

# SOME CONSIDERATIONS IN REALIZING A TeV LINEAR COLLIDER BASED ON PDPWA SCHEME

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## Abstract

Proton-driven plasma wakefield acceleration (PDPWA) has recently been proposed as an approach to bring the electron beam to the energy frontier in a single passage of acceleration. Particle-in-Cell (PIC) simulation shows that a TeV proton bunch, with a bunch intensity of  $10^{11}$ , and a bunch length as short as 100 microns can resonantly excite a large amplitude plasma wakefield and accelerate an externally injected electron bunch to 600 GeV in a single stage of 500 m long plasma. This novel PDPWA scheme may open a new path for designing a TeV linear lepton collider by using the currently available proton drivers. In this paper, we investigate some key issues, e.g. bunch length, centre-of-mass (CoM) energy, luminosity and dephasing in realizing a TeV linear collider based on the PDPWA scheme.

## INTRODUCTION

Plasma acceleration has made great progress in the last decade. The wakefields driven by either a short and intense laser pulse (LWFA) or by an intense and relativistic electron beam (PWFA) have already demonstrated the field amplitude over 50 GeV/m, which are more than two orders of magnitude higher than the field amplitude in conventional RF-based structure [1, 2]. However, to reach the TeV energy level, both LWFA and PWFA have some technical limitations, e.g. the synchronizing, timing and aligning of many acceleration modules to reach such high energy may be challenging.

Recently, Caldwell et al. has proposed a new acceleration scheme by employing the high energy proton bunch as the drive beam. This is so-called “proton-driven plasma wakefield acceleration” (or abbreviated as PDPWA) [3, 4]. The advantage of the PDPWA is the availability of the high energy proton beams from the current proton synchrotrons, e.g. Tevatron and the LHC. Meanwhile, the energies stored in such proton drivers are in the range of 10-100 kJ, 2-3 orders of magnitude higher than that of the SLAC electron beam of 100 J. If such high energies of the proton drivers can be efficiently coupled to the witness beams through plasmas, one can imagine achieving very high witness beam energies.

In this PDPWA scheme, a high energy proton bunch is sent into a uniform plasma, the space charge of the proton beam attracts the ambient plasma electrons towards to the vicinity of the beam. While the plasma ions remain stationary due to their greater mass compared to the plasma electrons. These plasma ions will then provide a restoring force to the plasma electrons and the electrons rush back and forth towards

the beam axis. This very quick movement of the plasma electrons corresponds to a very high frequency electric field (wakefield). Surfing on the right phase of the wakefield, an externally injected electron bunch can gain a significant amount of energies from this field. The simulation has shown that a 1 TeV proton bunch, with a bunch length of 100  $\mu\text{m}$  and a bunch intensity of  $10^{11}$  can excite a wakefield amplitude around 2 GeV/m. An externally injected 10 GeV electron beam can witness the field and gain energy up to 620 GeV after propagating a 500 m long plasma with density of  $6 \times 10^{14} \text{ cm}^{-3}$ . The final energy spread of the electron beam at the highest energy is about  $\sim 1\%$ . If demonstrated, this PDPWA may open a new research frontier, especially for application in the future high energy lepton colliders. We have recently submitted a proposed experiment for the PDPWA based on the CERN SPS beam [5, 6, 7]. The basic idea is to use the SPS proton beam as the driver pulse to study the interactions of proton beams and plasmas. The target of the experiment is to demonstrate the capability of the PDPWA and to realize one GeV energy gain for the electron beam with a ten-metre long plasma. The experience gained from the first round of experiment will provide valuable inputs for the future design of a high energy lepton collider based on the PDPWA scheme.

## SHORT PROTON DRIVER

Given the great potential that PDPWA can bring the electron beam to the energy frontier in a single stage plasma, a multi-TeV linear collider based on this scheme can be considered. However, one of the most critical issues in realizing a TeV collider is the short proton bunch production.

