# BEAM-BASED CONFIRMATION OF SKEW-QUADRUPOLE FIELD CORRECTION IN 10.8 M LONG UNDULATOR 

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## Abstract

The synchrotron radiation facility NewSUBARU is a 1.5 GeV storage ring which has two long straight sections. A permanent magnet, planar-type, out-of-vacuum 10.8 m Long Undulator is placed in one of the long straight sections. The longitudinal moments of the skewquadrupole field errors in the undulator were determined from the response of the stored beam. The method used was to measure the change of the horizontal closed orbit distortion produced by two types of vertical local bump orbit in the undulator. This method is more reliable than the global modeling of a ring, which uses the response matrix of the whole ring, since the present method is not sensitive to skew-quadrupole field errors in sections other than the target section.

## INTRODUCTION

The synchrotron radiation facility NewSUBARU [1] is an EUV and soft X-ray light source at the SPring-8 site. The Laboratory of Advanced Science and Technology for Industry (LASTI) at the University of Hyogo is in charge of its operation, in collaboration with SPring-8. The main parameters of the storage ring are listed in Table 1. The storage ring is a racetrack type, with two 14 m long straight sections. In one of the long straight sections, a permanent magnet, planar-type, out-of-vacuum 10.8 m Long Undulator (LU) [2] is in operation. The main parameters of LU are listed in Table 2. The ring has two operation modes. In the 1.5 GeV mode, the injected beam is accelerated to 1.5 GeV and stored. In the 1.0 GeV topup mode, which started in 2003, the beam current is kept at $250 \pm 0.15 \mathrm{~mA}$ by occasional injection with the undulator gap closed [3]. For top-up operation, adjustment of the skew-quadrupole field is necessary for good injection efficiency. In particular, the contribution of the undulator, which depends on the gap, requires correction. The correction is necessary not only for injection but also for operation with small coupling, which gives a good light spectrum from the undulator.

TABLE 1. Main parameters of the storage ring.

| Electron Energy | $0.5-1.5 \mathrm{GeV}$ |
| :--- | :---: |
| Injection Energy | 1.0 GeV |
| Circumference | 118.731 m |
| Type of Bending Cell | Modified DBA |
| Maximum Stored Current | 500 mA |
| Betatron Tune $v_{x} / v_{y}$ | $6.30 / 2.23$ |
| Natural Emittance at 1 GeV | 38 nm |
| Natural Energy Spread at 1 GeV | $0.047 \%$ |

[^0]TABLE 2. Main parameters of LU.

| Type | Planar, out of vacuum |
| :--- | :---: |
| Magnet | Permanent Nd-Fe-B |
| Number of Periods | 200 |
| Period Length | 54 mm |
| Gap | $119-25 \mathrm{~mm}$ |
| K | $0.3-2.5$ |
| Number of Units | 8 |
| Total Length | 10.8 m |

The skew-quadrupole field errors in LU were determined from the response of the stored beam. Beambased measurement is useful for an undulator which cannot be detached from the storage ring easily because of its length, such as 10.8 m LU of NewSUBARU. The changes of the horizontal closed orbit distortion (COD) produced by two types of vertical local bump orbits [4] in LU give sufficient information about the linear coupling. The distribution of the skew-quadrupole field errors can be expressed by their multipole moments along the beam axis. This method is more reliable than the global modeling of a ring, which uses the response matrix of the whole ring [5], since the present method is not sensitive to skew-quadrupole field errors in sections other than the target section.

In 2006, the control system of the insertion devices will be improved. The new system will have feed-forward local correction coils instead of ring correction magnets. This paper reports preparatory experiments in which the gap-dependent skew-quadrupole field errors were cancelled by shimming [6] and the existing correction coils.

## ANALYTICAL FORMULAS FOR LONGITUDINAL MOMENTS

The coordinates $x, y$ and $s$ used here are the horizontal and vertical displacements and the longitudinal coordinate, respectively. In order to express the distribution of the skew-quadrupole field along the beam axis, we use the moments along the $s$-axis. If the undulator is located from $s_{K}=-s_{0}$ to $s_{K}=+s_{0}$, the $N$ th moment $K_{N}$ is given by

$$
\begin{equation*}
K_{N} \equiv \int_{-s_{0}}^{s_{0}} s_{K}^{N} K\left(s_{K}\right) d s_{K} . \tag{1}
\end{equation*}
$$

Here $N$ is an integer, where $N=0, \ldots, \infty . K$ is the skewquadrupole field, given by

$$
\begin{equation*}
K \equiv(1 / B \rho)\left(\partial B_{x} / \partial x\right) \tag{2}
\end{equation*}
$$

where $B_{x}$ is the horizontal magnetic field and $B \rho$ is the momentum of reference electrons.

