Non-scaling Fixed Field Alternating Gradient Permanent Magnet Cancer Therapy Accelerator

Dejan Trbojevic and Vasily Morozov

Non-scaling Fixed Field Alternating Gradient Permanent Magnet Cancer Therapy Accelerator

1. Introduction → Scaling – Non-Scaling FFAG (NS-FFAG)

2. NS-FFAG

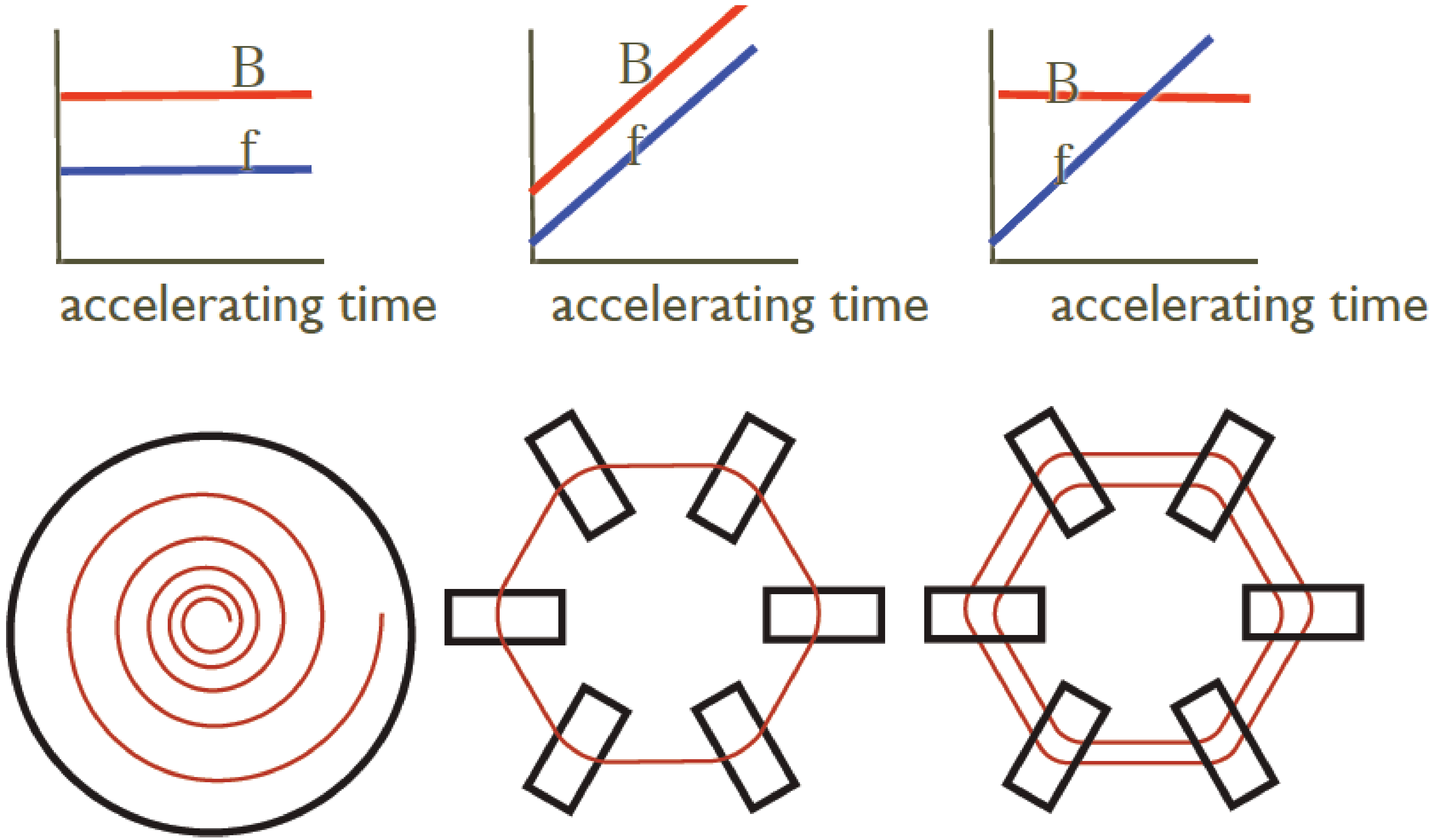
3. FFAG straight section solution P. Meads

4. NS-FFAG ring with permanent magnets

5. Racetrack

6. Acceleration

ACCELERATORS:



Cyclotron
*isochronous

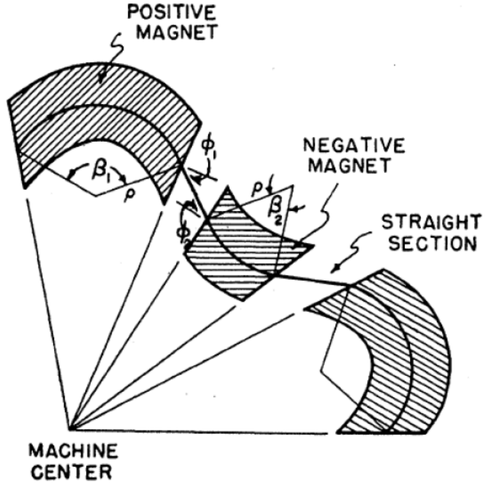
Synchrotron
*const. closed orbit
(varying mag. field)

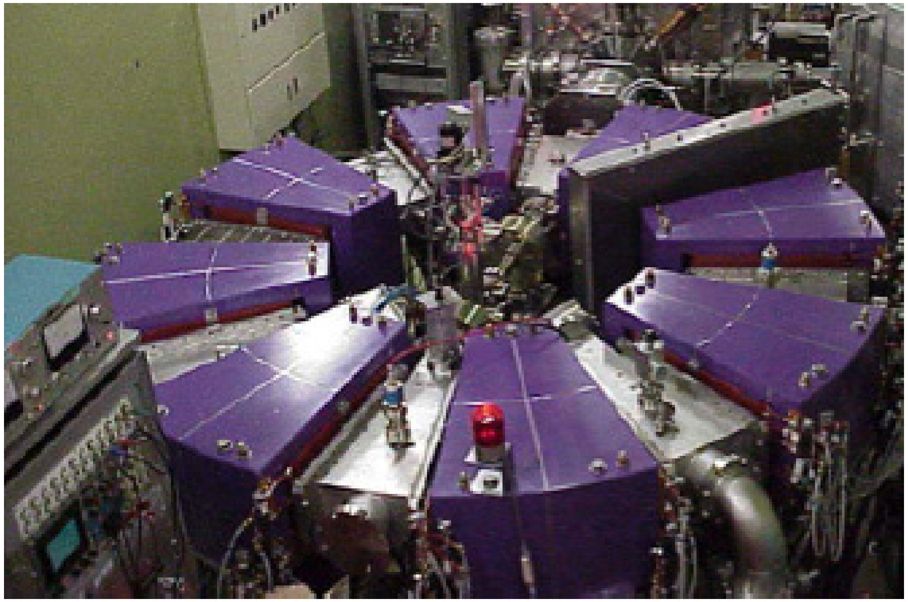
FFAG
*varying closed orbit
(const. mag. field)

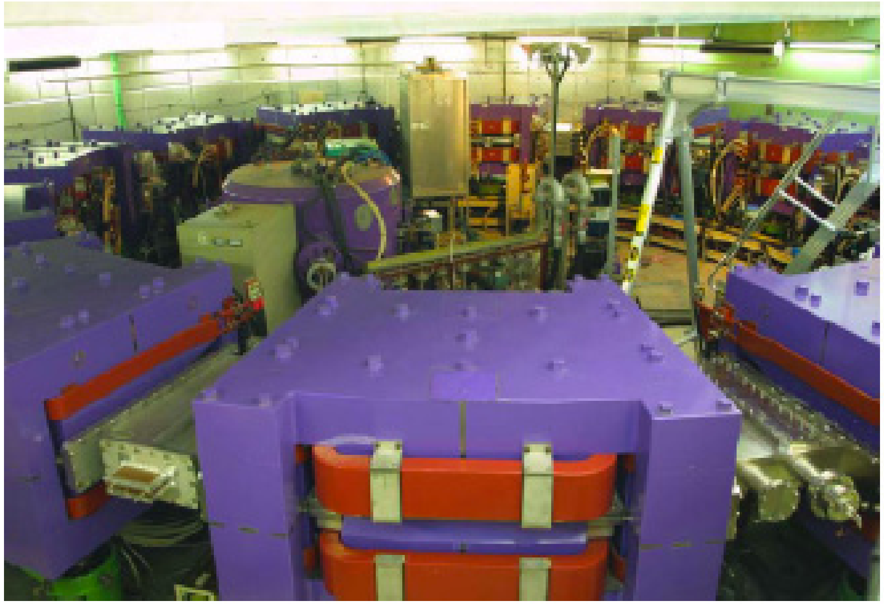
SCALING FFAG

The orbits follow non-linear magnetic field: $B_R \sim B_o (r/ro)^k$

$$p_o = eB_o r_o, \;\; p = eB_R r, \;\; p = p_o \left(\frac{r}{r_o}\right)^{k+1}$$







Due to restrictions on the particle motion stability in the vertical plane the length of the opposite bending magnet can not be shorter than 2/3 of the positive field bend. This increases the circumference.

Radial Sector FFAG

MURA-KRS-6 Phys. Rev. **103,** 1837 (1956)
November 12, 1954
K. R. Symon: The FFAG SYNCHROTRON – MARK I

$$N = \frac{2\pi}{(\beta_1 - \beta_2)}$$

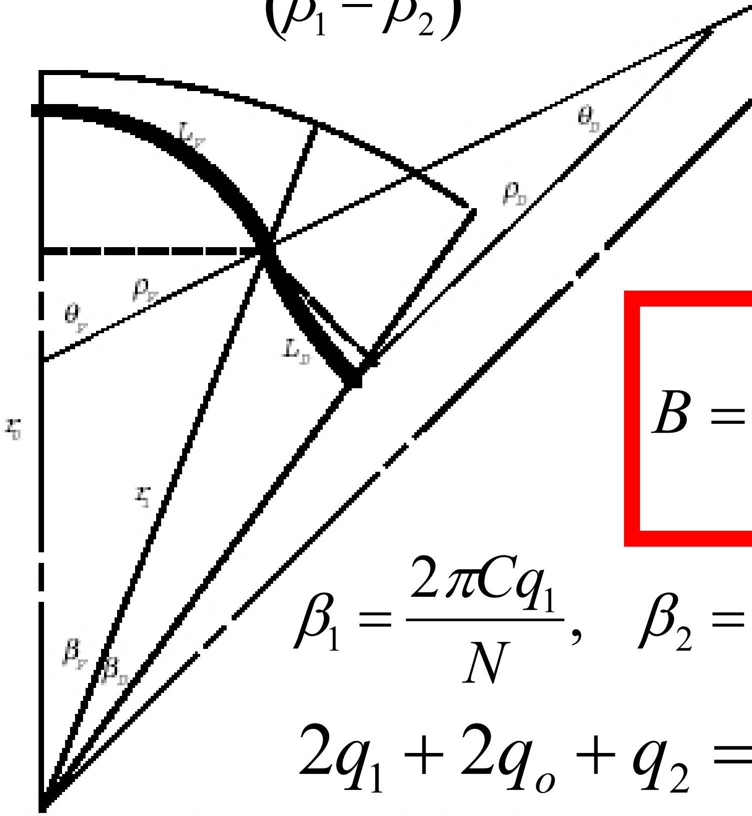
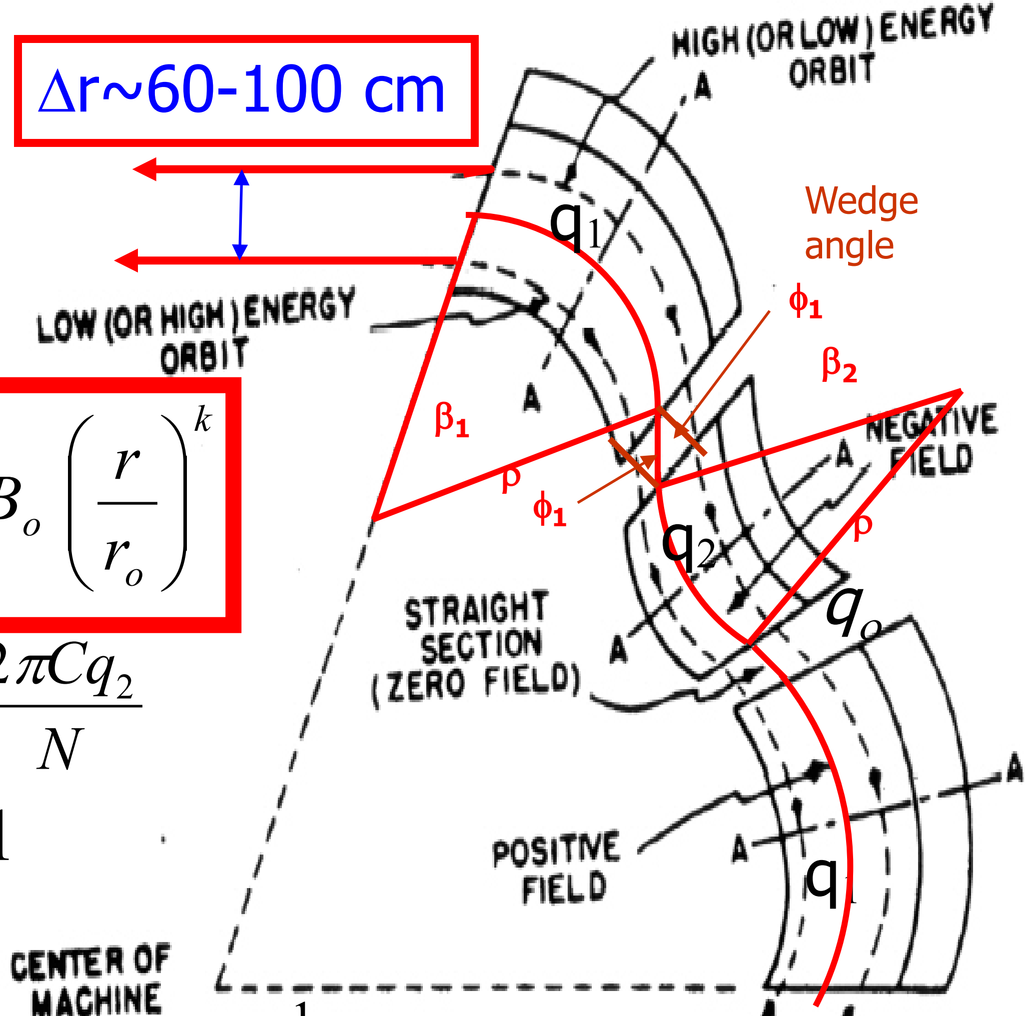


Figure 1: Closed orbit of a triplet focusing FF half cell; a half of F magnet, D magnet of one half straight section, is depicted.

$$B = B_o \left(\frac{r}{r_o}\right)^k$$

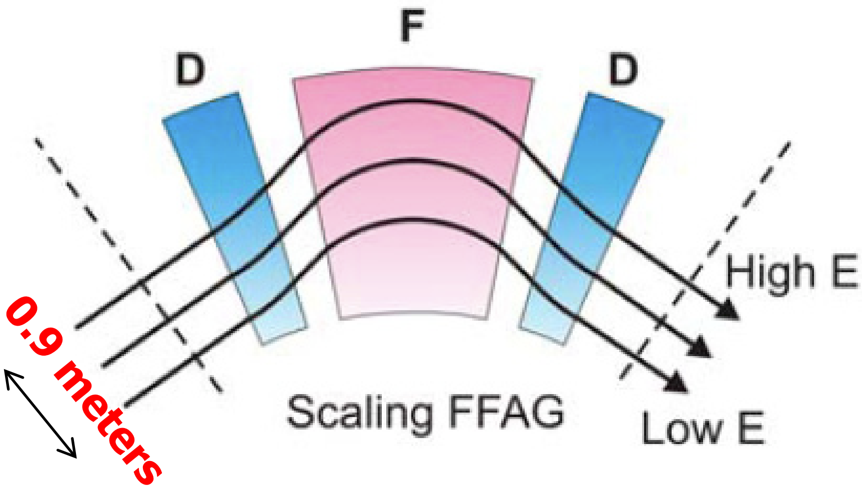
$$\beta_1 = \frac{2\pi C q_1}{N}, \quad \beta_2 = \frac{2\pi C q_2}{N}$$

