation (2), also scattering parameters, which are derived from either the electric or magnetic field by means of a modal development, can be written as

$$\underline{S}(\omega) = \sum_{i=1}^{\infty} \frac{\underline{a}_i}{\omega - \underline{p}_i}, \qquad (4)$$

with $\underline{a}_i$ resulting from an integration of the spatial field distribution $\mathbf{E}_i(\mathbf{r})$ over selected cross sections with a subsequent weighting. For most technical applications, the number of modes can be reduced to a finite number.

In the following considerations it is assumed that complex scattering parameters have been sampled at discrete frequencies:

$$\begin{aligned} \omega_i &= \{\omega_1, ..., \omega_n\} \\ \underline{S} &= \{\underline{S}_1, ...\underline{S}_n\}. \end{aligned} \qquad (5)$$

Initial Resonance Frequency Estimation

For the proposed pole fitting algorithm, an initial knowledge of the resonance frequencies' positions in the recorded data set is necessary. For this first estimate the following technique can be employed: With the discrete convolution kernel

$$g := \{1, -2, 1\} \qquad (6)$$

the second derivative of the measured spectrum can be approximated (neglecting a constant scaling factor) as

$$\underline{S}'' \approx g * \underline{S}. \qquad (7)$$

In the approximated second derivative S'', resonance peaks will prominently stand out against the background noise and can be automatically extracted.

* Work supported by the German Ministry for Education and Research (BMBF) under contract 05K10HRC.

Pole Fitting

With this first estimate of the resonance frequencies an initial fit can be performed. For this purpose, we assume that all poles are well separated. In this case the scattering parameters in the vicinity of the k-th pole can be approximated as:

$$\underline{S}(\omega) \approx \frac{\underline{a}_k}{\omega - \underline{p}_k} + \underline{R}_k, \tag{8}$$

with $\underline{R}_k$ summarizing the influence of all other poles as a constant term [1]. For the i-th sampled frequency point, (8) reads:

$$\underline{S}_i(\omega_i)(\omega_i - \underline{p}_k) - \underline{R}_k(\omega_i - \underline{p}_k) = \underline{a}_k. \tag{9}$$

By taking a subset of the data in the vicinity of the initial resonance frequency, an overdetermined system of equations can be assembled:

$$\begin{pmatrix} \omega_1 \underline{S}_1 \\ \vdots \\ \omega_n \underline{S}_n \end{pmatrix} = \begin{pmatrix} \underline{S}_1 & \omega_1 & 1 \\ \vdots & \vdots & \vdots \\ \underline{S}_n & \omega_n & 1 \end{pmatrix} \begin{pmatrix} \underline{p}_k \\ \underline{R}_k \\ \underline{a}_k - \underline{R}_k \underline{p}_k \end{pmatrix}. \tag{10}$$

Its least square solution yields an initial estimate of the unknown pole parameters.

In a subsequent step, the initial fit is successively corrected. It is assumed that all but the k-th pole are correct. In this case, equation (8) can be written as

$$\underline{S}(\omega) = \frac{\underline{a}_k}{\omega - \underline{p}_k} + \underline{R}_k(\omega), \tag{11}$$

with a frequency dependent residual $\underline{R}_k(\omega)$.

Similar to (8) an overdetermined system of equations can be assembled, whose solution is a new set of parameters $\underline{p}_k$ and $\underline{a}_k$ for the k-th pole. This process of correcting single poles is repeated successively for all poles until the pole parameters have converged and no further improvement can be achieved. The resulting poles are the best found approximation of the data set.

APPLICATION EXAMPLE

In this section we present an application example of the proposed pole fitting scheme. As input data we use complex scattering parameters measured for a 1.5 GHz seven-cell CEBAF upgrade-type SRF cavity (R100-3, [3]) at 2 K in a vertical test stand. The transmission was measured between the input coupler and one of the two higher order mode couplers with a resolution of 10 kHz. To reduce the number of modes necessary for the pole fit, the phase of the scattering parameters was purified from the spurious phase of the connected coaxial cables. This significantly reduces the number of necessary modes. Figure 1(a) shows the magnitude of the measure transmission in the TM_{110} pass band. Figure 1(b) shows the magnitude of the approximated second derivative using equation (7).

The peaks above -90dB were taken as initial guess for the resonance frequencies. For the subsequent fit, all data

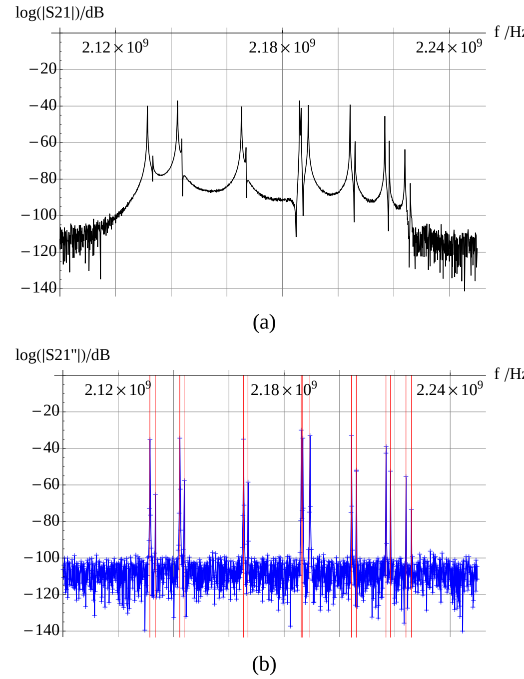
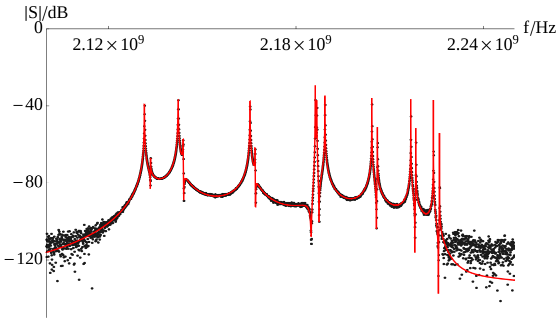


Figure 1: (a) Magnitude of transmission in the TM_{110} pass band measured at R100-3 at 2K in vertical test bench. (b) Magnitude of the second derivative of the transmission. Each peak indicates one resonance frequency and is additionally marked by a red line.

