SELF-CONSISTENT DYNAMICS OF ELECTROMAGNETIC PULSES AND WAKEFIELDS IN LASER-PLASMA INTERACTIONS *

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Abstract

In the present work we study the dynamics of laser pulses propagating in a cold relativistic plasma, which can be of interest for particle acceleration schemes. After obtaining a Lagrangian density from the one-dimensional equations for the laser pulse envelope and the wakefield, we define a trial function and apply the variational approach in order to obtain an analytical model which allows us to calculate an effective potential for the pulse width. Using this procedure, we analyze the stability of narrow and large laser pulses and then compare its results with numerical solutions for the envelope and density equations.

INTRODUCTION

Intense electromagnetic pulses displace plasma electrons and create a resulting ambipolar space-charge field with the associated density fluctuations, here known as the wakefield, which can be used as an accelerating structure [1, 2, 3, 5, 7, 8, 9, 11]. Investigation of the laser pulse and wakefield coupled dynamics has been done in the literature but, since focus has been mostly directed to fast pulses, frequently phase and group velocity are approximated by the speed of light c [6] and pulse distortions are sometimes neglected or treated under stationary wave assumptions [12].

As these approximations can be restrictive if one desires to follow the time dependent dynamics of laser pulses along the direction of modulation, we want to examine how the system behaves when they are relaxed. We shall investigate to what extent can a pulse retain its initial shape following its interaction with the wakefield. For a given pulse power, the dynamics is largely dictated by the pulse width [14]. One of the findings here is that while wider pulses with widths larger than c/ω_p may keep their shapes even in the presence of space-charge fields, narrow pulses with widths comparable to c/ω_p always tend to spread as time evolves.

THE MODEL

Following previous works [3, 4, 6], we consider our system as consisting of a mobile cold electronic fluid and a neutralizing fixed ionic background. All fields propagate along the x axis of our coordinate system. The laser field is described by the vector potential $\mathbf{A} = \hat{\mathbf{z}}\, a(x,t)e^{i(k_0x-\omega_0t)} + \ c.c.$, where a is the slowly varying complex amplitude of the field, with k_0 and ω_0 respectively as the wavevector and frequency of the high-frequency carrier. From the wave equation for the vector potential and the equations of motion for the electron, one can write the following equations (valid for $|ea/mc^2| \ll 1$)

$$2i\left(\omega_0\frac{\partial a}{\partial t} - k_0\frac{\partial a}{\partial x}\right) - \frac{\partial^2 a}{\partial t^2} - \frac{\partial^2 a}{\partial x^2} = \left(n - \frac{|a|^2}{2}\right)a, \quad (1)$$

$$\frac{\partial^2 n}{\partial t^2} + n = \frac{1}{2}\frac{\partial^2 |a|^2}{\partial x^2}, \quad (2)$$

where we have migrated to the dimensionless quantities $\omega_p x/c \to x$, $\omega_p t \to t$, $ea/mc^2 \to a$, and $(n-n_0)/n_0 \to n$. Here n_0 is the equilibrium density and $\omega_p \equiv 4\pi n_0 e^2/m$ is the plasma frequency. We note that k_0 and ω_0 are normalized likewise. We now introduce the wave frame coordinates $\tau = t$ and $\xi = x - v_g t$, where $v_g = k_0/\omega_0$ is the group velocity of the radiation. Defining a new wakefield potential function $\varphi \equiv v_g^2 n - |a|^2/2$ and $\kappa \equiv v_g^2 - 1$, if one moves to the new coordinates and realizes that due to the slow modulations $\partial/\partial\tau \ll v_g\partial/\partial\xi$, eqs. (1) and (2) can be written as [14]

$$-2i\omega_0\frac{\partial a}{\partial \tau} + \kappa\left(\frac{\partial^2 a}{\partial \xi^2} - \frac{|a|^2 a}{2v_g^2}\right) + \frac{\varphi a}{v_g^2} = 0, \quad (3)$$

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{1}{v_g^2}\varphi = -\frac{|a|^2}{2v_g^2}, \quad (4)$$

We note that, if we had chosen $v_g = c$, the coefficient κ would be zero and we would neglect terms in eq. (3) that, as we shall see, are substantial to determine the longitudinal dynamics of the interaction. From eq. (4) we have:

$$\varphi(\xi) = \frac{1}{2v_g}\int_{\xi}^{\infty} \sin\left(\frac{\xi - \xi'}{v_g}\right)|a(\xi')|^2 d\xi'. \quad (5)$$

Eqs. (3) and (4) incarnates our basic model and expression (5) shall be used in in the coming simulations but let us focus presently on two limits which can be examined.