Since TeV proton beam is very energetic and rigid, it is therefore difficult to use a conventional magnetic chicane or other means to compress the bunch length from initially tens of centimeters to sub millimetres (a compression ratio of 1000). Preliminary simulation shows that for a magnetic bunch compressor design, a formidable RF power is required to chirp the bunch and large dipole magnets are needed for the path modulation [8]. Some other bunch compression schemes such as emittance exchange, fast nanochopper or bunch compression via tuning the momentum compaction factor in synchrotron are still not mature [9].

Fortunately, plasma may help us achieve the short bunch length via modulation. When a long proton bunch is injected into a high density plasma, the particles in the bunch head excites the wakefield, the transverse wakefield (transverse instability) modulates the particle

distribution in the rear part of the bunch, and which in turn many ultra-short bunch slices are produced due to the transverse wakefield. The distance between the two adjacent bunch slices is one plasma wavelength. The plasma wavelength is inversely proportional to the square root of plasma density. Therefore, the higher the plasma density, the more numbers of the bunch slices. When these ultra-short bunch slices propagating further through a long plasma cell, they excite the wakefield coherently and eventually the wakefield saturate and reach up a high amplitude, which is comparable to the short driver case [10]. This wakefield can accelerate not only the proton bunch but also an externally injected electron bunch. This mechanism is very much like the self-modulated laser wakefield acceleration (SM-LWFA) in which a long laser pulse is split into many short pulses in a high density plasma [11].

More recently, simulation shows that a LHC beam, with a beam energy of 7 TeV, a bunch intensity of  $1.15 \times 10^{11}$  and an rms bunch length of 7.55 cm can excite a wakefield amplitude of 1.5 GeV/m by working in self-modulation regime. An externally injected electron bunch can be accelerated to 6 TeV after propagating a 10 km plasma [10]. These exciting results can potentially pave a way to realize a multi-TeV linear collider based on the self-modulation regime. In doing so, acceleration of positron beam in the self-modulation regime needs to be investigated as well. A recent preliminary PIC simulation has demonstrated that a positron beam can also be accelerated via the self-modulated wakefield driven by an SPS proton beam [12].

## LUMINOSITY

A next generation TeV lepton collider is in principle needed to complement the results from the Large Hadron Collider (LHC), which is now in full swing at CERN. However, due to synchrotron radiation, the high energy  $e^+/e^-$  collider has to be linear, so as to avoid the severe energy loss in a curved path when the beam energy goes up. For a linear collider, the accelerating gradient will determine the scale of the machine. Since plasma can sustain very high electric field, it is foreseen that the plasma-based accelerator can minimize the scale and therefore the cost of future linear collider significantly.

There are two figures of merit for a collider. One is the CoM energy and the other is the luminosity. The CoM energy indicates the interesting working region of the machine, while the luminosity determines the production rate for a particle of interest. For two Gaussian beams of electrons and positrons, the luminosity is given by

$$L = f \frac{N^+ N^-}{4\pi\sigma_x^* \sigma_y^*} \quad (1)$$

Here  $f$  denotes the collision rate (frequency) of beam,  $N^+$  and  $N^-$  the bunch population for electrons and positrons,  $\sigma_x^*$  and  $\sigma_y^*$  the horizontal and vertical beam sizes at the interaction point (IP). Using the SPS beam as the drive beam, we calculate the luminosity which would be achieved. The bunch intensity of the SPS beam is around

$1.15 \times 10^{11}$ . The SPS provides 288 bunches with a cycle time around 14 s, the frequency of the beam is therefore around 20. Assume that we can focus the electron and positron beam sizes to the CLIC level at the IP, that is, the beam sizes are 60 nm and 0.7 nm in horizontal and vertical directions, respectively. If the loaded electron (positron) bunch charge is 10% of the driver charge, namely, the bunch intensity of the electron and positron is  $1.15 \times 10^{10}$ . The estimated luminosity is around  $5 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ , which is two orders of magnitude lower than the specific luminosity of the ILC.