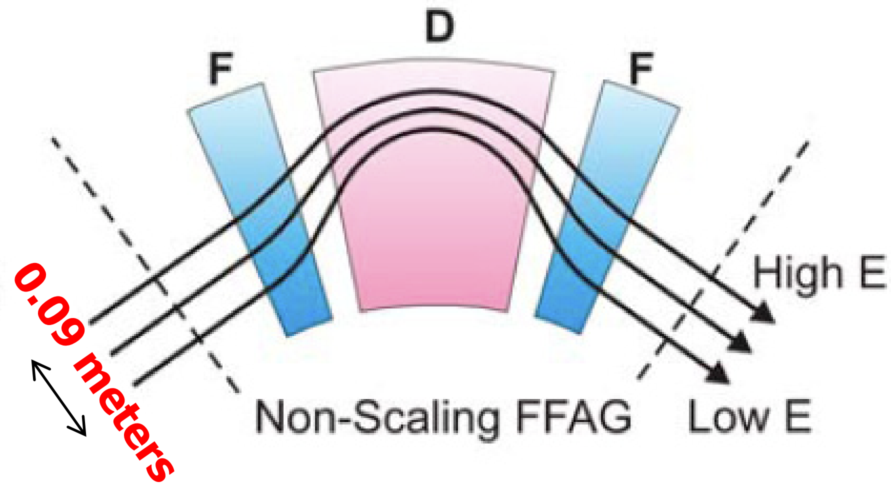
$$2q_1 + 2q_o + q_2 = 1$$

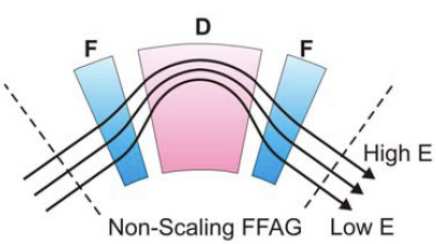


$$C = \frac{1}{q_1 - q_2}$$ *the circumference factor*

NON-SCALING FFAG







- Orbit offsets are proportional to the dispersion function:

$$\Delta x = D_x * \delta p/p$$

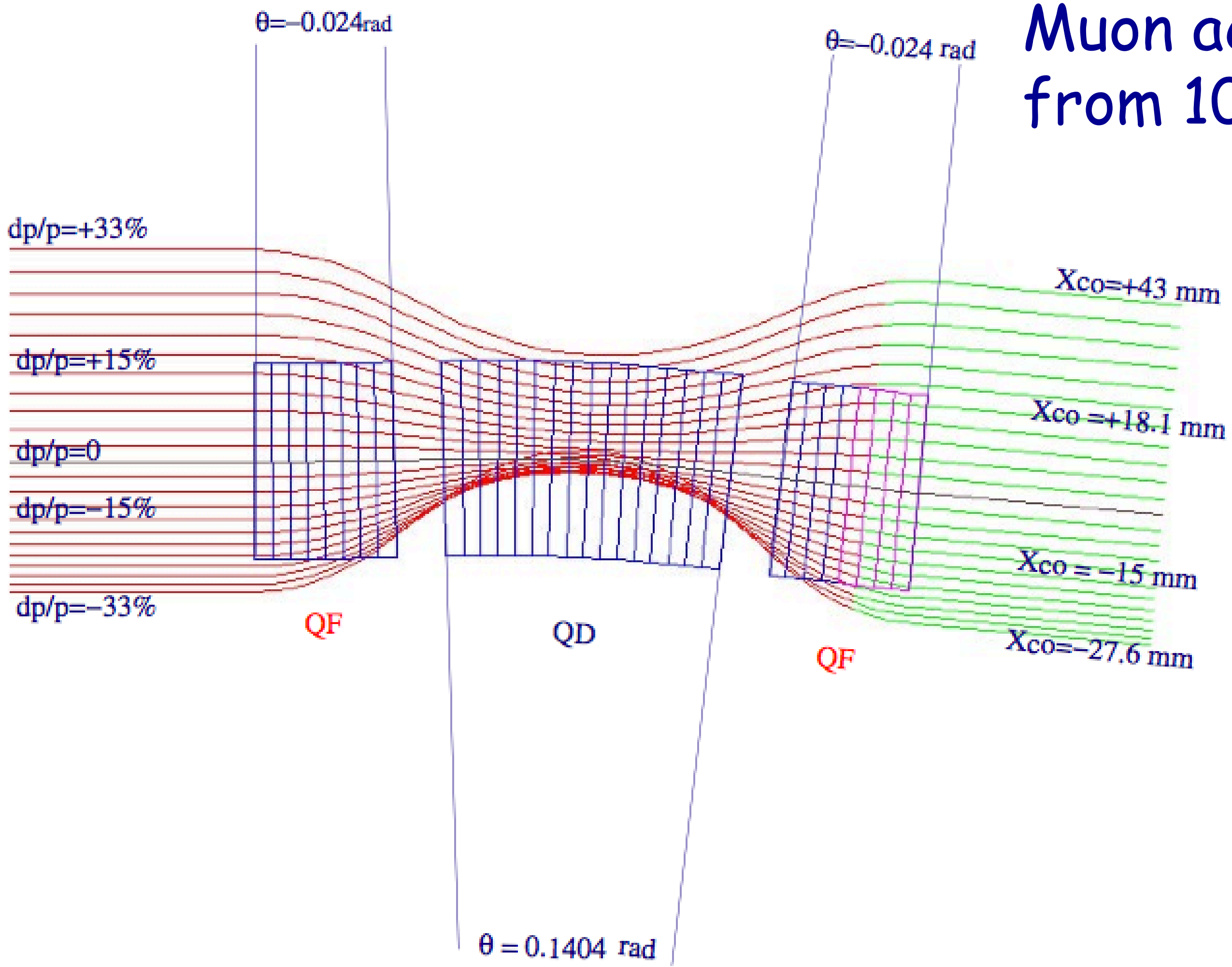
- To reduce the orbit offsets to ±4 cm range, for momentum range of δp/p ~ ± 50 % the dispersion function D_x has to be of the order of:

$$D_x \sim 4\ cm / 0.5 = 8\ cm$$

Triplet NS-FFAG for muon acceleration

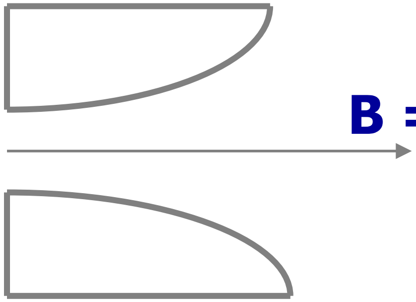
TRBOJEVIC, COURANT, AND BLASKIEWICZ Phys. Rev. ST Accel. Beams **8,** 050101 (2005)

Muon acceleration from 10 to 20 GeV



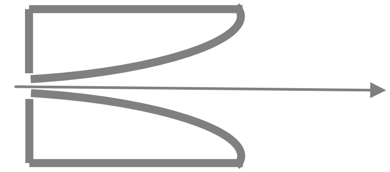
BROOKHAVEN NATIONAL LABORATORY

Scaling FFAG – Non scaling FFAG



$B = B_o(r/r_o)^k$

$B = B_o + r\,G_o$



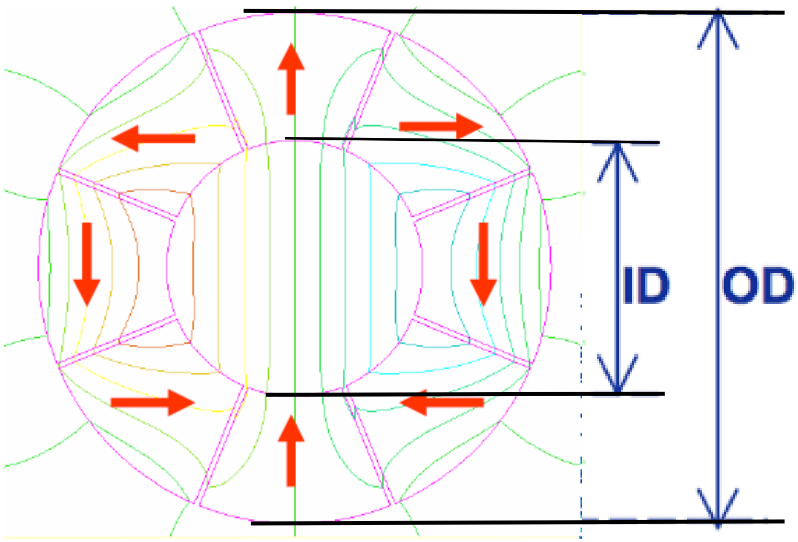
Scaling FFAG properties:

- Zero chromaticity.
- Orbits parallel for different $\delta p/p$
- Relatively large circumference.
- Relatively large physical aperture (80 cm – 120 cm).
- RF - large aperture
- Tunes are fixed for all energies no integer resonance crossing.
- Negative momentum compaction.
- $B = B_o(r/r_o)^k$ non-linear field
- Large acceptance
- Large magnets
- Very large range in $\Delta p/p = \pm 90\%$
- could be isochronous CW operation

Non-Scaling FFAG properties:

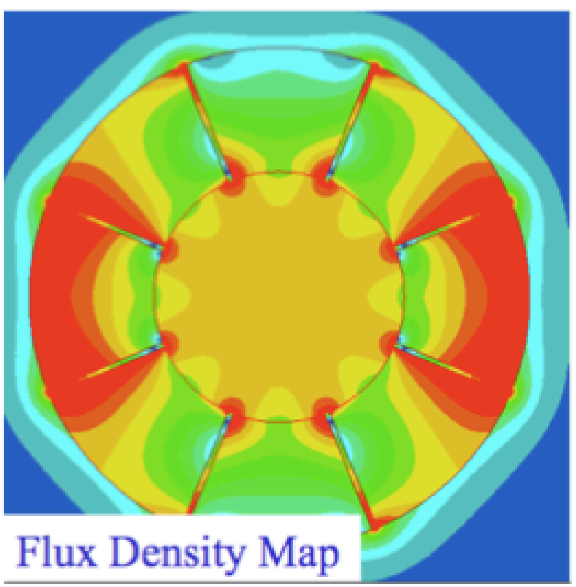
- Chromaticity is changing.
- Orbits are not parallel.
- Relatively small circumference.
- Relatively small physical aperture (0.50 cm – 10 cm).
- RF - smaller aperture.
- Tunes move 0.4-0.1 in basic cell resonance crossing for protons
- Momentum compaction changes.
- $B = B_o + x\,G_o$ linear field
- Smaller acceptance
- Small magnets
- Large range in $\Delta p/p = \pm 60\%$
- Very difficult to be isochronous

Permanent magnets



Halbach PM Dipole Structures:

$$B_g = B_r \ln(OD/ID)$$



There is no upper limit for air gap flux density in Halbach dipole structures according to equation.

But in reality it would be limited by:
(1)The realistic size
(2)The demagnetization effect

Permanent magnets

Electron Energy Corporation - courtesy of : Jinfang Liu and Peter Dent
924 Links Ave. Landisville PA 17538

B_r = 1.35 T

Nd-Fe-B Type Rare Earth Magnets

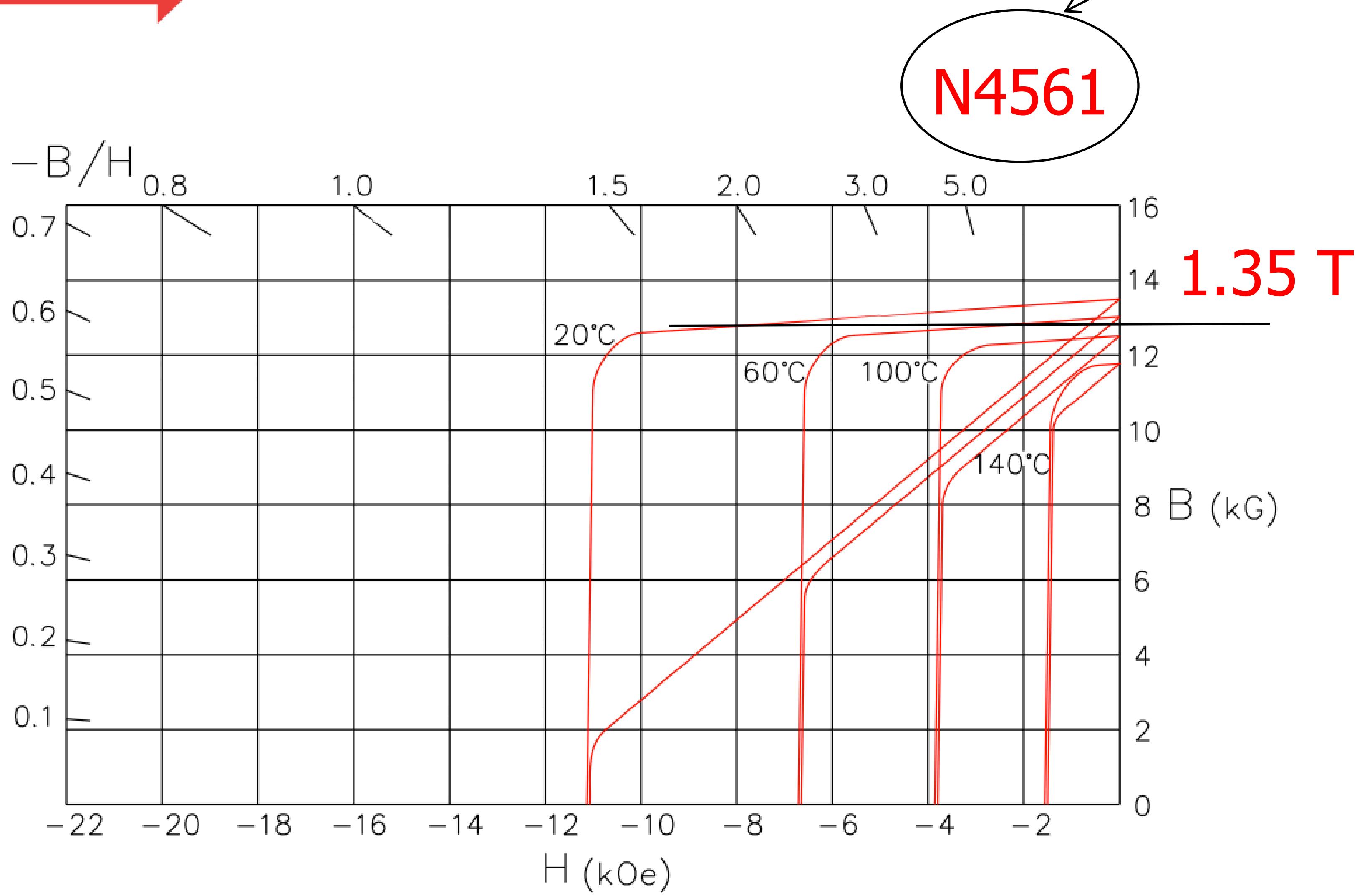
PM Grades	B_r (kG) (kG)	$(BH)_{max}$ (MGOe)	Max. operating temp (°C)
N50	14-14.5	48-51	70
N45	13.2-13.8	43-46	70
N45M	13.2-13.6	43-46	100
N42SH	12.8-13.2	40-43	120
N33UH	11.3-11.7	31-34	180

