Electron Beam Focusing by an Anisotropic Solid-State Plasma

Atsushi Ogata, Koichi Kan, Kimihiro Norizawa, TakaFumi Kondoh, Jinfeng Yang, Yoichi Yoshida

Abstract

This paper proposes the use of a solid-state plasma lens in the fs linacs, in which the bunch radii of the beams are much larger than the lengths. Although gas plasmas degrade vacuum, solid-state plasmas do not. Another problem is that a return current flowing in the beam region of plasmas decreases the focusing power of the lens. By orientating anisotropic plasmas, we can constrain the return current, while maintaining the focusing power.

INTRODUCTION

Electron bunches having sizes of the order of fs enhance the collective ionization and coherent radiation by the factor of $N^2$ for wavelengths shorter than any previously realized[1], in which $N$ is the number of electrons in a bunch. Although the bunch lengths and radii should be of the same order for this purpose, the radii remain much longer than the lengths in present-day linacs.

Plasma lenses, which will reduce the bunch radii, show the promise for providing a focusing strength that is several orders of magnitude higher than that which can be produced by conventional magnets. The omni-directional focusing in plasma lenses is in contrast to that in magnetic lenses, in which focusing and defocusing directions are always orthogonal to each other.

However, on introducing the gas plasmas into the linac beam line, we encounter with a serious problem: the deterioration of high vacuum. One solution to this problem is the use of solid-state plasmas.

The physical mechanism for focusing of particle beams using passive (no external current) plasmas is the expulsion of plasma electrons by the beam, and the self-focusing of the beam through its own magnetic field. Solid-state plasmas in electron beam lines are overdense: $n_p > n_b$, where $n_p$ and $n_b$ denote electron densities in the plasma and in the beam, respectively. The space charge of the electron beam is fully neutralized and the resulting focusing force is determined solely by $n_b$[2].

SOLID-STATE PLASMA LENS FOR SHORT AND WIDE BUNCHES

The plasma lens for a bunched beam has to satisfy two conditions. One condition is that the plasma response time should be shorter than the bunch length. It is represented by

$$
\frac{c}{\omega_p} < t,
$$

where $t$ is the bunch length, $c$ is the speed of light, $\omega_p = \left(\frac{e^2 n_p}{m_0}\right)^{1/2}$ is the plasma frequency, $m$ is the electron rest mass, and $\epsilon_0$ is the dielectric constant.

The return current causes the second condition:

$$
r < \frac{c}{\omega_p},
$$

where $r$ is the bunch radius. This condition arises from the following mechanism. The beam induces a longitudinal return current in the plasma that, by Lenz’ law, will flow in the direction opposite to the beam current direction[3]. The scale length of the radius over which the plasma return current flows is of the order of $c\omega_p^{-1}$. The radial force acting on the electron beam is determined by the difference between the beam and the plasma return current densities in the beam region. Therefore, fully overlapping of the beam and return currents in the beam region lead to a significant reduction in the radial force. Govil et al. have experimentally observed return current effects in a gas plasma lens[4].

Most present-day fs linacs used for studying fast phenomena[5] have bunches in which

$$
t < r.
$$

Inequalities (1) and (2) are in contradiction when this third condition is considered. We can avoid this contradiction using anisotropic solid plasmas. Consider conductive layers that are insulated in the direction normal to the layers. On orientating such layers normal to the beam current, we will have high transverse $\omega_p$ in the absence of the return current.

One of the materials fabricated from the anisotropic solid-state plasma is graphite. Figure 1 shows the structure of the graphite. We orient the graphite layers normal to the beam, as shown in the figure. There are different data for the frequency of the bond plasmas of the graphite. We assume $h\omega_p = 5$ eV, or $\omega_p = 7.69 \times 10^{15}$ s$^{-1}$ and $n_p = 1.81 \times 10^{22}$ cm$^{-3}$ in the following[6, 7], where $h$ denotes Plank’s constant.

Few data are available on the Van der Waals bonds between graphite layers. According to a data sheet for commercial graphite, the resistances parallel and perpendicular to the layer are $4 \times 10^5$ ohm cm and 0.15 ohm cm, respectively[8]. If we consider the configuration shown in Fig.2, the return current will flow in the conducting pipe outside the plasma.

Because the electron density in solid-state plasma is higher than that in the beam, the plasma lens is in the so-called “overdense” regime. The focusing strength $K$ of an
overdense plasma lens is given by the equation:

\[ K = \frac{2\omega_b^2}{c^2}, \quad (4) \]

where \( \omega_b \) denotes the plasma frequency determined by the electron density in the beam \( n_b \), and \( \gamma \) denotes the Lorentz factor\(^2\). If we introduce the equivalent magnetic gradient \( G(r) = W_\bot (r) r^{-1} \), the focusing strength becomes

\[ K = eG(r)(mc\gamma)^{-1}, \]

where \( W_\bot (r) \) denotes the transverse wake field of the beam.

ENVELOPE EQUATION OF THE ISIR LINAC BEAMS

In this section, we will develop the discussion using specific but typical linac parameters for the ISIR linac used for pulse radiolysis experiments, which has a photocathode RF gun. The parameters are summarized in Table 1 for typical operations.