Wide Pulses

The first case is the one where the width Δ of the laser pulse is much larger than the plasma wavelength c/ω_p or, in our dimensionless variables, $\Delta \ll 1$. Under this condition $|a|^2$ varies slowly and, if one approximates $\partial|a|^2/\partial\xi \to 0$, the wide pulse dynamics can be described by a Nonlinear Schrödinger Equation (NLS) of the form

$$-2i\omega_0\frac{\partial a}{\partial \tau} + \kappa\frac{\partial^2 a}{\partial \xi^2} - \frac{|a|^2 a}{2} = 0. \quad (6)$$

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Modeling the pulse as a Gaussian with time dependent amplitude and width, Lagrangian average methods [6, 10, 13] quickly reveal that under the present circumstances a stable pulse solution to eq. (6) does exist, which can be found as the minimum of the effective potential

$$U_{eff}^{wide} = \frac{\kappa}{2\pi^2\omega_0^2\Delta}\left(\frac{4\kappa}{\Delta}+W\right), \tag{7}$$

$W = \int |a|^2 dx$ measuring the photon number within the pulse. U_{eff}^{wide} has one minimum, and its general form is illustrated in fig. 1, panel (a). One can write the equilibrium width Δ_w at the potential minimum and the oscillatory frequency Ω of slightly perturbed pulses around it as:

$$\Delta_w = 8|\kappa|/W \ \ , \ \ \Omega = W^2/(32|\kappa|\pi\omega_0). \tag{8}$$

If $\Delta_w \gg 1$ the stable solution is located in the wide pulse region and the wide pulse approximation should be expected to remain valid for all times if the initial condition lies sufficiently close to the stable solution and satisfies $\dot{\Delta} = 0$. When $\Delta_w \lesssim 1$, even if one starts with an initially wide pulse $\Delta(0) > 1$ the off-equilibrium pulse will drift towards the equilibrium Δ_w. However, since Δ_w is now out of the wide pulse regime , we shall resort to numerical simulations to investigate the dynamics there.

Narrow Pulses

Narrow pulses act like delta functions. Therefore $\varphi \approx 0$ inside the pulse, although it can be large behind where it reads $\varphi(\xi,\tau) \approx (1/2v_g)\, W \sin\left[(\xi-\xi_0(\tau))/v_g\right]$ with $\xi_0(\tau)$ as the pulse position. In this regime, the full expression (3) can be approximated by the following NLS

$$-2i\omega_0\frac{\partial A}{\partial\tau} + \kappa\left(\frac{\partial^2 A}{\partial\xi^2} - \frac{|A|^2 A}{2v_g^2}\right) = 0. \tag{9}$$

Lagrangian average method now reveals that regardless of the pulse power no static solution is to be found, with any pulse-like initial condition always spreading out as time evolves. The effective potential in this case reads

$$U_{eff}^{narrow} = \frac{\kappa^2}{2\pi^2 v_g^2\omega_0^2\Delta}\left(\frac{4v_g^2}{\Delta}+W\right). \tag{10}$$

As U_{eff}^{narrow} has no local minimum along the positive Δ axis, the width grows until it reaches the transition region $\Delta \sim 1$ where the approximation again fails and numerical work is required. U_{eff}^{wide} is represented in fig. 1, panel (b).