Permanent magnets

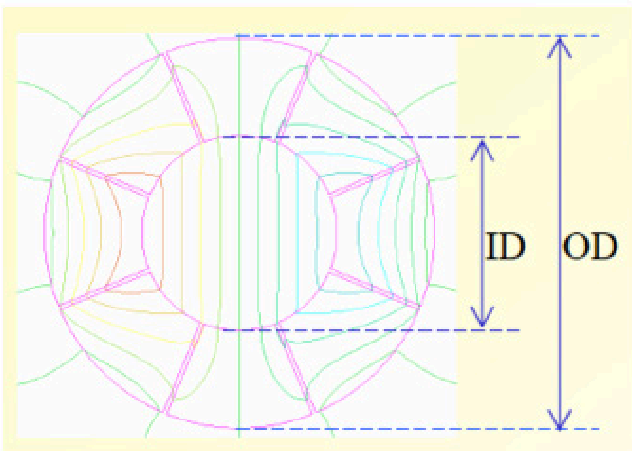
MCE

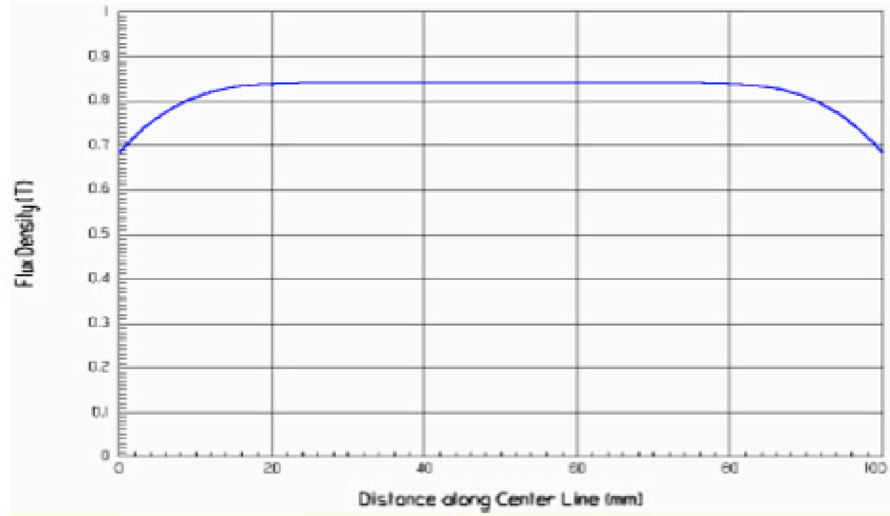
MAGNETIC COMPONENT ENGINEERING, INC.
2830 Lomita Blvd., Torrance CA 90505

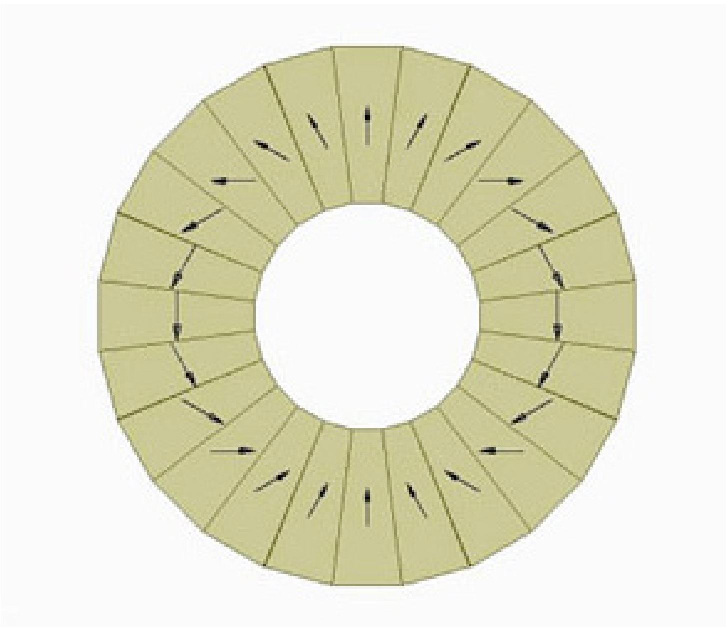
N4561



Permanent magnets







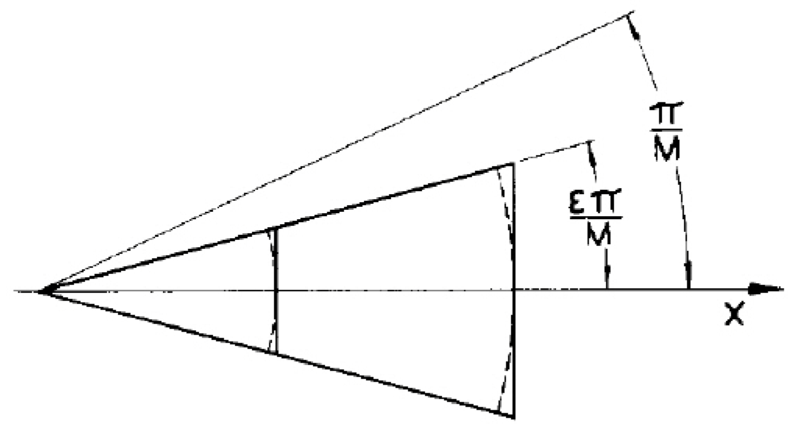
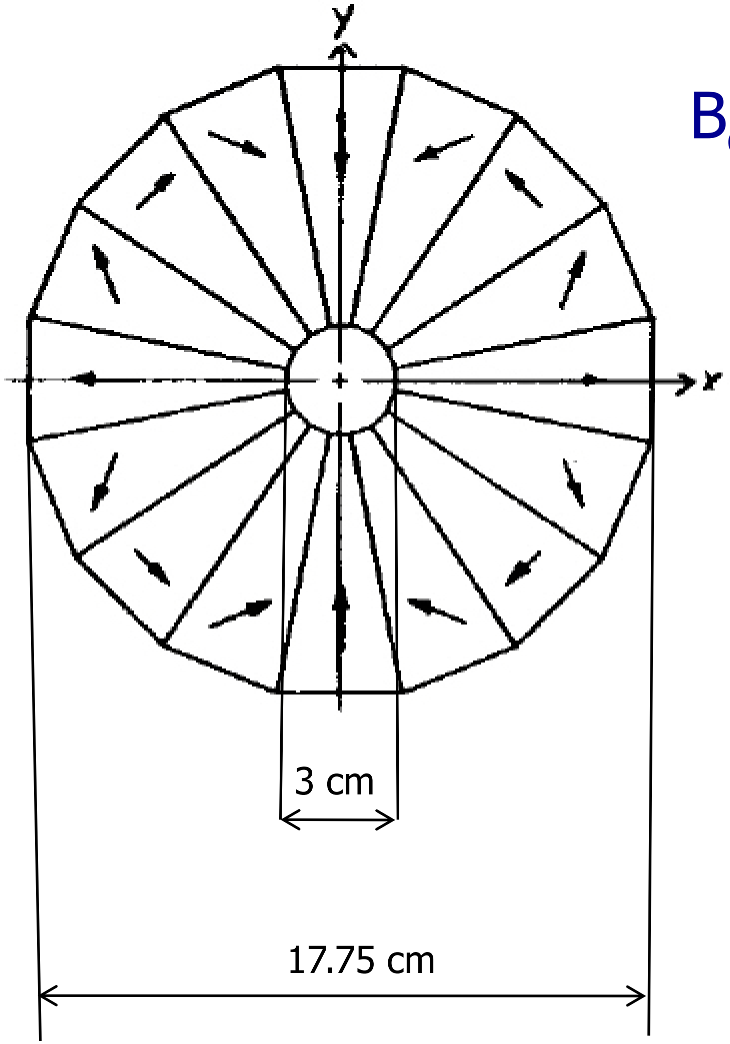


Fig. 4. One piece of a segmented REC multipole.

Halbach magnet-from his original publication



$B_g = B_r \ln(OD/ID)$

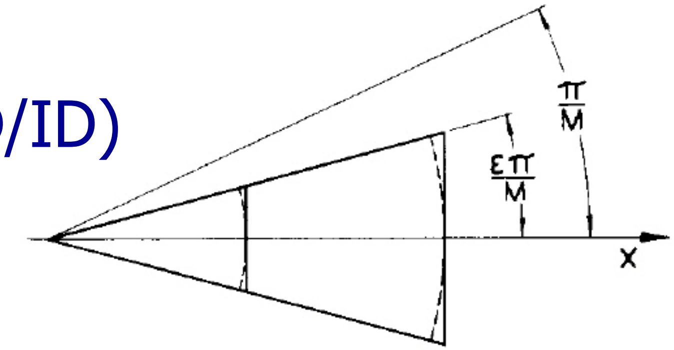


Fig. 4. One piece of a segmented REC multipole.

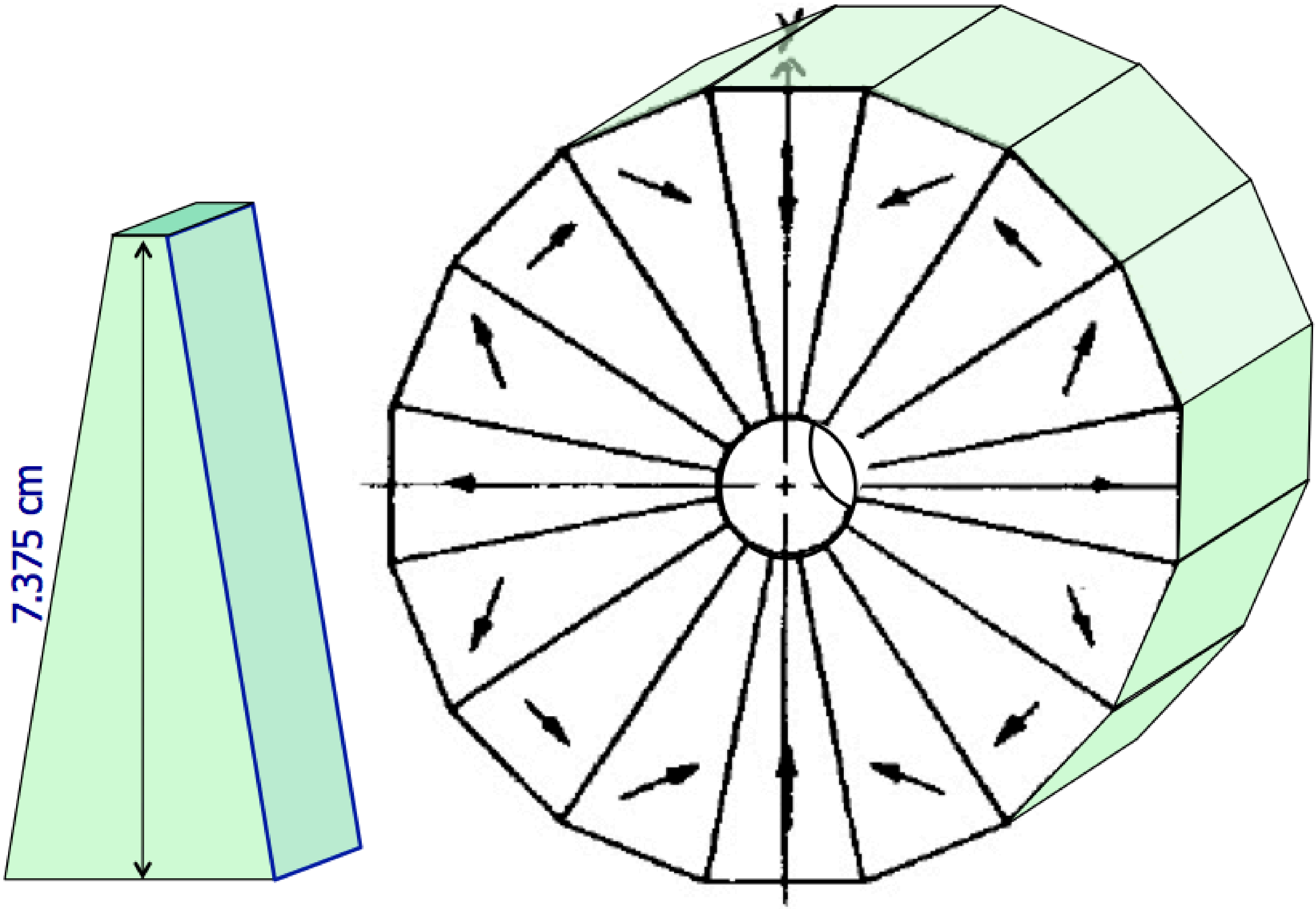
B_r=1.35 T

OD=17.75 cm
ID =3.0 cm
ln(OD/ID)=1.78

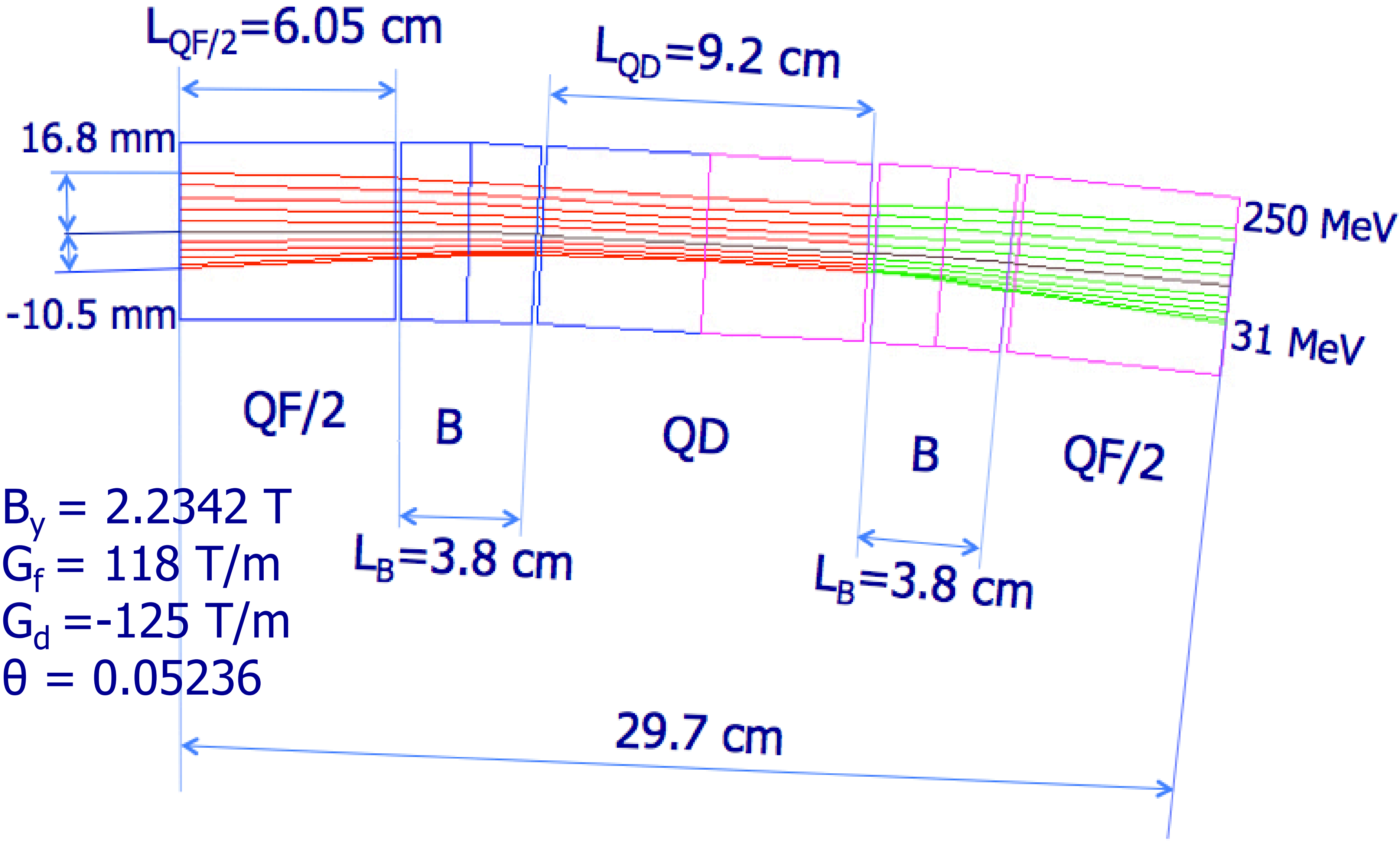
QLD=8.0 cm
BL= 4.8 cm
QLF=11 cm
GF = 2.2 T/0.015 m = 150.0 T/m
GD= -2.2 T/0.013 m=-170.0 T/m