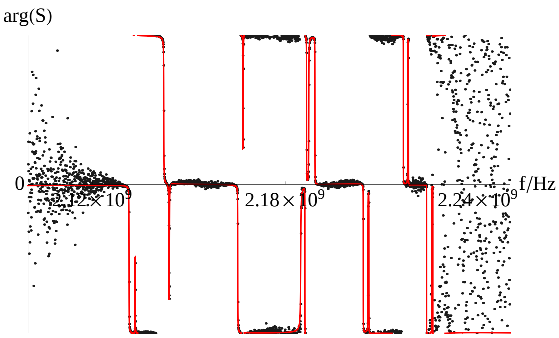
points around the initial resonance frequencies within an interval of 10dB were considered. After the initial fit, ten subsequent iterations were performed for error correction.

Figure 2 shows the measured data (black dots) and the spectrum reconstructed from the fitted poles (equation (4)). Both the measured data and the spectrum reconstructed from the fitted poles are in excellent agreement, indicating that the pole fitting algorithm is working properly. Figure 3 shows the external Q-factors computed from the fitted poles. Here, the red circles indicate the external Q-values measured directly using the -3dB method. The second, forth and sixth poles (indicated by green circles) are overlapping with other poles and could therefore no be measured directly. In such cases more error-prone measurement techniques need to be applied, here a worst case approximation using only half of the 3dB bandwidth. The successful solution of this experimental issue is a main benefit of the used pole fitting algorithm. In fact, the algorithm can resolve overlapping poles. A comparison of the measured and reconstructed spectra in Fig. 2 shows that overlapping poles are excellently fitted thereby delivering both, loaded Q-values and resonance frequencies, precisely.

Another discrepancy between measured and fitted data is obvious for the last two poles amounting to approximately 15% for the Q-values. Here, the sampling resolution of 10 kHz was 6 times larger than the bandwidth of the two poles ($\approx$ 1.6 kHz). Yet, even with this rather strong undersampling, the fit is able to deliver reasonable Q-values.



(a)



(b)

Figure 2: Magnitude (a) and phase (b) of the measured (black) and fitted (red) transmission in the TM_{110} passband. The fitted and measured spectra are in excellent agreement except for a resonance at 2.23 GHz, which was hardly detectable since residing in the noise level below -110 dB. This mode was therefore not included in the fitting process.

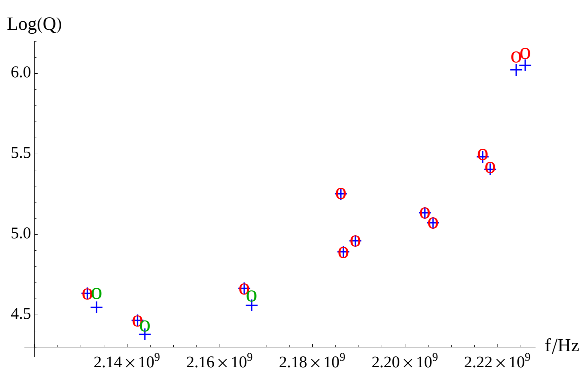


Figure 3: External Q-values: Blue crosses indicate fitted q-values, red circles indicate resonances measured with the 3dB method. Green circles indicate external Q values measured with the 3dB half method.

CONCLUSIONS

In this contribution we presented an automatic pole extraction algorithm which allows for the computation of external Q-values from measured spectra. The pole fitting is a two step process with first step being an initial fit, which is improved by successive corrections in the second phase.

We demonstrated the powerful potential of the algorithm by means of fitting complex scattering parameters as measured for a multi-cell SRF cavity at JLAB. Even with a very moderate sampling resolution, the fitted Q values and the values extracted from the pole fit are in excellent agreement. This can be linked to the higher dynamic range of the proposed method: While measurement techniques typically rely on data within the 3dB bandwidth of the resonance, the proposed method can use a considerably larger range for the fitting process.

A second advantage results from the ability to accurately fit poles residing in very close proximity and resolving even overlapping modes. Usually in this case, the -3dB method is not directly applicable to measure these poles correctly and thus other less precise methods have to be used. We have shown that the proposed correction scheme can even resolve overlapping poles. Thus, information on resonance frequencies and Q values can be extracted precisely which would otherwise be inaccessible by experimental measures.

Last but not least, one major advantage results from massive time savings that can generally be expected by utilizing the automatic fitting routine. As in the example presented here, a laborious quality assurance program is carried out at JLab for each production type cavity to characterize its broadband damping efficiency. While the usual experimental time effort results in several hours per cavity, the fitting process is performed in a few seconds on a conventional workstation utilizing merely the measured transmission spectra, which takes a few minutes at most. This grants to perform even more detailed studies not practicable under usual laboratory constraints, e.g. allowing to record a wider spectral range with better resolution in a reasonable time scale. For instance, by employing our newly developed routine, we were able to perform a detailed HOM survey for the first fabricated CEBAF upgrade type cryomodule R100 housing eight seven-cell cavities [3]. Hereby, we have completed full-string measurements in 8 different experimental configurations within less than three working days, which otherwise would have resulted in a prohibitively vast laborious effort of several weeks or months with present experimental techniques.

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