

DOUBLE RESONANT PLASMA WAKEFIELDS*

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Abstract

Present work in Laser Plasma Accelerators focuses on a single laser pulse driving a non-linear wake in a plasma. Such single pulse regimes require ever increasing laser power in order to excite ever increasing wake amplitudes. Such high intensity pulses can be limited by instabilities as well engineering restrictions and experimental constraints on optics. Alternatively we present a look at resonantly driving plasmas using a laser pulse train. In particular we compare analytic, numerical and VORPAL simulation results to characterize a proposed experiment to measure the wake resonantly driven by four Gaussian laser pulses. The current progress depicts the interaction of 4 CO₂ laser pulses, $\lambda_{laser} = 10.6\mu m$, of 3 ps full width at half maximum (FWHM) length separated peak-to-peak by 18 ps, each of normalized vector potential $a_0 \simeq 0.7$. Results confirm previous discourse and show, for a given laser profile, an accelerating field on the order of 900 MV/m, for a plasma satisfying the resonant condition $\omega_p = \frac{\pi}{t_{FWHM}}$.

INTRODUCTION

The attractiveness of plasmas as facilitators of particle acceleration is their ability to support extremely high field gradients, notably on the order of $E_{WB} = c\omega_p m_e/e \simeq 96\sqrt{n_0}(cm^{-3})$, the linear wavebreaking limit. Using a plasma density of $n_0 \simeq 10^{15} cm^{-3}$ this relation yields a field of $E_0 = 3$ GV/m, much higher than the ~ 100 MV/m available in typical radio-frequency linear accelerators. Well travelled projects concerning Laser Plasma Wakefield Accelerators (LWFA) focus on the application of a single high intensity laser pulse to generate wakes [1]. In this paper we focus on the use of multiple laser pulses to resonantly excite a wake larger than one constituent laser pulse is capable of creating. Such double resonant systems offer the advantage of scalability of wakefield amplitude through number of pulses as well as lower intensity lasers for ease of light transport and decreased exposure to laser-plasma instabilities.

For the purposes of this paper we define an LWFA system which is "double resonant" as a system in which each laser pulse is individually resonant ($T_p \simeq 2t_{FWHM}$, $\omega_p = \pi/t_{FWHM}$) and in which the train of pulses is also resonant ($t_{separation} = nT_p$). This allows laser pulses behind the initial pulse to further increase the magnitude of

the plasma's response, building on the wake created by the preceding pulses. It has been shown [2] that the maximum possible accelerating gradient in a plasma wake is proportional to $\sqrt{n_0}$, $Ez \propto \sqrt{n_0}$, but for a given intensity the total energy gain possible is $\Delta W \propto \sqrt[3]{n_0}$, thus it is prudent to design a system which works at lower density.

The use of lower plasma densities requires longer laser pulses in order to meet the resonance condition. A density of 10^{15} lends itself readily to the use of laser pulses produced by CO₂ laser systems. Furthermore, CO₂ laser systems naturally produce pulse trains due to accessing several rotational lines for amplification [3].

In order to be as general as possible, choice of specific pulse length and separation will be neglected until necessary, as in the case of simulation or numerical integration. To begin the analysis we start with a linear analytic framework and numerical integration of the 1D nonlinear differential equation for the evolution of the wake's potential, ϕ , in a cold neutral plasma. Analytic and numerical analysis are followed by 1D time explicit PIC simulations using the commercially available VORPAL [4] code to model both the plasma response and the evolution of the laser pulses.

ANALYTIC AND NUMERICAL ANALYSIS

We begin with a linear analysis in which the driving pulse train is a series of four gaussian laser pulses, of equal duration σ_t and pulse separation S peak-to-peak, but different amplitude a_n , as given by Eq. 1.

$$a(t) = \sum_{n=0}^N a_n e^{-\frac{(t+nS)^2}{2\sigma_t^2}} \quad (1)$$

As is customary, a is the normalized vector potential and is defined as $a = eA/m_e c^2$. The spacing as given allows the pulses to be considered non-overlapping so that $a^2(t)$ is the sum of the squares. Linear analysis gives the plasma response as [1]:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\phi = \frac{\omega_p^2 a^2(t)}{2}, \quad (2)$$

where $\phi = e\Phi/m_e c^2$. In this paper the plasma response to each pulse is assumed to be small enough such that density modulations due to prior pulses are ignored. While this assumption is not entirely physical, especially in the non-linear analysis, it allows a quick survey of possible plasma parameters for future simulation. Using an appropriate Green's function to integrate the source term on the right hand side of Eq. 2, the potential of the plasma wake is