B_g=2.4 T

Halbach permanent magnets



Arc design NS-FFAG



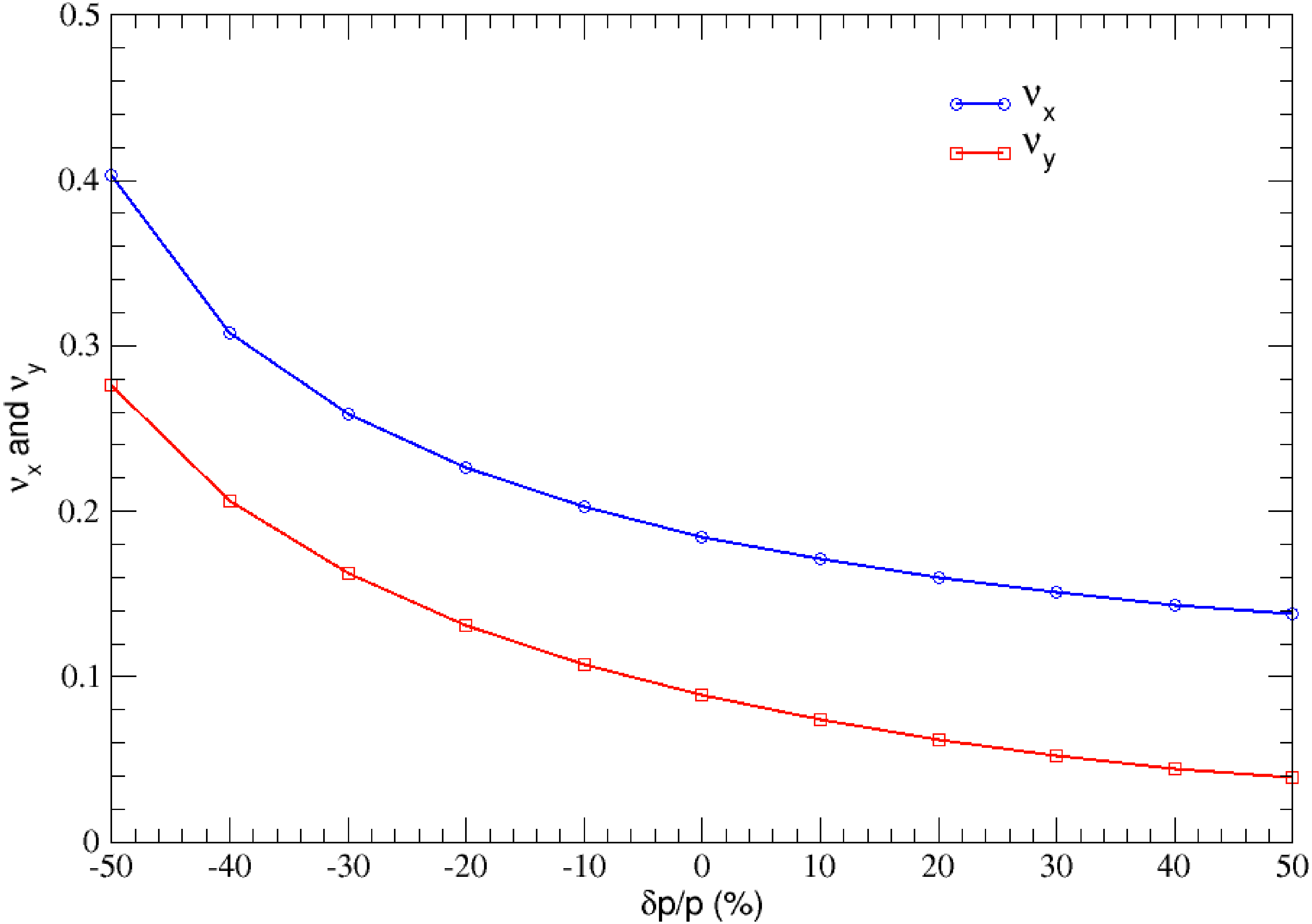
B_y = 2.2342 T

G_f = 118 T/m

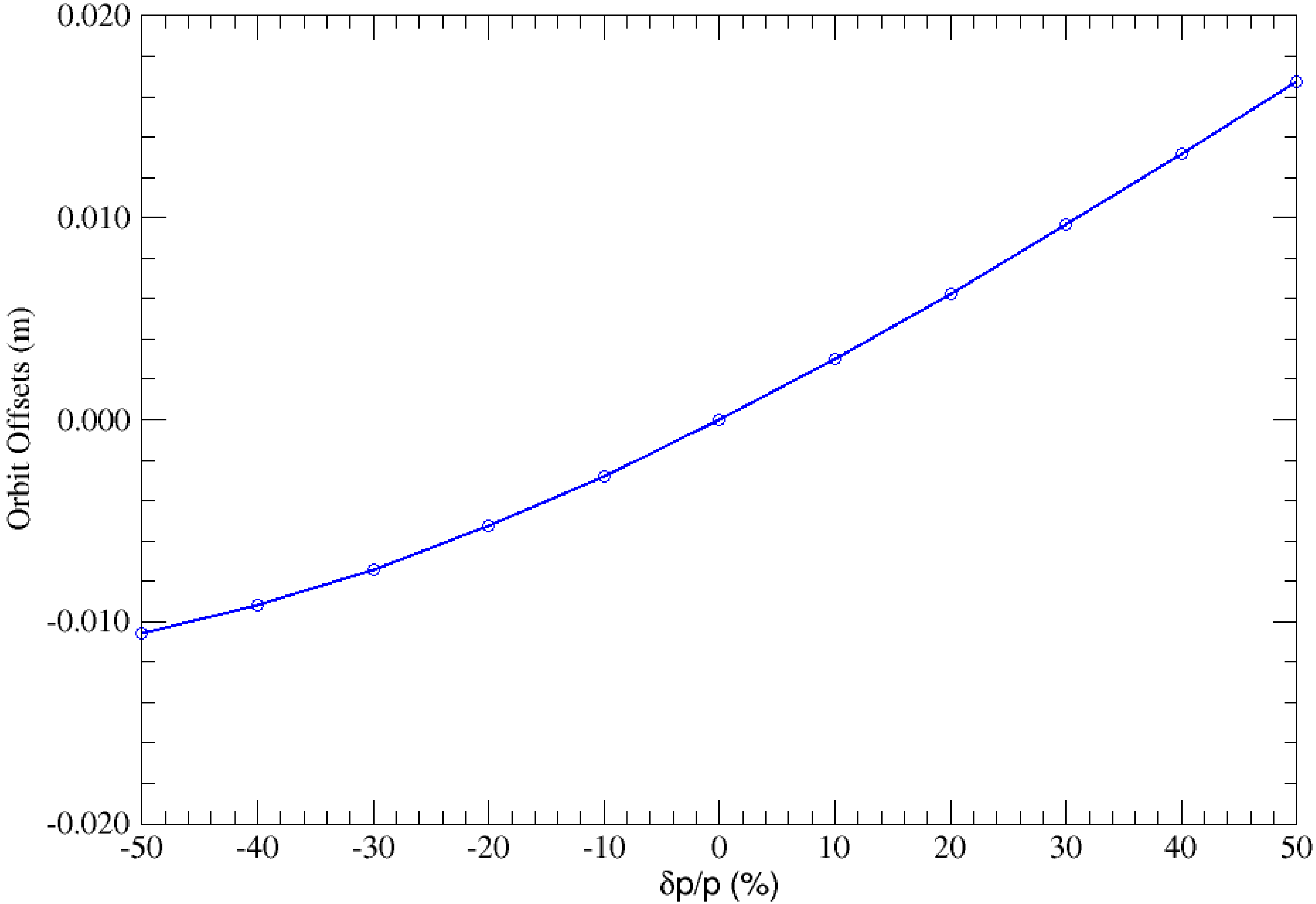
G_d =-125 T/m

θ = 0.05236

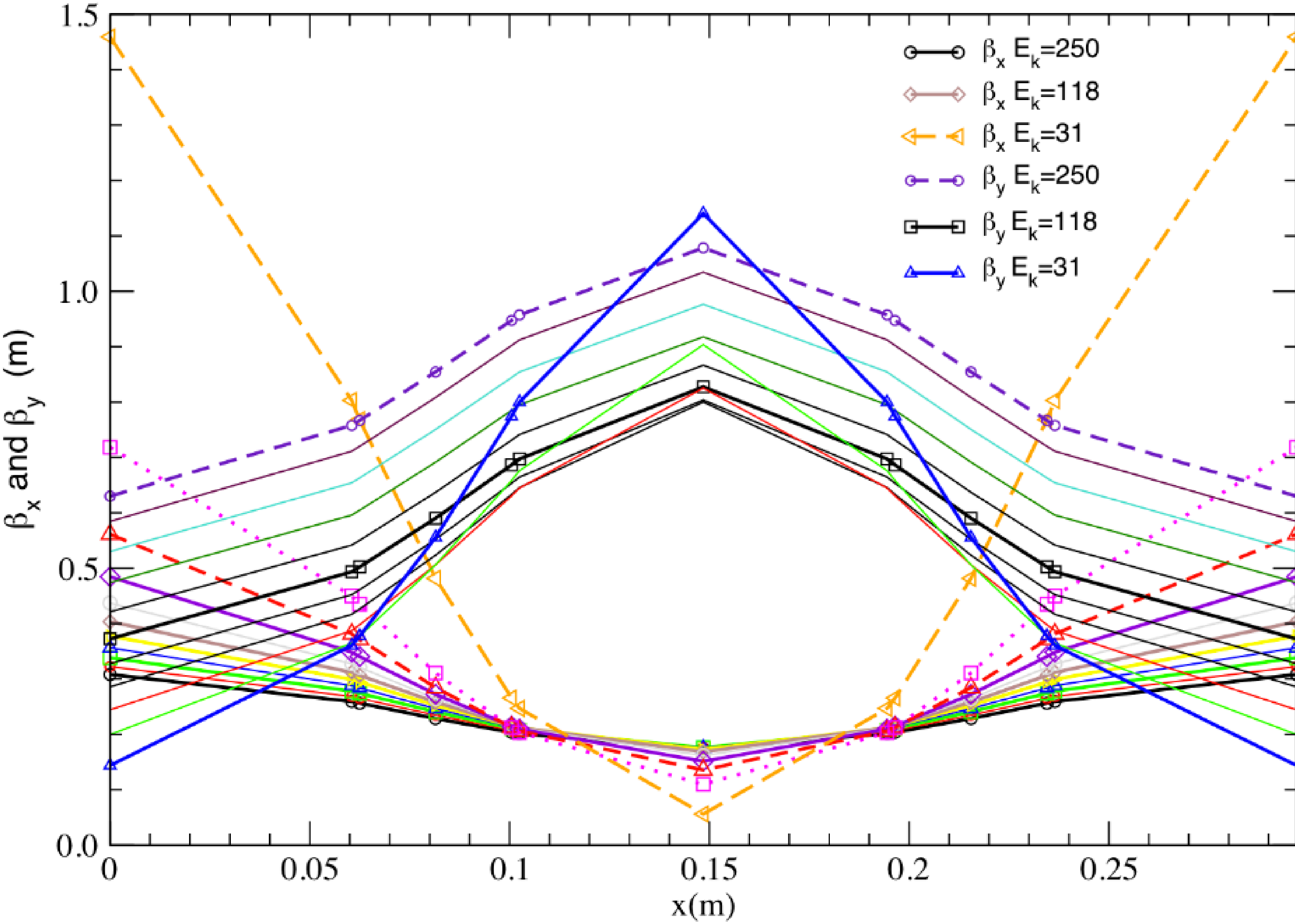
Arc design NS-FFAG: Tunes vs. momentum



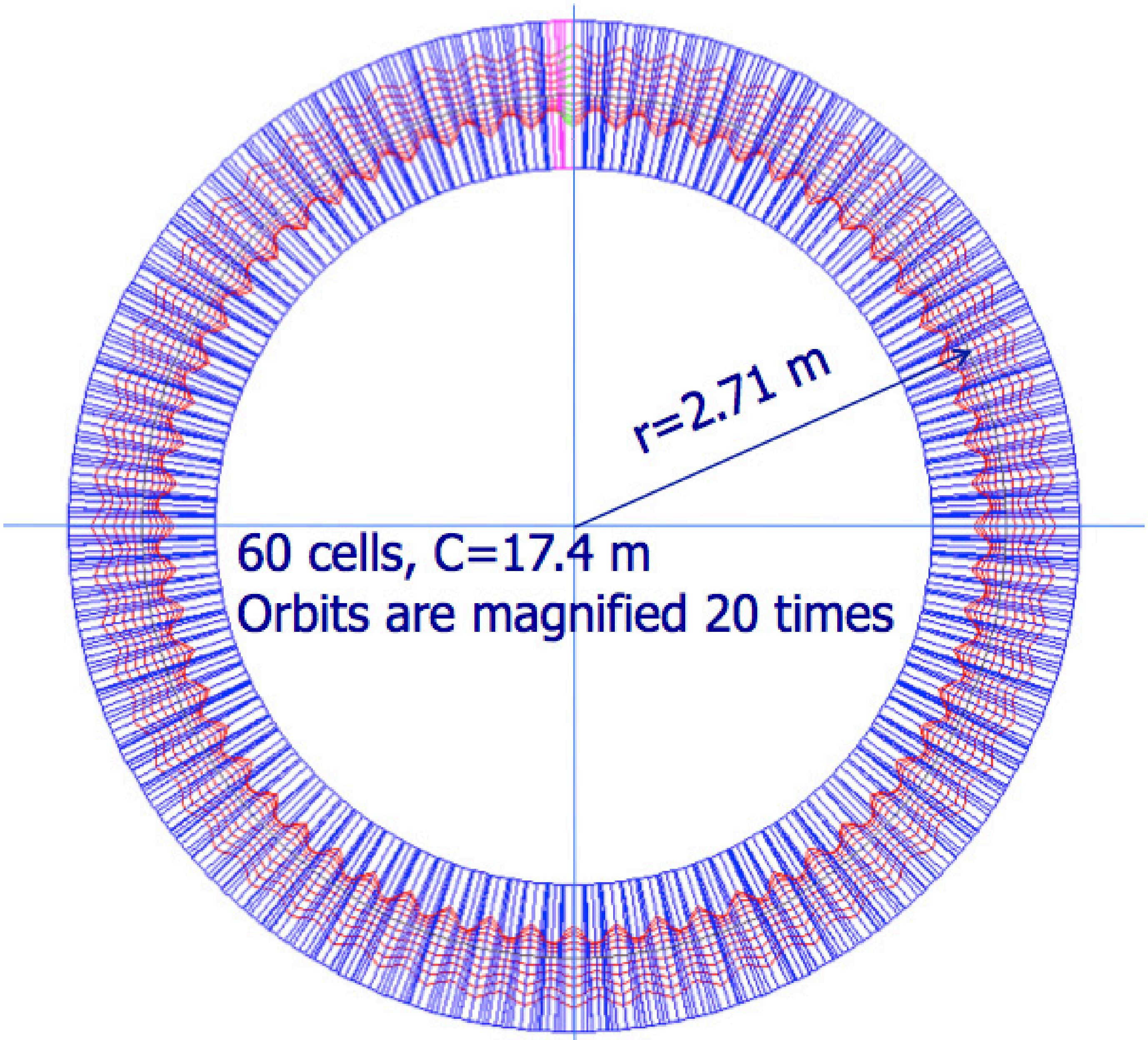
Maximum orbit offsets vs. momentum



β_x and β_y vs. momentum

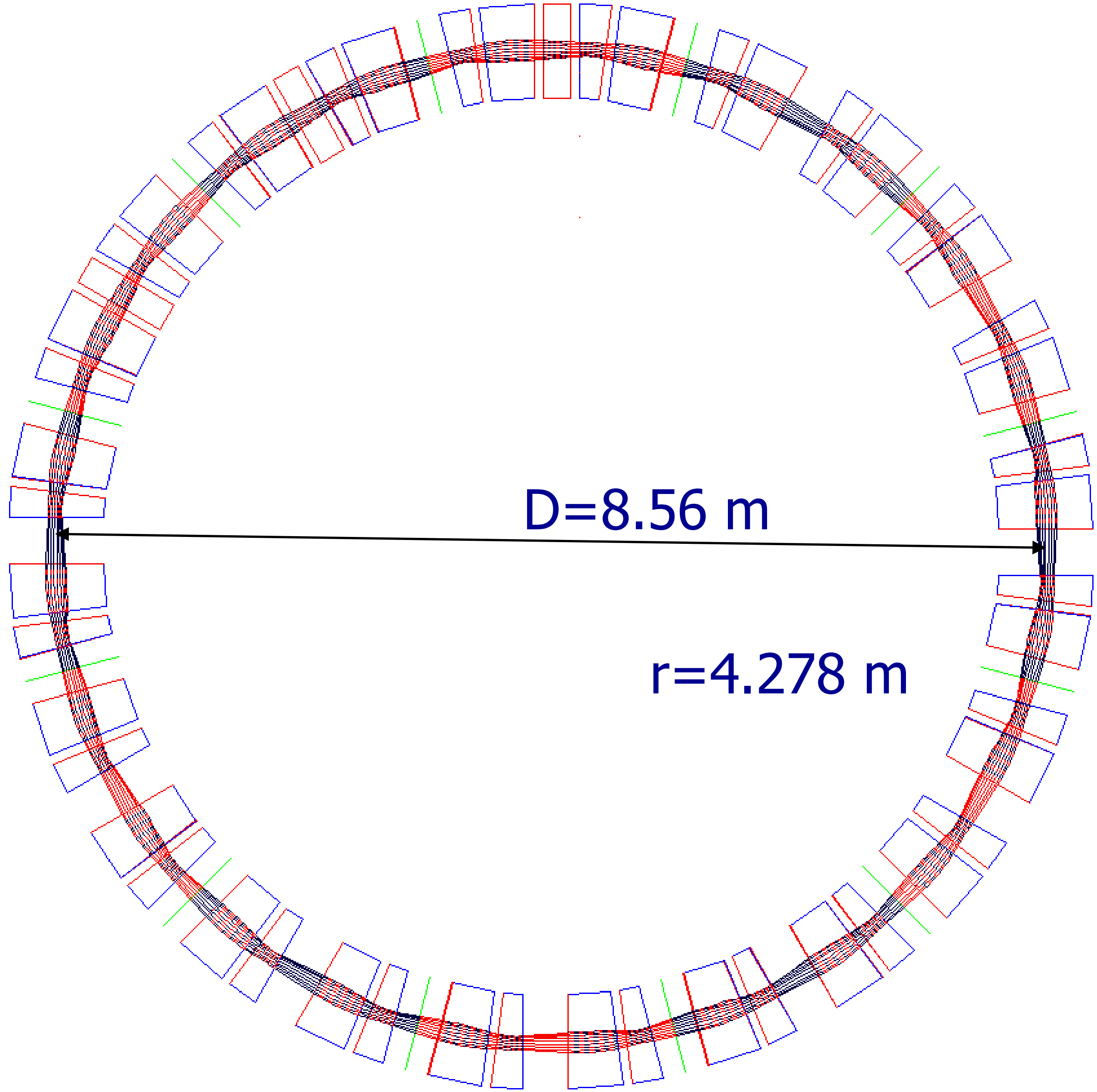


Arc design NS-FFAG



The ring with all elements:

24 doublets
12 cavities
Three kickers



C= 26.88 m
-50%<δp/p<+50%
E_{k_inj} =31 MeV
E_{k_max} = 250 MeV

Magnetic Properties:

L_{BD} = 22 cm
L_{BF} = 30 cm
G_d = -14.3 T/m
G_f= 8.73 T/m
B_{do}= 0.804 T
B_{fo}= 0.563 T

Minimum horizontal aperture:

A_{min}=0.140638+0.101838+6σ ~26 cm

Values of the magnetic fields at the maximum orbit offsets:

$B_{d\ max-}$= 0.804 + (-14.3)·(–0.0484) = **1.496 T**
$B_{d\ max+}$= 0.804 + (-14.2)·(0.107) = -0.715 T

$B_{f\ max+}$= 0.563 + 8.73 · 0.141 = **1.794 T**
$B_{f\ max-}$ = 0.563 + 8.73 · (–0.102) = -0.327 T

Offsets at F

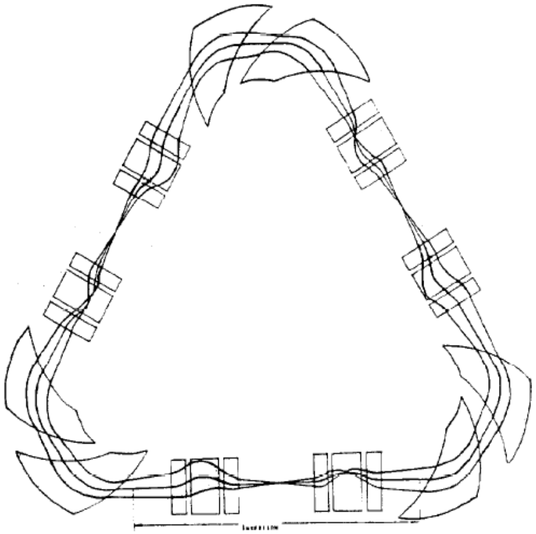
δp/p	x_{0ff} (m)
50	0.140638
40	0.111097
30	0.082114
20	0.053819
10	0.026376
0	0.000000
-10	-0.025024
-20	-0.048317
-30	-0.069370
-40	-0.087506