The envelope equation of a linac is given by\(^{10}\)

\[ \frac{d^2}{dz^2} \sigma_r(z) + K \sigma_r(z) = \frac{(z)^2}{\sigma_r(z)^2}, \quad (5) \]

where \( z \) denotes emittance. Equation (4) together with the parameters in the table gives \( K = 975 \times 10^3 \text{ m}^{-2} \). This value is substantially larger than that of conventional magnetic lenses, but we have to design the lens with a length of the order of \( \text{mm} \) or less, instead of \( \text{m} \).

In the calculation, we should consider the emittance increase due to multiple Coulomb scattering and bremsstrahlung in the lens. The scattering angle is given by the equation\(^9\):

\[ \theta(z) = \frac{1}{2} \left[ 1 + 0.038 \log \left( \frac{z}{X_0} \right) \right], \quad (6) \]

where the radiation length \( X_0 \) is defined as

\[ X_0 = \frac{716.4 A}{Z(Z + 1) \log(287/Z^{1/2})} \text{ g cm}^{-2}, \quad (7) \]

where \( A \) and \( Z \) denote the mass and atomic numbers, respectively. In graphite having a density \( 2.26 \text{ g cm}^{-3} \), \( X_0 = 42.9 \text{ g cm}^{-2} \) or \( 19.0 \text{ cm} \).

Table 1: Parameters of ISIR linac.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>( E_{\text{beam}} )</td>
<td>30</td>
<td>MeV</td>
</tr>
<tr>
<td>Bunch length</td>
<td>( l )</td>
<td>60</td>
<td>m</td>
</tr>
<tr>
<td>Bunch length</td>
<td>( t/c )</td>
<td>200</td>
<td>fs</td>
</tr>
<tr>
<td>Bunch radius</td>
<td>( r )</td>
<td>200</td>
<td>m</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>( Q )</td>
<td>400</td>
<td>pC</td>
</tr>
<tr>
<td>Electrons in a bunch</td>
<td>( N )</td>
<td>2.5</td>
<td>( 10^9 )</td>
</tr>
<tr>
<td>Electron density</td>
<td>( n_b )</td>
<td>8.29</td>
<td>( 10^{14} ) cm(^{-3} )</td>
</tr>
<tr>
<td>Plasma frequency</td>
<td>( \omega_p )</td>
<td>1.62</td>
<td>( 10^{12} ) s(^{-1} )</td>
</tr>
<tr>
<td>Emittance</td>
<td>( \epsilon )</td>
<td>1</td>
<td>( \mu \text{m} )</td>
</tr>
</tbody>
</table>

\( \sigma_r \) value is substantially larger than that of conventional magnetic lenses, but we have to design the lens with a length of the order of \( \text{mm} \) or less, instead of \( \text{m} \).

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Figure 3 shows the angle of the 30MeV electron beam scattered by graphite as a function of graphite thickness \( z \). Substituting the emittance with

\[ \theta(z) = \theta_0 + \theta(z), \quad (8) \]
in eq. (5), we can calculate the evolution of the beam envelope. The results are shown in Fig. 4 in two scales. Calculations were performed under the assumptions of the initial conditions \( \sigma_r(0) = 200 \, \mu m, \sigma_r'(0) = 1.5 \times 10^{-3}, \sigma_r(0), \sigma_r'(0) = 0 \). The solid lines show the case in which the plasma occupies the whole region, while the dashed lines show the case in which the plasma occupies the region between 0 and 0.5 mm, which is indicated by the pale blue color.

Once the envelope, i.e., the bunch radius, reaches its minimum value, it increases monotonously in the free space if the lens is appropriately thin, while it increases with the betatron oscillation in the lens if the plasma is long enough. This increase is due to the emittance growth. The thin lens causes a reduction of the bunch radius from 200 \( \mu m \) to 75 \( \mu m \) at the focal point.

One phenomenon that has not been considered is the relaxation of the plasma oscillation. Solid-state plasmas have shorter relaxation time than the gas plasmas. Johnson and Dresselhaus reported that the relaxation time of the graphite plasma is \( 10^{-14} \) s \cite{11}. Although the beam electrons force the plasma electrons to oscillate, the relaxation reduces the amplitude, and, in turn, the focusing strength.

Experiments are planned at ISIR linac, Osaka University.

**FOIL FOCUSING: AN ALTERNATIVE**

An alternative method involves the use of thin metal foils. This is an old technique known as foil focusing \cite{12-14}. It uses a periodic array of transverse conducting foils. The image charges in the foils reduce the average radial electric field acting on the beams, and as a result they can be confined by self-magnetic fields.

Foil focusing was originally used for dc beams, in which the return current is considered not to flow in the thin foils. To apply foil focusing to beams of short bunches, the foil thickness should be much shorter than the bunch length. In the present case, the thickness should be approximately a few \( \mu m \) or less. If this condition is satisfied, the configuration of Fig.2 should function satisfactorily.

As shown in Fig. 4, the lens thickness should be at least a few hundred \( \mu m \) or more. We have to stack a hundred foil sheets with insulating spacing. Although the foil material is more easily available than graphite, this task is not easy.

**REFERENCES**