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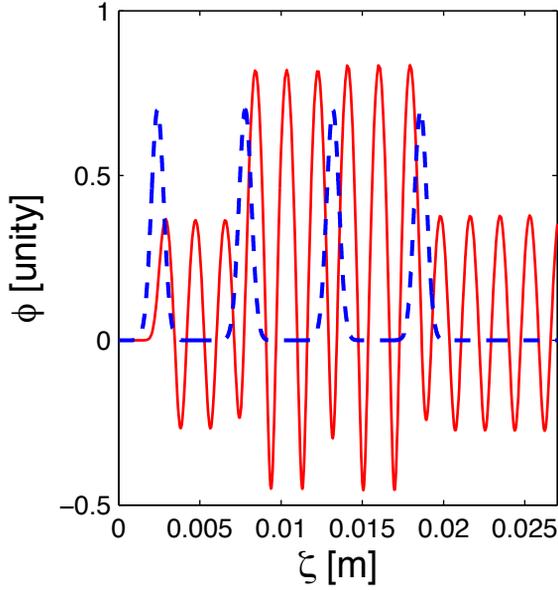


Figure 1: The potential (red), as calculated by Eq. 4, generated in a plasma due to four linearly polarized laser pulses (blue, dashed) of amplitude $a_0 = 0.7$ in a plasma of density of $\omega_p = \pi/t_{FWHM}$. Of note is that laser pulse 3 is spread evenly across the potential at its minimum (red curve) which results in no net energy exchange between the pulse and plasma wake. Furthermore, laser pulse 4 is biased towards the decreasing side of the negative swing in potential, indicating that energy is removed from the plasma wake.

found, in the limit that t is taken to be long after all pulses have passed, to be

$$\phi(t) \simeq \frac{\omega_p \sqrt{\pi \sigma_t}}{2} e^{-\frac{\omega_p^2 \sigma_t^2}{4}} \sum_{n=0}^N a_n^2 \sin \omega_p \left(t + \frac{nS}{2} \right). \quad (3)$$

The preceding result indicates the possibility of using a plasma whose plasma frequency is related to the pulse separation by $\frac{\omega_p S}{2} = 2\pi$, instead of $\omega_p = \frac{\pi}{t_{FWHM}}$. This observation is unsurprising as S is an integer number of t_{FWHM} , specifically the ratio of the former to the latter plasma frequencies is $\frac{2}{3}$. In fact, these two are not the only possibilities as $\frac{\omega_p S}{2} = 2\pi m$ can be used in conjunction with $S = qt_{FWHM}$ to yield $\omega_p = 4\pi m/qt_{FWHM}$, returning, non-uniquely, the single pulse resonance condition for $m=1$ and $q=4$.

In deriving the nonlinear 1D Poisson equation the quasistatic approximation with the further approximation that the drive beams are non-evolving is used [1]. These assumptions allow us to write Poisson's equation as a function of the beam coordinate $\zeta = z - v_p t$ only;

$$\frac{\partial^2 \phi}{\partial \zeta^2} = \frac{k_p^2}{2} \left[\frac{(1+a^2)}{(1+\phi)^2} - 1 \right]. \quad (4)$$

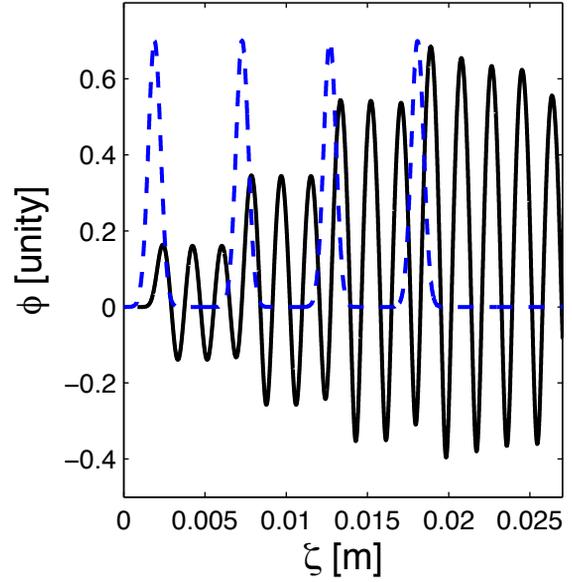


Figure 2: The potential (black), as calculated by Vorpál, generated in a plasma due to four linearly polarized laser pulses (blue, dashed) of amplitude $a_0 = 0.7$, $t_{FWHM} = 3$ ps, $t_{sep} = 18$ ps for a plasma density of $\omega_p = \pi/t_{FWHM}$. In contrast to Fig. 1, the laser pulses are concentrated at the correct phase for strong coupling between plasma wakefield and laser pulse, $d\phi/d\zeta > 0$ and $\phi < 0$.

While analytic solutions for the above equation exist for the case of square pulses [5], direct solutions for gaussian pulses prove troublesome so numerical integration must be used.

The result of numerical integration are plotted in Fig. 1, showing a cold plasma's response to the ponderomotive force generated by 4 laser pulses, in which all pulses are of amplitude $a_0 = 0.7$. The parameters chosen for the numerical integration of Eq. 4 are $t_{FWHM} = 3$ ps, $S = 6 \cdot t_{FWHM}$. The expected steepening and period lengthening of the nonlinear response electric field are present. Comparison with PIC simulation shows that simple numerical integration of Poisson's equation is inadequate for complete characterization of a resonant system.