-50	-0.101838

Offsets at D

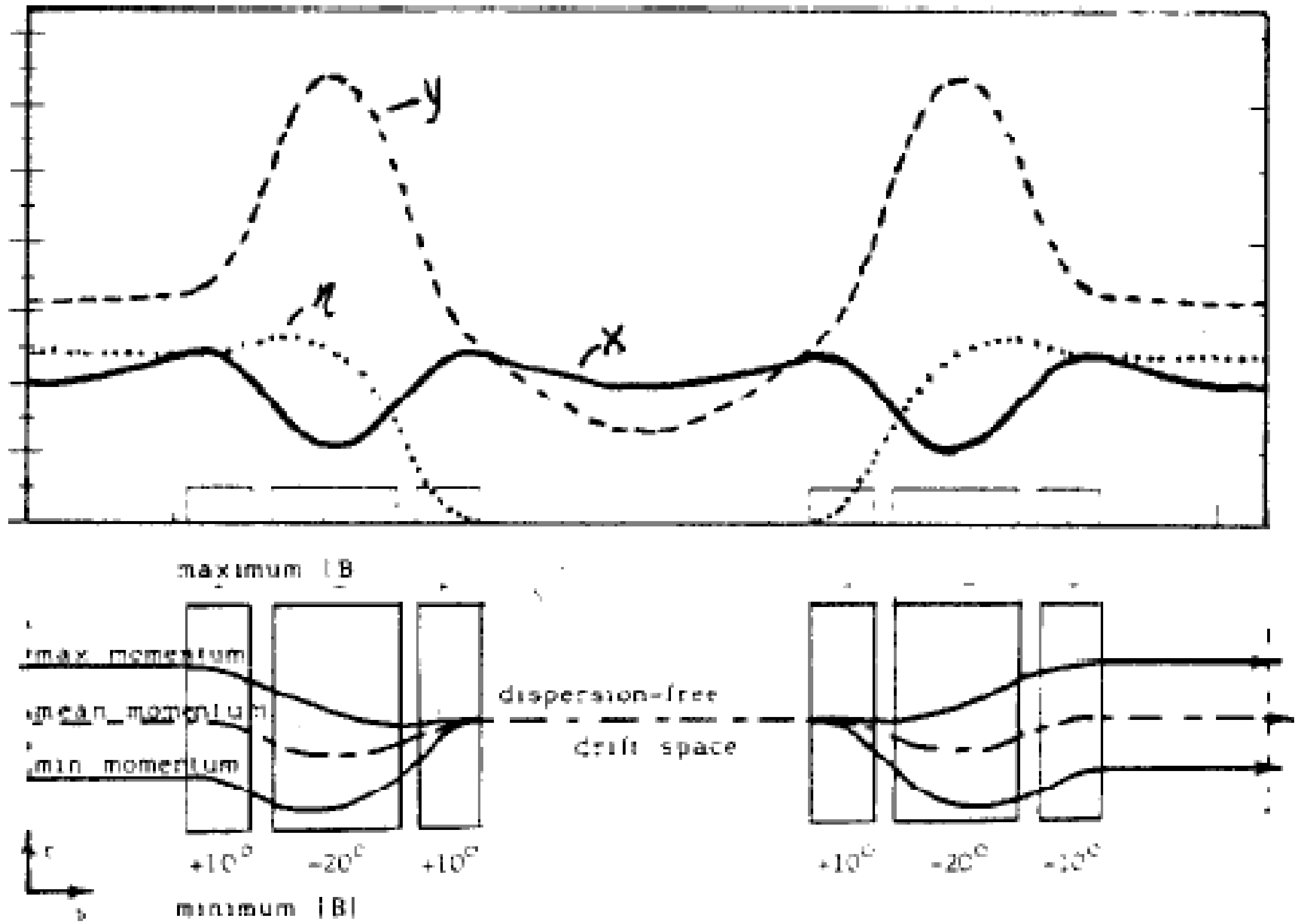
δp/p	x_{0ff} (m)
50	0.107354
40	0.083583
30	0.060737
20	0.039014
10	0.018662
0	0.000000
-10	-0.016560
-20	-0.030484
-30	-0.041077
-40	-0.047447
-50	-0.048481

Previous solution for the FFAG straight section

P.F. Meads Jr., IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4. (1983) pp. 2448-2450.



FFAG With Insertion: Three-Fold Periodicity



Matching arcs to the straight section

Input parameters are:
x_{max} and x_{min} from the arc NS-FFAG
p_{max}, p_o, and p_{min}, D_x, β_x, β_y,

Unknowns:
B_D , B_F , F_{fo} , F_{do} , and l_o

$$\rho_{do} = \frac{p_c}{eB_D} \quad \rightarrow p_c = 1.62141 \frac{MeV}{c}$$

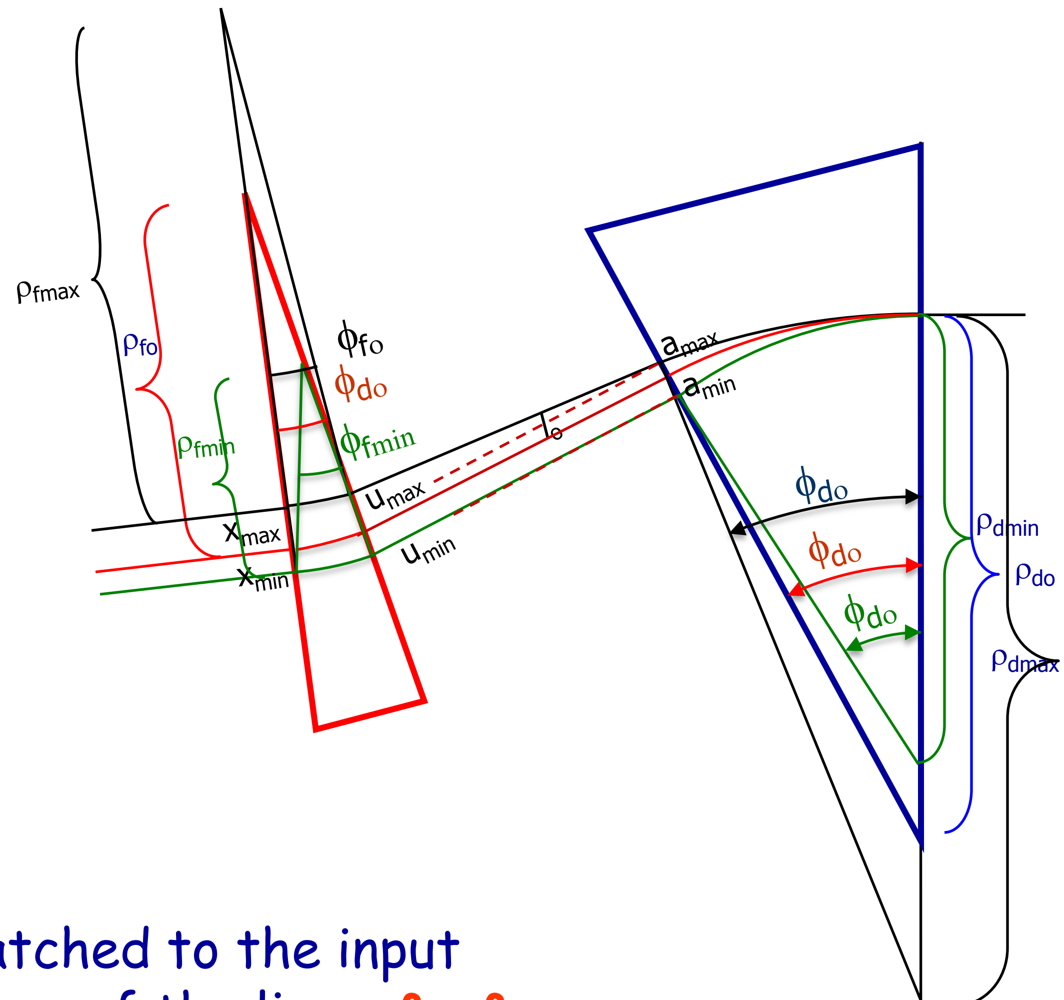
$$\rho_{d\,\max} = \frac{p_{\max}}{eB_D} \rightarrow p_{\max} = 2.43213 \frac{MeV}{c}$$

$$\rho_{d\,\min} = \frac{p_{\min}}{eB_D} \rightarrow p_{\min} = 0.81071 \frac{MeV}{c}$$

$$\rho_{fo} = \frac{p_c}{eB_F}$$

$$\rho_{f\,\max} = \frac{p_{\max}}{eB_F}$$

$$\rho_{f\,\min} = \frac{p_{\min}}{eB_D}$$



To be matched to the input parameters of the linac: β_x, β_y, α_x,α_y

Matching arcs to the straight section

$$\frac{\rho_{d\min}}{\sin\phi_{do}} = \frac{\rho_{do} - a_{\min}}{\sin\phi_{d\min}}$$