FULL SYSTEM *VS*. ESTIMATES

With the previous estimates at our disposal, we now proceed to investigate the full system defined by eqs. (3) and (4) or (5) using shaded contour plots for the laser intensity $|a(\xi,\tau)|^2$, with brighter shades corresponding to higher amplitudes. In fig. 2 we display the case where a fixed

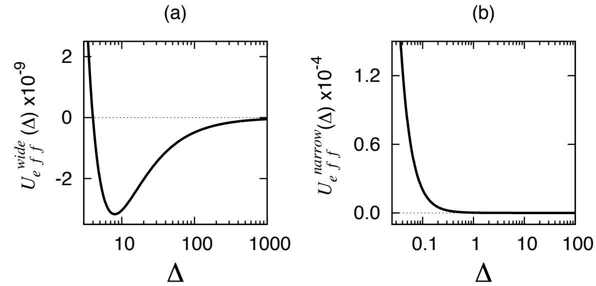

Figure 1: (a) U_{eff}^{wide} and (b) U_{eff}^{narrow} for $v_g^2 = 0.99$ and $W = 0.01$.

point can be found in the wide pulse region. Panel (a) depicts the approximation obtained from the NLS eq. (6), and in panel (b) we plot the full solution to the set (3), (5). We take $W = 0.01$ with $v_g^2 = 0.99$, for which $\kappa = 0.01$ and $\Delta_w = 8$. We launch a pulse with the shape of a hyperbolic secant of width $\Delta(0) = 10$ and initial momentum (associated with its expansion or contraction) null.

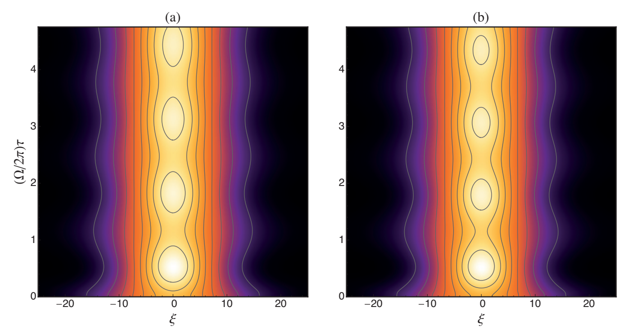

Figure 2: Countour plots of $|a(\xi,\tau)|^2$ for $W = 0.01$, $v_g^2 = 0.99$ and $\Delta(0) = 10$ (wide condition).

Fig. 3 is plotted for the same parameters, but now starting from a narrow width $\Delta(0) = 0.25$. Comparison of the dynamics generated by the approximated NLS (9) as shown in panel (a), and by the full system (3) and (5) in panel (b), reveals that for short times the pulse starts opening up, as predicted by the narrow pulse approximation. For later times panel (c) shows that the pulse crosses the transition region $\Delta \sim 1$ mostly unhindered, and keeps opening up as it reaches and moves further into the wide region.

We next investigate the case where $W = 0.1$, keeping $v_g^2 = 0.99$. Now $\Delta_w \sim 0.8 < 1$, which means that no stable solution exists in the wide region either. Pulses starting from wide configurations initially have their width reduced as commented earlier but, when $\Delta \sim 1$, they are strongly affected by the wakefield as demonstrated in fig. 4. Panels (a) and (b) once again respectively refer to the NLS (6) and the full system for short times, while panel (c) depicts the full system for long enough times that allow the pulse to reach the transition region. In this case, the initial effective potential is not large enough to provide sufficient “momentum” with which the pulse could clear the transition region, and pulse distortion is thus appreciable.

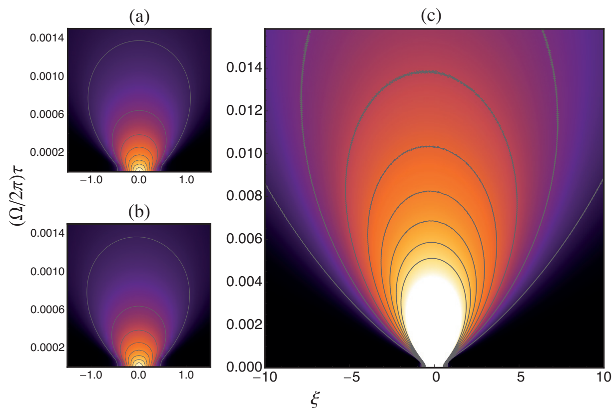

Figure 3: Contour plots of $|a(\xi,\tau)|^2$ for $W = 0.01$, $v_g^2 = 0.99$ and $\Delta(0) = 0.25$ (narrow condition).