One can easily draw the conclusion according to Eq. (1) that the small beam sizes at IP or a high collision repetition rate are needed to achieve a high luminosity. SPS+ will be one of the LHC injector upgrade projects. It will provide maximum beam energy of 1 TeV. The cycle time for the 288 proton bunches is around 2 s. It may increase the luminosity of a collider by a factor of a few if a SPS+ beam is used as the drive beam.

## DEPHASING ISSUE

Surfing on the right phase of the wakefield, the electrons can quickly be accelerated to the relativistic energy regime. Due to the heavy mass of proton, the relativistic factor  $\gamma$  of a TeV proton beam is smaller than that of an electron beam with energy of 1 GeV. Therefore the electrons may overrun the wakefield (the group velocity of the wakefield is the same as the velocity of the driver) and the acceleration process will cease. Dephasing therefore becomes a limiting factor for PDPWA, especially for the  $e^+/e^-$  acceleration to reach the energy frontier in a single passage over a very long plasma channel.

We estimate in the following the conditions to avoid the significant dephasing in a PDPWA. To simplify the problem, we assume the wakefield structure in the co-moving frame does not evolve in time. It means that the protons (electrons) experience a constant deceleration (acceleration) field of magnitude  $E_{dec}$  ( $E_{acc}$ ). The rate of change of proton and electron energy are written as

$$\frac{d(\gamma_i m_i c^2)}{dt} = -q E_{dec} v_i$$

$$\frac{d(\gamma_e m_e c^2)}{dt} = e E_{acc} v_e$$

where the subscript  $i$  and  $e$  denote proton and electron, respectively.

The relative position change between an electron and a proton at a given time  $T$  is given by [13]

$$\Delta d = \int_0^T (v_e - v_i) dt = \frac{m_e c^2}{e} \left[ \frac{\gamma_e - \gamma_{e0}}{E_{acc}} + \frac{m_i e}{m_e q} \frac{\gamma_i - \gamma_{i0}}{E_{dec}} \right]$$

here  $\gamma_{e0}, \gamma_e$  are the relativistic factor of the initial and final energies of electron,  $\gamma_{i0}, \gamma_i$  are the relativistic factor of the initial and final energies of proton.

The equations for the momentum are as following

$$\frac{d(\gamma_i m_i v_i)}{dt} = -q E_{dec}$$

$$\frac{d(\gamma_e m_e v_e)}{dt} = q E_{acc}$$

Integrating the above momentum equations from 0 to  $T$ , we have

$$m_e c (\sqrt{\gamma_i^2 - 1} - \sqrt{\gamma_{i0}^2 - 1}) = -q E_{dec} T$$

$$m_e c (\sqrt{\gamma_e^2 - 1} - \sqrt{\gamma_{e0}^2 - 1}) = e E_{acc} T$$

Combining the above two equations, we get

$$\Delta d = \frac{m_e c^2}{e E_{acc}} (\gamma_e - \gamma_{e0}) \left[ 1 - \frac{(\sqrt{\gamma_e^2 - 1} - \sqrt{\gamma_{e0}^2 - 1})(\gamma_i - \gamma_{i0})}{(\sqrt{\gamma_i^2 - 1} - \sqrt{\gamma_{i0}^2 - 1})(\gamma_e - \gamma_{e0})} \right]$$

For the case  $\gamma_e \gg \gamma_i$ , the above equation can be written as

$$\Delta d \approx \frac{m_e c^2}{e E_{acc}} (\gamma_e - \gamma_{e0}) \left[ 1 - \frac{(\gamma_i - \gamma_{i0})}{(\sqrt{\gamma_i^2 - 1} - \sqrt{\gamma_{i0}^2 - 1})} \right]$$

We can rewrite it as the phase slippage formula

$$\delta = k_p \Delta d \approx \frac{1}{e E_{acc} / m_e c \omega_p} (\gamma_e - \gamma_{e0}) \left[ 1 - \frac{(\gamma_i - \gamma_{i0})}{(\sqrt{\gamma_i^2 - 1} - \sqrt{\gamma_{i0}^2 - 1})} \right]$$