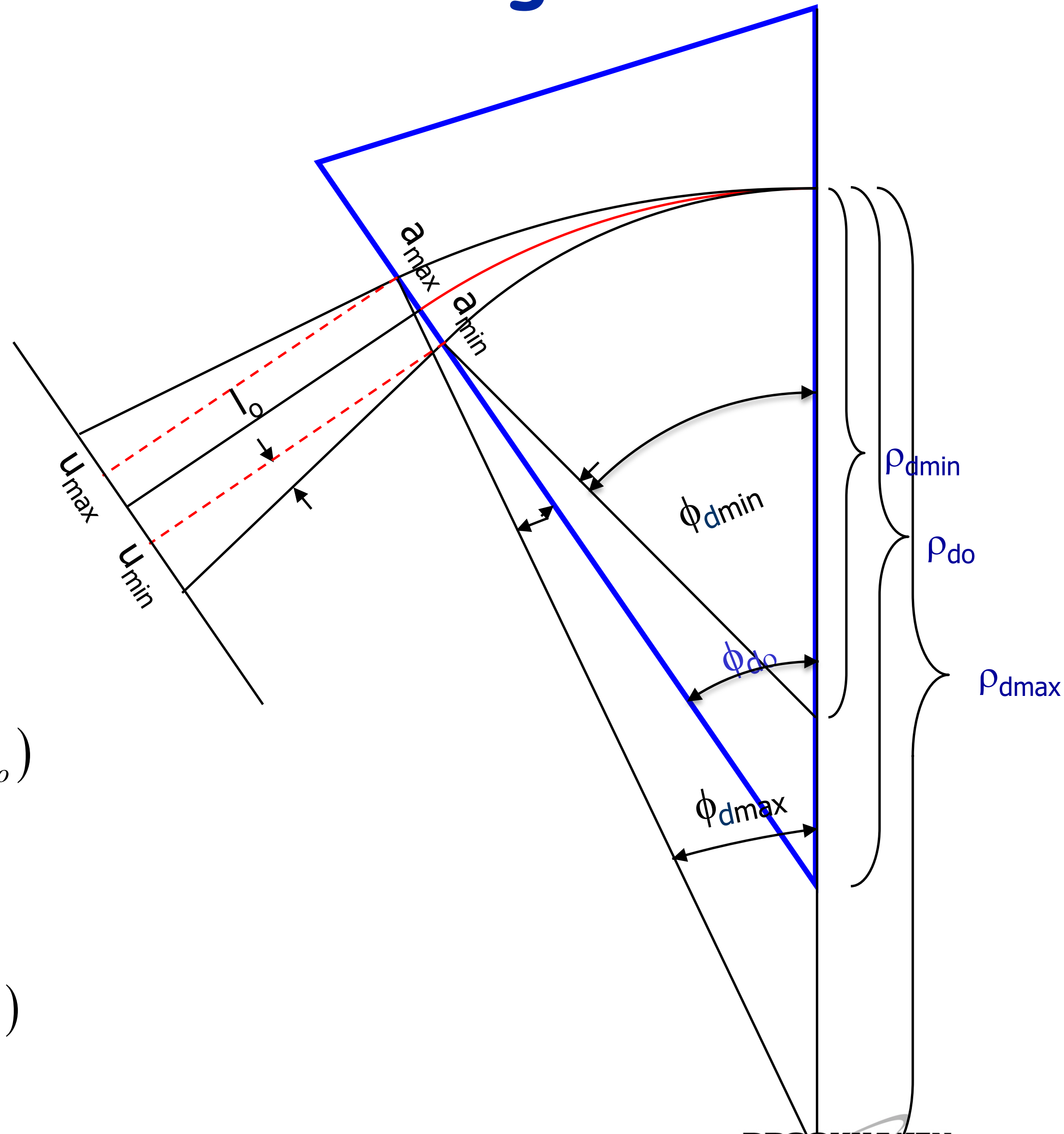
$$\frac{\rho_{d\max}}{\sin\phi_{do}} = \frac{\rho_{do} + a_{\max}}{\sin\phi_{d\max}}$$

$$a_{\min} = \rho_{do} - \rho_{d\min}\frac{\sin\phi_{d\min}}{\sin\phi_{do}}$$

$$u_{\min} = a_{\min} + \ell_o \tan\left(\phi_{d\min} - \phi_{do}\right)$$

$$a_{\max} = \rho_{d\max}\frac{\sin\phi_{d\max}}{\sin\phi_{do}} - \rho_{do}$$

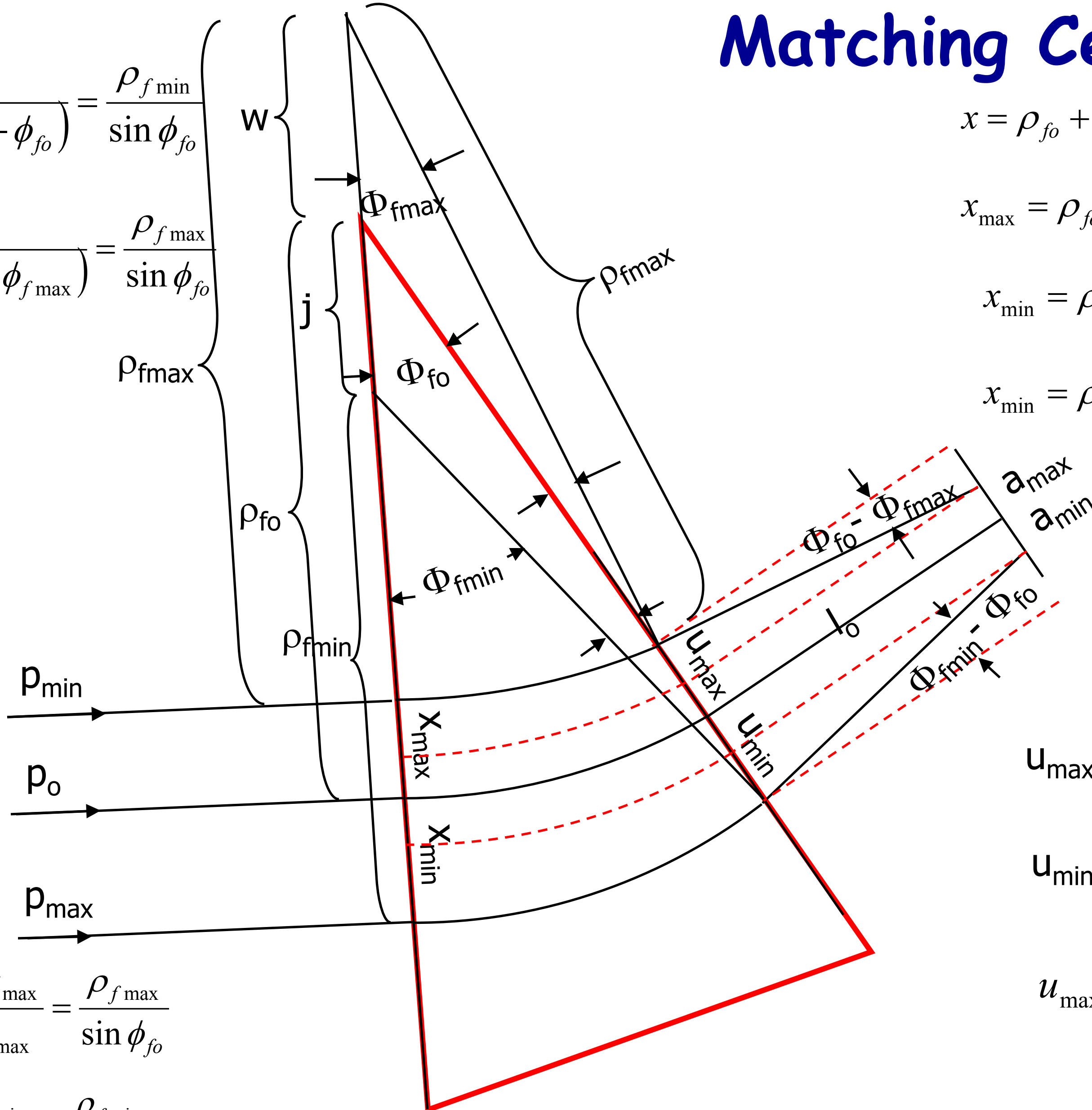
$$u_{\max} = a_{\max} + \ell_o \tan\left(\phi_{do} - \phi_{d\max}\right)$$



Matching Cell @ entrance

$$\frac{j}{\sin(\phi_{f\min}-\phi_{fo})}=\frac{\rho_{f\min}}{\sin\phi_{fo}}$$

$$\frac{w}{\sin(\phi_{fo}-\phi_{f\max})}=\frac{\rho_{f\max}}{\sin\phi_{fo}}$$



$$\frac{\rho_{fo}-u_{\max}}{\sin\phi_{f\max}}=\frac{\rho_{f\max}}{\sin\phi_{fo}}$$

$$\frac{\rho_{fo}+u_{\min}}{\sin\phi_{f\min}}=\frac{\rho_{f\min}}{\sin\phi_{fo}}$$

$$x=\rho_{fo}+w-\rho_{f\max}$$

$$x_{\max}=\rho_{fo}+\rho_{f\max}\frac{\sin(\phi_{fo}-\phi_{f\max})}{\sin\phi_{fo}}-\rho_{f\max}$$

$$x_{\min}=\rho_{f\min}+j-\rho_{fo}$$

$$x_{\min}=\rho_{f\min}\left[1+\frac{\sin(\phi_{f\min}-\phi_{fo})}{\sin\phi_{fo}}\right]-\rho_{fo}$$

$$\Phi_{fo}-\Phi_{fmax}=\Phi_{do}-\Phi_{dmax}$$

$$\Phi_{fmin}-\Phi_{fo}=\Phi_{dmin}-\Phi_{do}$$

$$u_{max}=a_{max}+l_o\tan(\Phi_{fo}-\Phi_{fmax})$$

$$u_{min}=a_{min}+l_o\tan(\Phi_{fmin}-\Phi_{fo})$$

$$u_{\max}=\rho_{fo}-\rho_{f\max}\frac{\sin\phi_{f\max}}{\sin\phi_{fo}}$$

$$u_{\min}=\rho_{f\min}\frac{\sin\phi_{f\min}}{\sin\phi_{fo}}-\rho_{fo}$$

Matching arcs to the straight section

$$X_p = \frac{r_0}{2n_0}\left\{(1-n_0)+\sqrt{(1-n_0)^2-4n_0(\Delta p/p_0)}\right\}$$

$$X_{dp} = \frac{\rho_o}{2n_d}\left[(1-n_d)+\sqrt{(1-n_d)^2-4n_d(\Delta p/p_o)}\right]$$

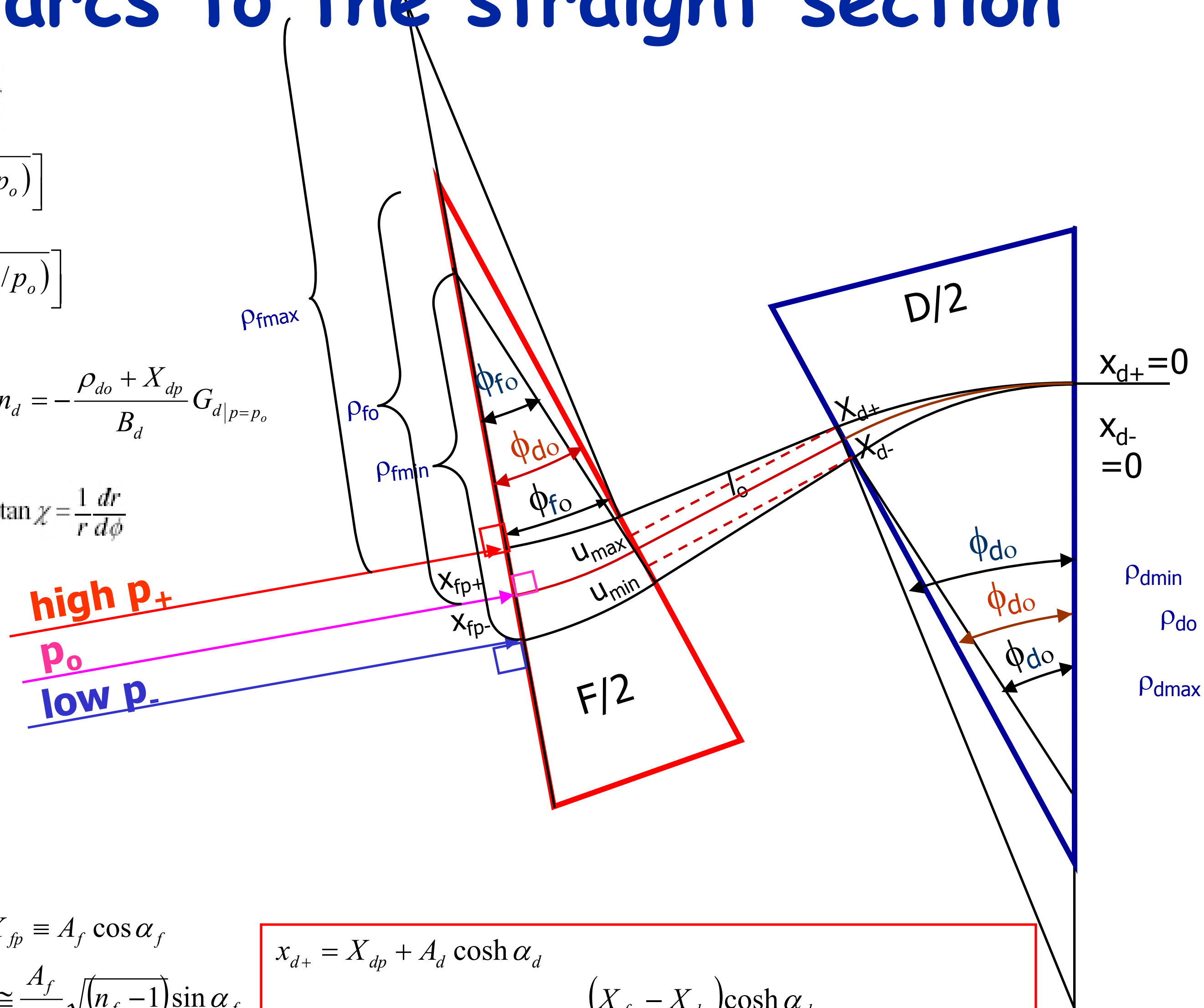
$$X_{fp} = \frac{\rho_o}{2n_f}\left[(1-n_f)+\sqrt{(1-n_f)^2-4n_f(\Delta p/p_o)}\right]$$

$$n_0 = -\frac{r}{B}\frac{dB}{dr}\Big|_{p_0} \qquad n_f = -\frac{\rho_f + x_{f\perp}}{B_f}G_{f|p=p_o} \qquad n_d = -\frac{\rho_{do}+X_{dp}}{B_d}G_{d|p=p_o}$$

$$\alpha_f \equiv \sqrt{1-n_f}\,\phi_f \qquad \alpha_d \equiv \sqrt{n_d-1}\,\phi_d \qquad \tan\chi = \frac{1}{r}\frac{dr}{d\phi}$$

$$\frac{A_f}{A_d} \equiv \sqrt{\frac{n_d-1}{1-n_f}}\,\frac{\rho_{f0}\sinh\alpha_d}{\rho_{d0}\sin\alpha_f}$$

$$A_d = \frac{X_{fp}-X_{dp}}{\cosh\alpha_d + \frac{\sqrt{n_d-1}}{\rho_{d0}}\sinh\alpha_d\left[\ell - \frac{\rho_{f0}}{\sqrt{1-n_f}\tan\alpha_f}\right]}$$