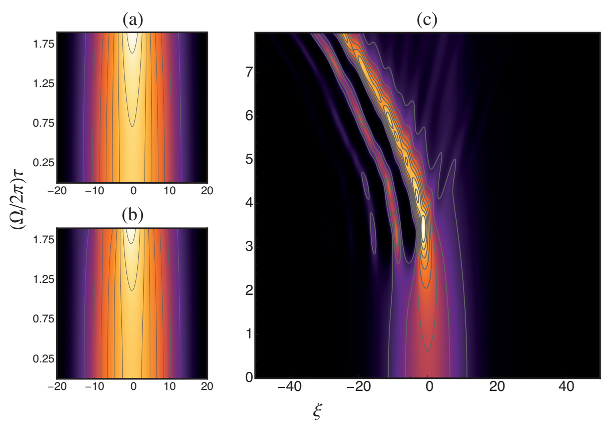

Figure 4: Contour plots of $|a(\xi,\tau)|^2$ for $W = 0.1$, $v_g^2 = 0.99$ and $\Delta(0) = 10$ (wide condition).

Narrow pulses however have the same sort of behavior observed for $W = 0.01$. Fig. 5 depicts the space-time history of a pulse starting from $\Delta(0) = 0.25$: panels (a) and (b) representing the NLS narrow approx. eq. (9) and the full system respectively show that, for short times, both dynamics coincide; panel (c) shows the pulse crossing the transition $\Delta \sim 1$. At later times the distortion is more prominent, as compared with that of fig. 3, because of the higher intensity wakefields generated here.

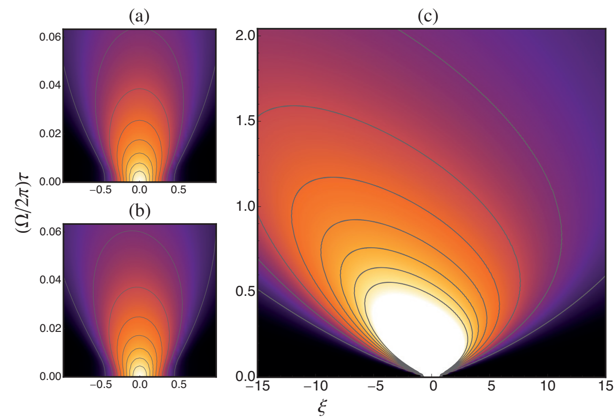

Figure 5: Contour plots of $|a(\xi,\tau)|^2$ for $W = 0.1$, $v_g^2 = 0.99$ and $\Delta(0) = 0.25$ (narrow condition).

CONCLUSIONS

This work was devoted to the study of the coupled dynamics of laser pulses and wakefields in laser-plasma systems. Average Lagrangian methods have been employed to create estimates used to guide the investigation. Stable laser pulses were found in low power regimes where pulse width is much larger than the plasma wavelength, $\Delta \gg c/\omega_p$. In that case estimates and full simulations of the coupled system agree to a large extent.

In cases of high power pulses, stable solutions are absent. While pulses launched from wide initial conditions shrink until reach the transition region $\Delta \sim c/\omega_p$, where they are heavily distorted, pulses with narrow initial conditions $\Delta \ll c/\omega_p$ traverse the transition region and keep spreading as they move deeper into the wide regimes. The asymmetry is credited to the fact that narrower pulses always depart from higher effective potentials, enabling them to cross the transition region due to inertial effects. If $\Delta \sim c/\omega_p$, wakes are strongly excited. When narrow pulses traverse the transition region, wakes are briefly excited for as long as the pulse stays in the transition region. When the pulse comes from the wide pulse side and is allowed to reach the transition in higher power regimes such that $\Delta_w \sim c/\omega_p$, it remains partially trapped there. Wakes are excited for longer stretches of time, albeit in an incoherent form.

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