For a single stage PDPWA with a 1 TeV proton drive beam that accelerates electrons to 500 GeV energy (assuming electron injection energy far less than 500 GeV),  $\gamma_{i0} = 1000$ ,  $\gamma_e - \gamma_{e0} \approx 10^6$ . If we assume that the wakefield amplitude is  $e E_{acc} / m_e c \omega_p \sim 1$ , then the phase slippage is  $k_p \Delta d = 10^6 \left[ 1 - \frac{(\gamma_i - 1000)}{(\sqrt{\gamma_i^2 - 1} - \sqrt{1000^2 - 1})} \right]$

which has to be smaller than  $\pi$ . Fig.1 shows the phase slippage as a function of the final energy of the proton for a 1 TeV drive beam. It can be seen that the final energy of a 1 TeV proton beam has to be larger than 160 GeV in order to satisfy the phase slippage requirement. Using the average accelerating (decelerating) field of  $\sim 1.4$  GV/m, the maximum dephasing length is about 600 m. And the transformer ratio of such a single stage PDPWA is about 0.6. This provides the basic parameter to design such an acceleration stage.

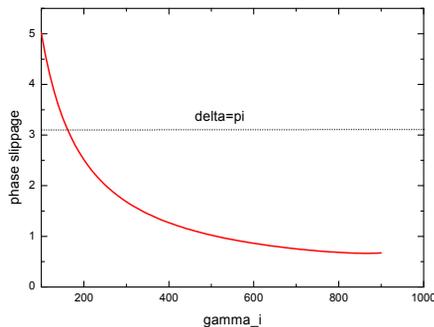


Figure 1: Phase slippage of a 1 TeV proton beam vs.  $\gamma_i$  of the proton beam.

### OTHER NOVEL IDEAS

Many novel ideas have emerged since the PDPWA has been proposed. The recent simulation shows that a 10 ~ 100 GeV proton bunch with a bunch length less than 100  $\mu$ m can be generated in a laser intensity of  $10^{22}$  W/cm<sup>2</sup> via a so-called snowplow regime of the laser-driven wakefield acceleration [14]. We may think about to inject such short and high energy proton bunch into a fast cycling synchrotron to boost the beam energy quickly (up to  $\sim$  TeV). Then this high energy, short proton bunch

may be used as an ideal driver to resonantly excite a large amplitude plasma wakefield for the electron beam acceleration and for a collider based on PDPWA scheme. This method may also serve as a preparation for the TeV regime acceleration of protons over centimeters with  $10^{23}$  W/cm<sup>2</sup> with a laser of ELI [15].

Seryi raised a multi-TeV upgrade concept for the ILC based on the PDPWA [16]. In this concept the proton bunches are accelerated employing the ILC technology (1.3 GHz superconducting RFs) with electrons and positrons. A special beamline arrangements allow control of proton phase slippage, separation and merging of proton and electron (positron) bunches (via dual path chicane), as well as ballistic compression of the proton bunches. This approach may open a path for the ILC to a much higher energy of several TeV.

Yakimenko et al also discussed a possible solution for a TeV CoM collider design based on PDPWA. Such collider may fit into a 6.3 km Tevatron tunnel for saving cost. In this scheme, a high average beam power proton driver is required and the spent proton beam may be recycled by the FFAG fast cycling rings [17].

### CONCLUSION

We investigate some key issues in an  $e^+e^-$  collider design based on the PDPWA scheme. Simulation shows that a short proton bunch can be achieved via the self-modulation of the plasma wakefield driven by a long proton bunch. It is possible to reach a few TeV electron beam acceleration from a PDPWA scheme by a long LHC-like proton. To reach a high luminosity, the beam sizes at the IP need to be reduced significantly (strong focus) and a high bunch charge and high collision rate are required as well. Dephasing between the proton beam and electron (positron) beam may be a limiting factor for a future high energy linear collider design based on the PDPWA scheme.

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