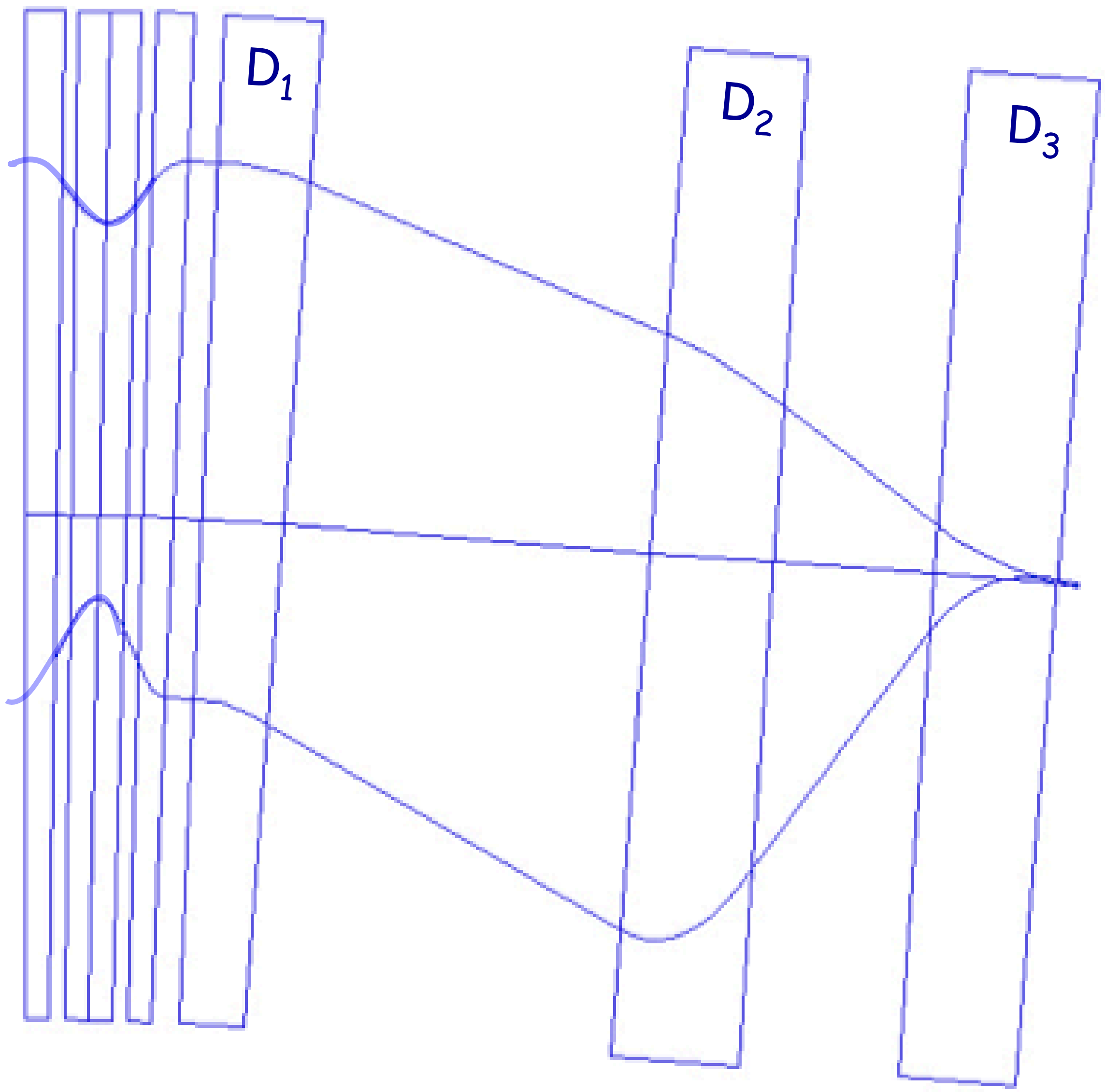
$$x_{d\perp} - X_{dp} \equiv A_d\cosh\alpha_d \qquad x_{f\perp} - X_{fp} \equiv A_f\cos\alpha_f$$

$$\tan\chi_d \cong \frac{A_d}{\rho_{do}}\sqrt{(n_d-1)}\sinh\alpha_d \qquad \tan\chi_f \cong \frac{A_f}{\rho_{fo}}\sqrt{(n_f-1)}\sin\alpha_f$$

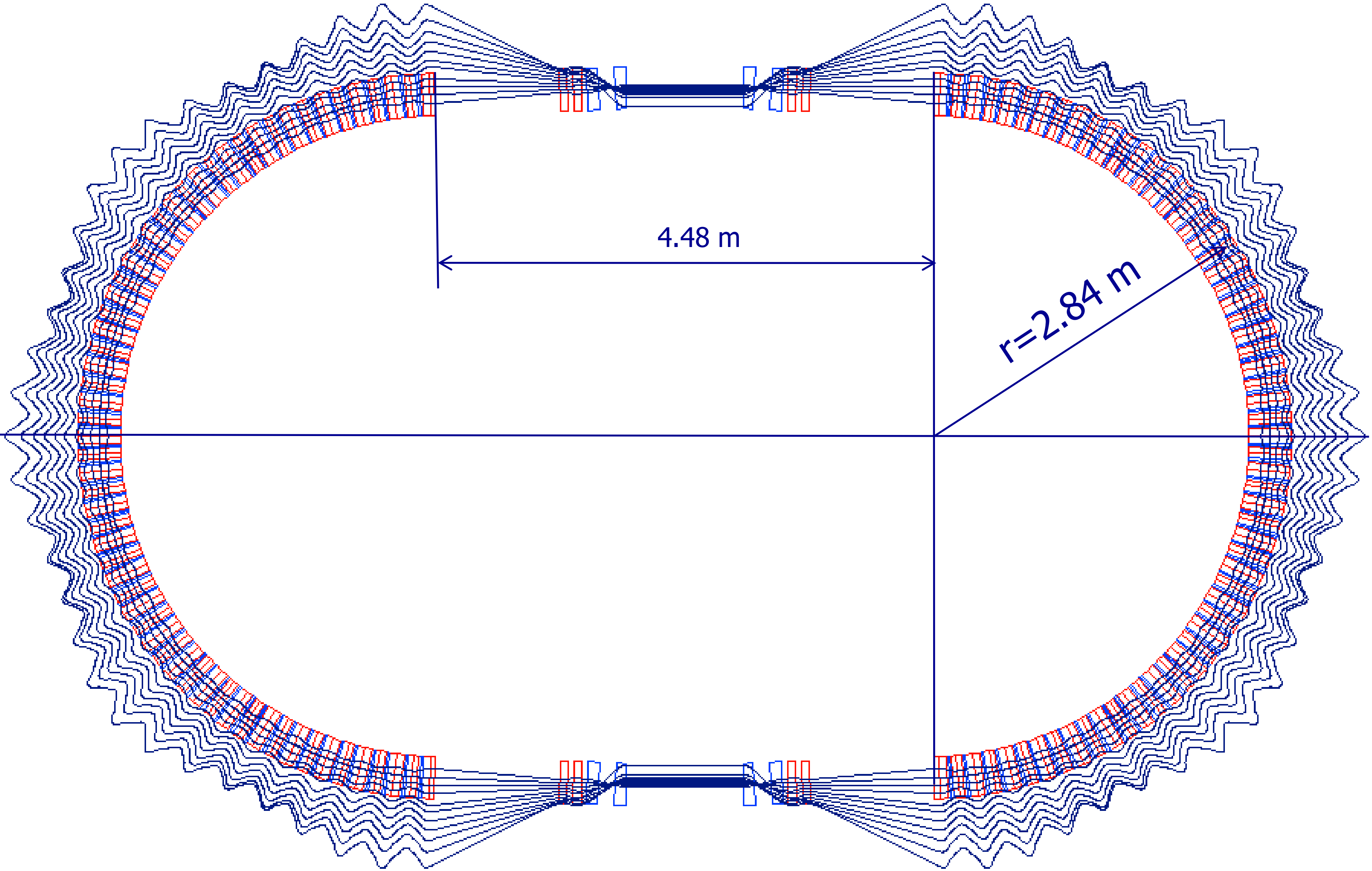
$$x_{d+} = X_{dp} + A_d\cosh\alpha_d$$

$$x_{d+} = X_{dp} + \frac{(X_{fp}-X_{dp})\cosh\alpha_d}{\cosh\alpha_d + \frac{\sqrt{(n_d-1)}}{\rho_{do}}\sinh\alpha_d\left[l_o - \frac{\rho_{fo}}{\sqrt{1-n_f}\tan\alpha_f}\right]} = 0$$