The difficulties in reproducing simulation results, using Eq. 4, stem from errors in the predicted plasma wavelength after the second pulse, which contribute to both modification of the group velocity of subsequent laser pulses and correct phasing of subsequent laser pulses. As described in [6], proper phasing of laser pulses to contribute constructively to the plasma wake requires $d\phi/d\zeta > 0$ and $\phi < 0$.

SIMULATION

Simulations were conducted using VORPAL to model the response of the plasma to the various laser beams as well as the beam's response to the plasma. For the simulations a pulse length of $t_{FWHM} = 3$ ps, pulse separation

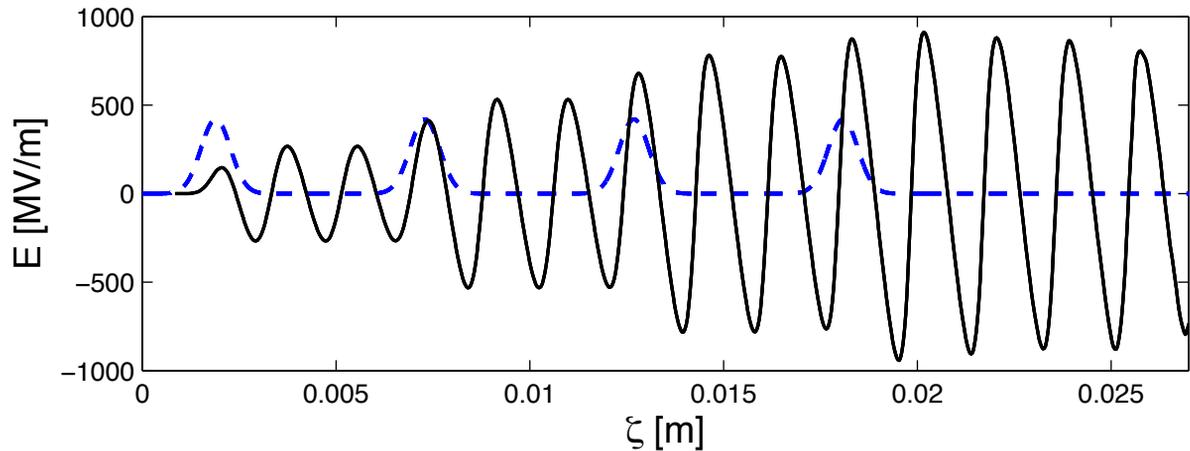


Figure 3: The electric field of the plasma response, in the longitudinal direction, as calculated by VORPAL. The resulting field is 900 MV/m, much greater than that predicted by numerical integration of Poisson's equation in 1D. The laser pulse train profile is shown in dashed blue and has been rescaled.

$S = 6 \cdot t_{FWHM}$, laser waist size $w_0 = 400 \mu\text{m}$ and a laser wavelength of $\lambda_{laser} = 10.6 \mu\text{m}$ were used, this results in an intensity at the focus of $I_0 = 6 \cdot 10^{15} \text{ W/cm}^2$. The simulation domain used is approximately 3.2 cm long using 32 cells per laser wavelength. The domain itself was 1D but time explicit in order to properly model effects at the laser wavelength. For the present studies the ion motion was ignored which is justified by the expected use of argon, $m_i/m_e \approx 7.3 \cdot 10^4$ in experiment. Simulation results are plotted in Fig. 3 and show a maximum accelerating field of 900 MV/m.

EXPERIMENT

The purpose of this work is to characterize a planned experiment to be conducted at the UCLA Neptune Laboratory in which the wakefield generated by multiple laser pulses is characterized. The experiment takes advantage of the high power MARS amplifier's [7] production of a train of CO_2 pulses [3] to excite a wakefield in a plasma for electron acceleration. To generate the plasma a Hollow Cathode Arc (HCA) system is used to ionize Argon and direct the plasma along the beam path. For laser pulses produced by MARS; $a_0=0.7$, $P=15 \text{ TW}$, $w_0=400 \mu\text{m}$ yielding an intensity of $I_0=6 \cdot 10^{15}$, well in range for secondary and tertiary ionization of argon in the HCA.

To characterize the wake generated in the plasma an electron beam is scanned through the wake by varying the delay between the MARS CO_2 pulse excitation of the plasma and the arrival of the electron beam. The beam is then sent through a spectrometer magnet and the resulting energy change of the beam is measured and correlated with distance behind the laser pulse train.

CONCLUSION

Current work reveals great promise in the ability to produce large wakes through use of a pulse train. It has been shown that a wake of 900 MV/m can be constructed by resonantly driving a plasma of appropriate density. While simulations do reveal that it is possible to drive a double resonant plasma response, a more robust analytic theory is required to trim down the space of possible plasma densities as simulation times prohibit an iterative approach using codes alone. Future work will focus on developing a more in depth non-linear theory as well as simulation using VORPAL to model the problem in a 3D PIC domain. The expansion of the simulations to 3D will allow proper modeling of beam based effects like self focusing and pump depletion.

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