Orbits of the maximum and minimum energy

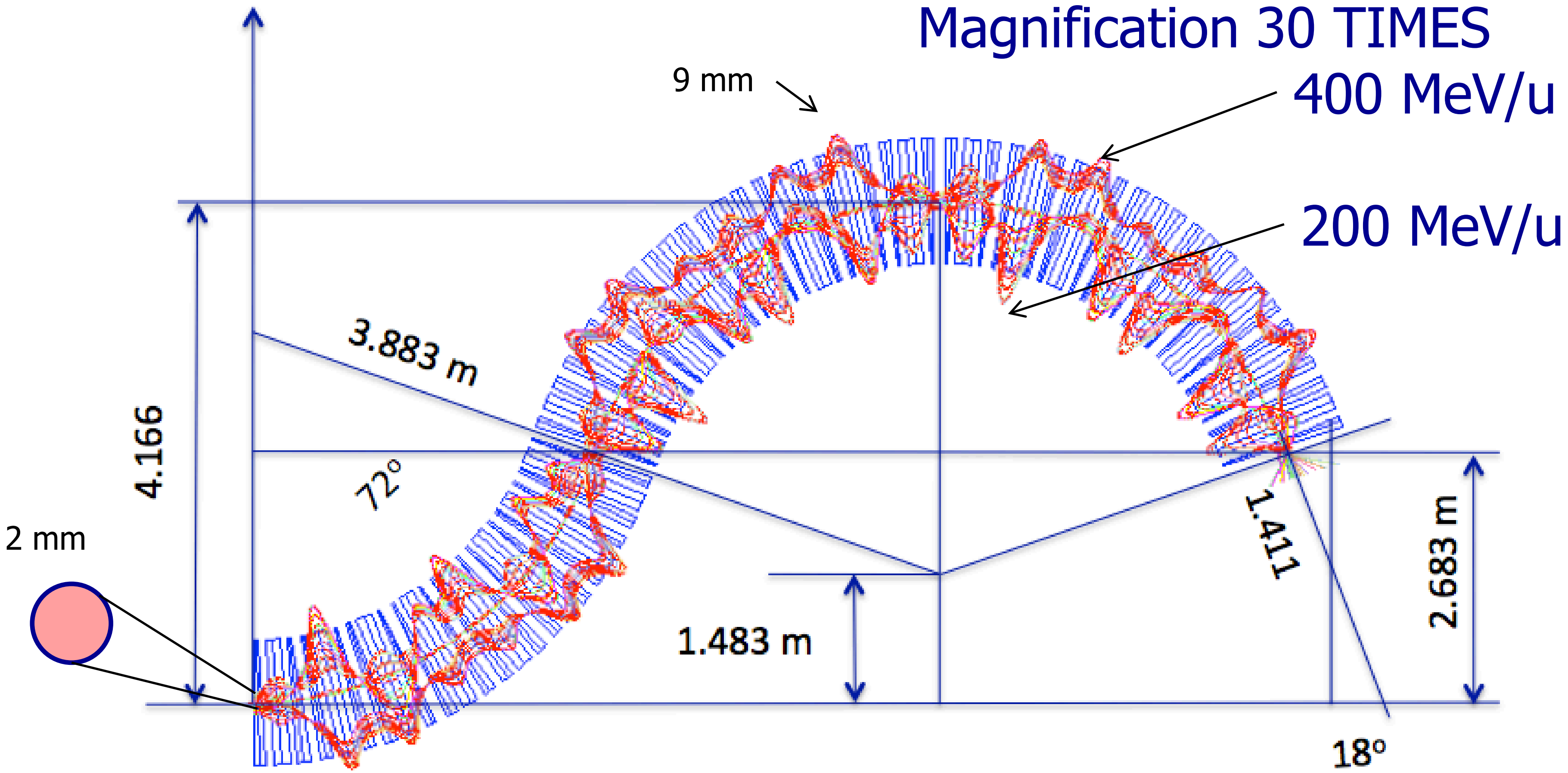


Race track NS-FFAG

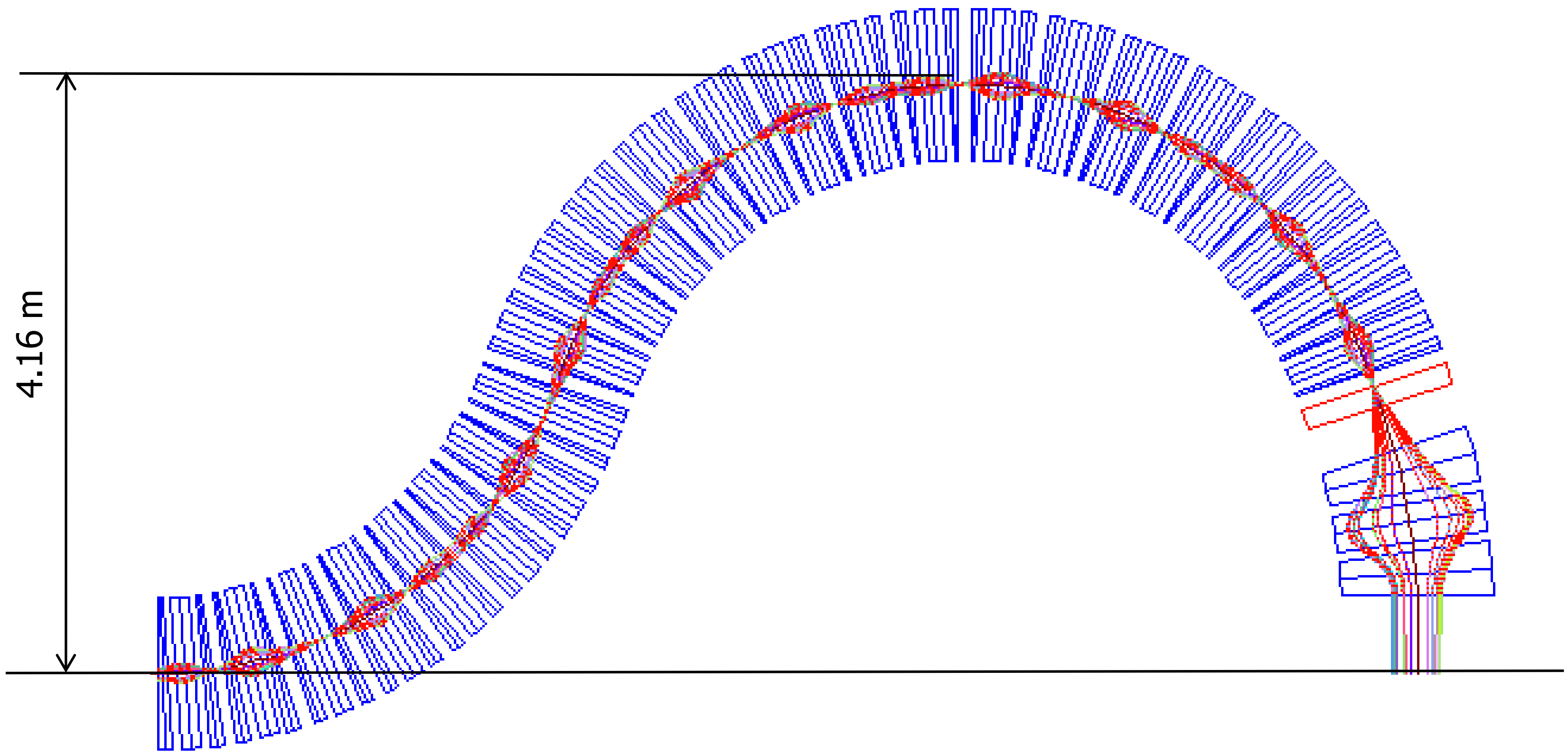


BROOKHAVEN NATIONAL LABORATORY

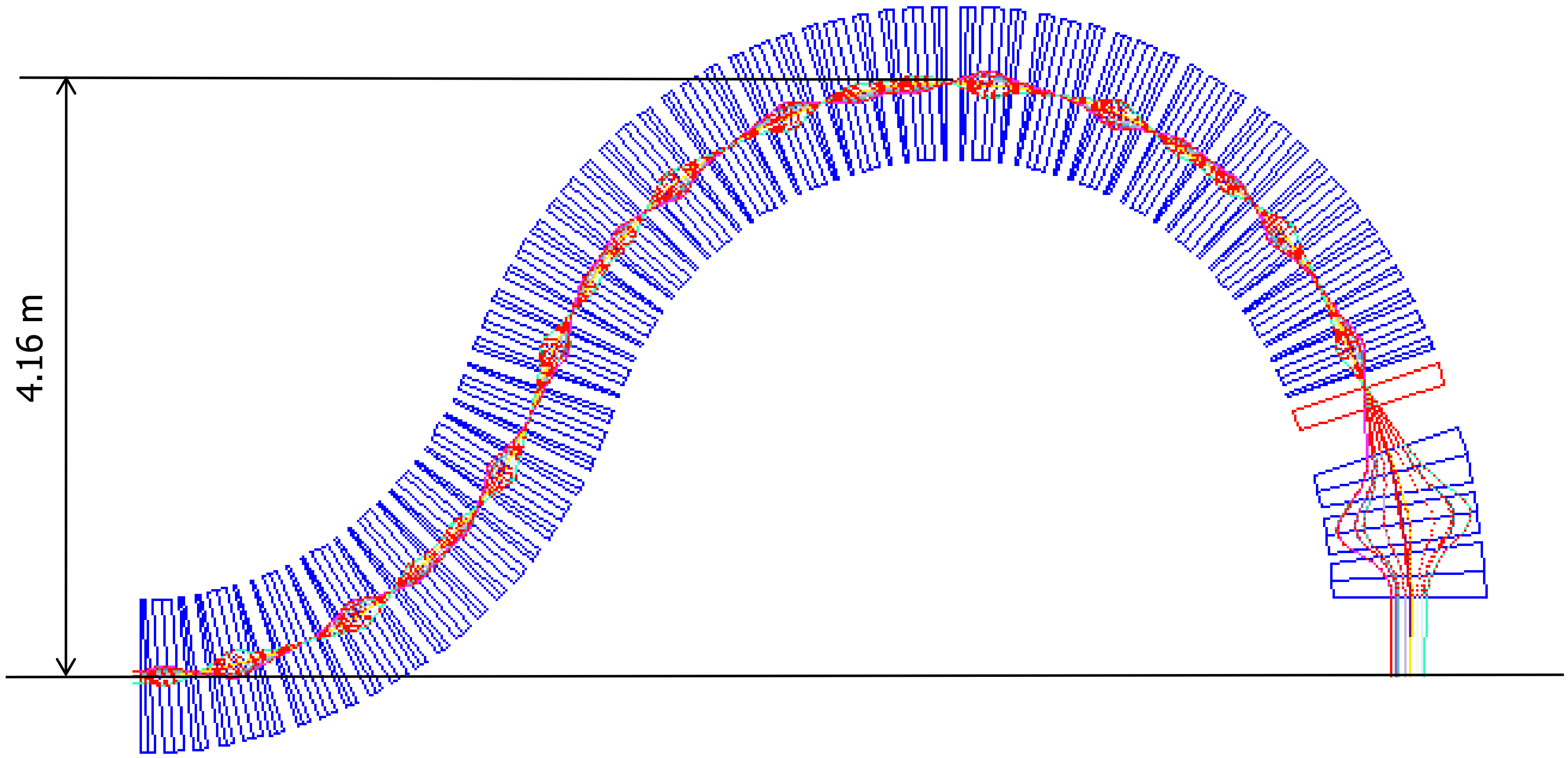
All at once: Fixed field & fixed focusing



Reaching the patient with parallel beams



Reaching the patient with parallel beams



Vasily Morozov - Dejan Trbojevic
NS-FFAG 10 fixed gradients

KBF1 = 212.7332 T/m

KBD1 = -179.260 T/m

KBF2 = 214.650 T/m

KBD2 = -173.543 T/m

KBF3 = 216.805 T/m

KBD3 = -171.042 T/m

KBF4 = 220.030 T/m

KBD4 = -178.477 T/m

KBD5 = -182.891 T/m

KFTRP1 = 25.5 T/m
KDTRP2 = -25.5 T/m
KFTRP3 = 25.5 T/m

LBFTRP = 0.20 m
LBDTRP = 0.34 m
LBFTRP = 0.20 m

BFtr = 1.905 T
BDtr = 0.4035 T

Acceleration PHASE JUMP each turn

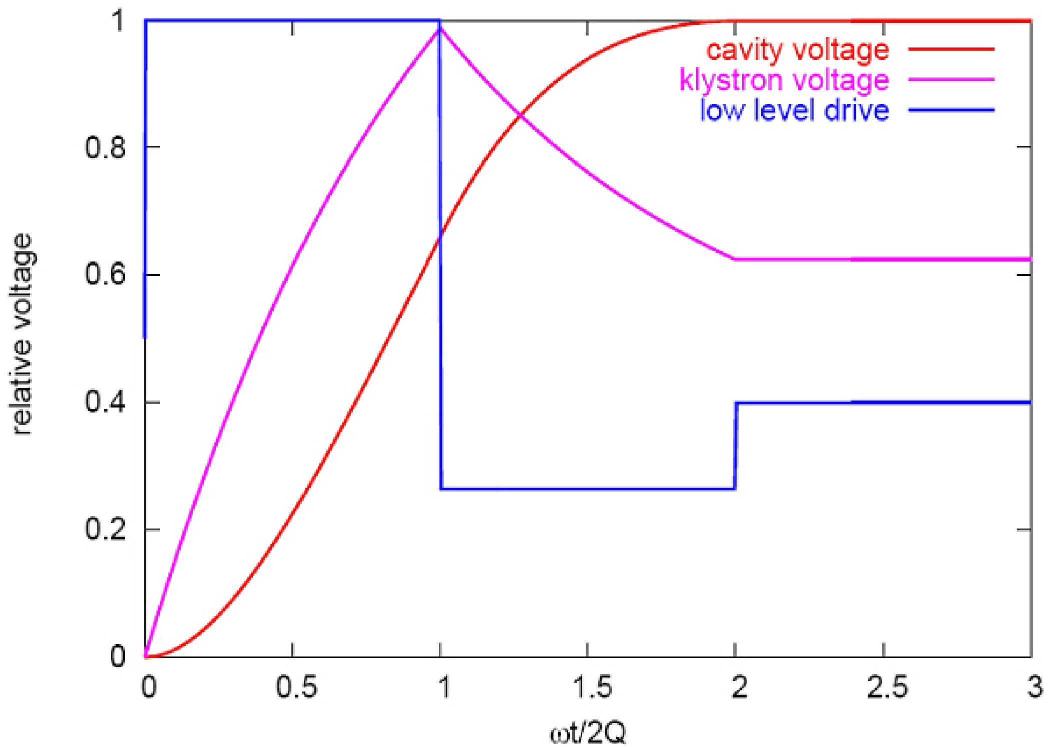
Acceleration is performed with the phase jump after each turn. The phase jump during acceleration with the fixed frequency by M. Blaskiewicz.

- The RF frequency needs to be in a high range, 370 MHz because of required large number of RF cycles between the passages of bunches in order to achieve higher values of Q and to limit the frequency swing. The total stored energy in the cavity is related to the amplitude V_{RF} of the RF voltage as:

$$U = \frac{V_{RF}^2}{2\omega_r \frac{R}{Q}}$$

where ω_r is the angular resonant frequency, Q is the quality factor, and R is the resistance.

The cavity voltage dependence on the klystron voltage (driven by the low level drive) →



ACCELERATION:

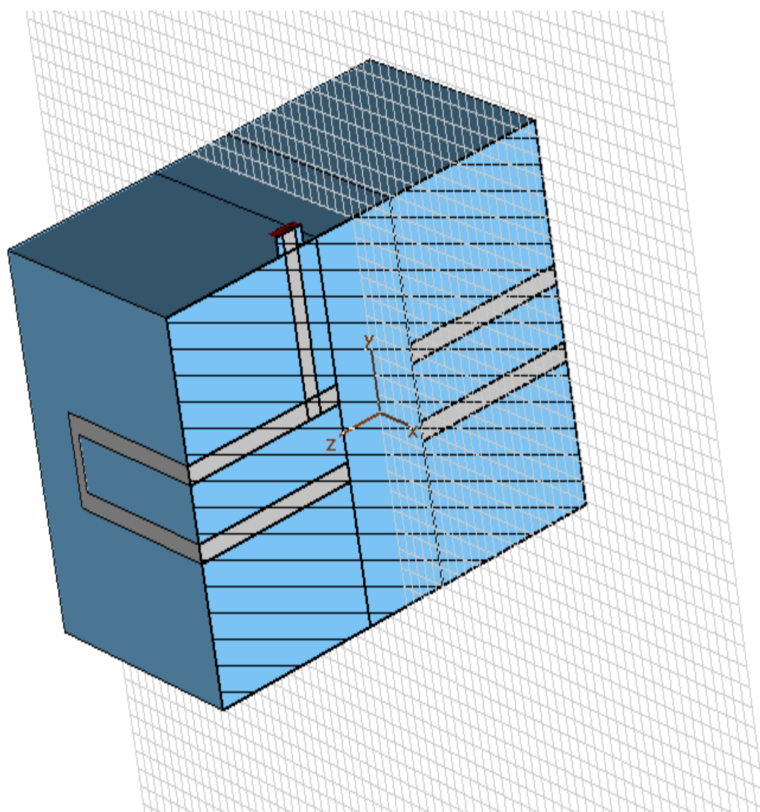
Requires a loaded quality factor Q=50

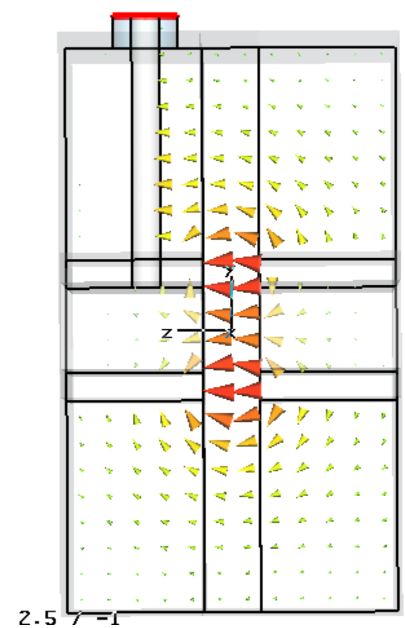
Full horizontal aperture 28 cm

Full vertical aperture 3 cm, R/Q = 33 Ohm (circuit)

for β=0.24

$$U = \frac{V_{accel}^2}{2\omega_r (R/Q)}$$





PROTON ACCELERATION 31-250 MeV

The bunch train lls half of the ring at the injection. The changes from injection to the maximum energy of 250 MeV between β_{inj}= 0.251 to $\beta_{extr.}$= 0.614. There is 80 ns time to change the cavity frequency when there is no beam. With Q=50 and f=374 MHz the exponential decay time for the eld is 43 ns, where R=Q 33 at =0.25. If the synchronous voltage is 22 kV, a number of turns required for acceleration of protons is:

$$N_{turns} = \frac{(250-31)[MeV]}{20\,[keV/cav]*n_{cav}} \approx 912$$

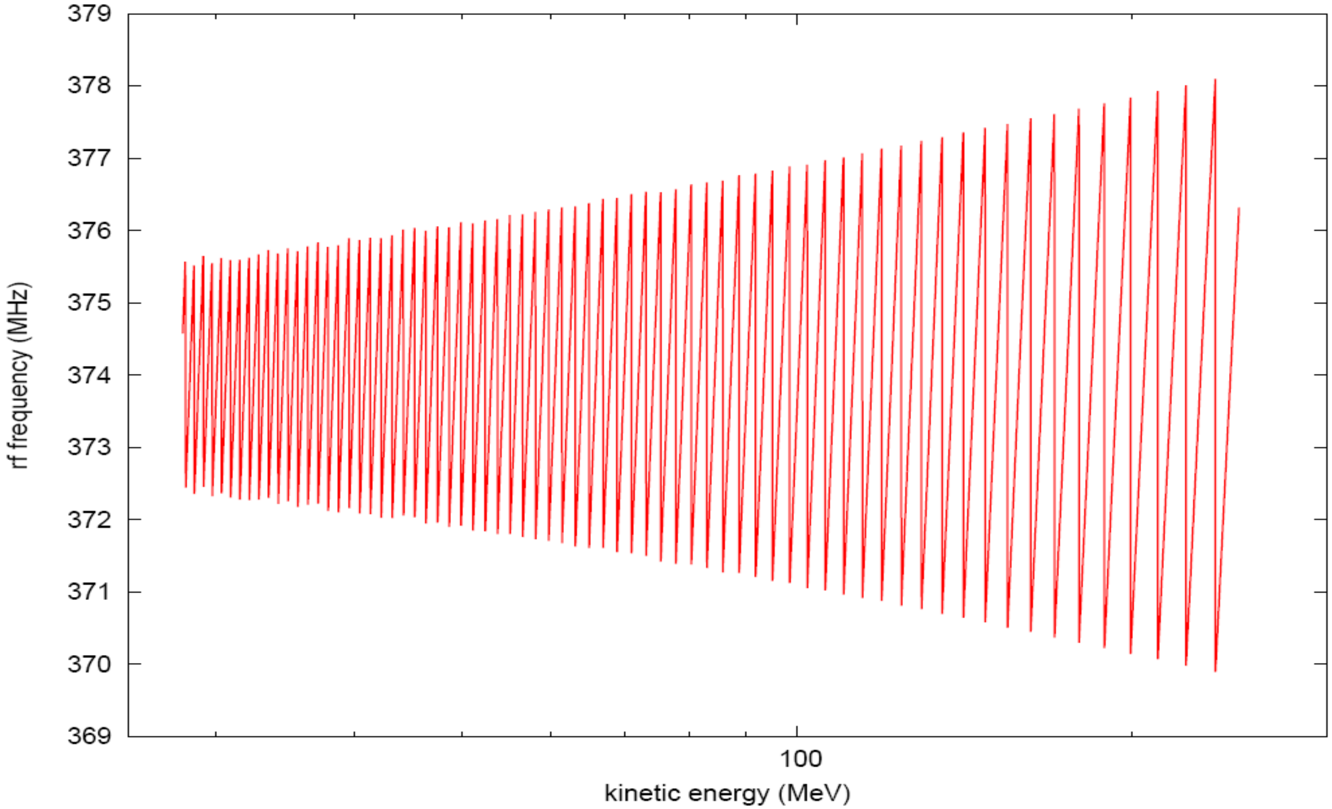
for twelve cavities (ncav=12). It is very clear that higher effective voltage on the cavities could improve the the resonance crossing problem as well as the patient treatment time. A total power for one RF driver is 100 kW, for twelve cavities this make 1.2 MW and this is a price for the fast acceleration.

Acceleration:

26.88 meter circumference

31 MeV < proton kinetic energy < 250 MeV, $0.24 < \beta < 0.61$

Central rf frequency = 374 MHz



Summary

• Today most of the proton cancer therapy accelerators are cyclotrons or slow extraction synchrotrons.

• An example of racetrack NS-FFAG was described were fast acceleration is assumed with a total number of turns less then 1000 as the integer resonance crossing can be a problem.

• To simplify the solution the permanent separated function magnets of the Halbach structure are proposed.

•The orbit offsets in the example presented are within 11mm<Δx<17 mm. This allows use of an aperture of 30 mm. With the outside diameter of 17.75 cm from the available Nd-Fe-B (for temperatures less than 70 C) materials a bending dipole field of 2.4 T could be obtained.

•Advantage of the accelerator is very small magnets and simplified operation, as the magnets are permanent. Acceleration is assumed to be with a fast phase jump scheme where the voltage on the cavities is changed within one turn of the circulating bunch.