## Linear Transfer Matrix for Small SkewQuadrupole Field

We assume that the undulator is in free space and that the beam focusing caused by the undulator is negligibly small. If there is a local skew-quadrupole field with strength $K \Delta s$ at $s=s_{K}$, the $4 \times 4$ linear transfer matrix through the undulator is

$$
\begin{align*}
& \left(\begin{array}{c}
x\left(s_{0}\right) \\
x^{\prime}\left(s_{0}\right) \\
y\left(s_{0}\right) \\
y^{\prime}\left(s_{0}\right)
\end{array}\right)=\left(\begin{array}{cc}
1 & 2 s_{0} \\
0 & 1 \\
K \Delta s\left(s_{0}-s_{K}\right) & K \Delta s\left(s_{0}+s_{K}\right)\left(s_{0}-s_{K}\right) \\
K \Delta s & K \Delta s\left(s_{0}+s_{K}\right) \\
K \Delta s\left(s_{0}-s_{K}\right) & K \Delta s\left(s_{0}+s_{K}\right)\left(s_{0}-s_{K}\right) \\
K \Delta s & K \Delta s\left(s_{0}+s_{K}\right) \\
1 & 2 s_{0} \\
0 & 1
\end{array}\right)\left(\begin{array}{c}
x\left(-s_{0}\right) \\
x^{\prime}\left(-s_{0}\right) \\
y\left(-s_{0}\right) \\
y^{\prime}\left(-s_{0}\right)
\end{array}\right) .
\end{align*}
$$

Here, a prime (') means a derivative with respect to $s$. In the case where $K$ is distributed along the length of the undulator, we can assume that $K$ is very small, and then the higher order terms with respect to $K$ can be ignored. The transfer matrix is now given by

$$
\begin{align*}
& \left(\begin{array}{c}
x\left(s_{0}\right) \\
x^{\prime}\left(s_{0}\right) \\
y\left(s_{0}\right) \\
y^{\prime}\left(s_{0}\right)
\end{array}\right)=\left(\begin{array}{ccc}
1 & 2 s_{0} \\
0 & 1 \\
s_{0} K_{0}-K_{1} & s_{0}{ }^{2} K_{0}-K_{2} \\
K_{0} & s_{0} K_{0}+K_{1} \\
s_{0} K_{0}-K_{1} & s_{0}{ }^{2} K_{0}-K_{2} \\
K_{0} & s_{0} K_{0}+K_{1} \\
1 & 2 s_{0} \\
0 & 1
\end{array}\right)\left(\begin{array}{c}
x\left(-s_{0}\right) \\
x^{\prime}\left(-s_{0}\right) \\
y\left(-s_{0}\right) \\
y^{\prime}\left(-s_{0}\right)
\end{array}\right) .
\end{align*}
$$

This shows that moments of order higher than $K_{2}$ are not necessary for expressing the linear coupling in free space.

## Horizontal COD Produced by Vertical Displacement

When a vertical orbit is displaced by $\Delta y$ in LU, a local skew-quadrupole field $K \Delta s$ at $s=s_{K}$ produces a horizontal COD given by

$$
\begin{align*}
& \Delta x(s)=\frac{\sqrt{\beta_{x}(s) \beta_{x}\left(s_{K}\right)}}{2 \sin \pi v_{x}} \times \\
& \quad \cos \left(\pi v_{x}-\left|\psi_{x}(s)-\psi_{x}\left(s_{K}\right)\right|\right) \Delta y\left(s_{K}\right) K \Delta s . \tag{5}
\end{align*}
$$

Here $\nu_{x}, \beta_{x}$ and $\psi_{x}$ are the horizontal betatron tune, the beta function and the betatron phase, respectively. When $K$ is distributed along the undulator, the horizontal COD at other than the undulator section ( $s_{0}<s<C-s_{0}$ ) is given by

$$
\begin{align*}
& \Delta x(s)=\frac{\sqrt{\beta_{x}(s)}}{2 \sin \pi v_{x}} \int_{-s_{0}}^{s_{0}} \sqrt{\beta_{x}\left(s_{K}\right)} \times \\
& \quad \cos \left(\psi_{x}(s)-\psi_{x}\left(s_{K}\right)-\pi v_{x}\right) \Delta y\left(s_{K}\right) K d s_{K}, \tag{6}
\end{align*}
$$

where $C$ is the circumference.
To obtain the skew-quadrupole components, we produced two types of vertical local bump orbit in the undulator section and measured the changes of the horizontal COD for each of them. Fig. 1 shows the two types of local bump orbit used, a flat bump and a tilt bump. The vertical displacement of the orbit in the undulator is expressed by $\Delta y=\Delta y_{0}$ for the flat bump and
$\Delta y=s \Delta y_{0}{ }^{\prime}$ for the tilt bump. Here $\Delta y_{0}$ is the height of the flat bump and $\Delta y_{0}{ }^{\prime}$ is the angle of the tilt bump in the undulator. We assume that the beam focusing caused by the undulator is negligibly small. In this case Eq. (6) becomes

$$
\begin{align*}
& \frac{\Delta x(s)}{\sqrt{\beta_{x}(s)}}=\frac{\Delta y_{0}}{2 \sin \pi v_{x}} \times \\
& \quad\left[\left(K_{0} \sqrt{\beta_{x 0}}\right) \cos \psi_{x}(s)+\left(K_{1} / \sqrt{\beta_{x 0}}\right) \sin \psi_{x}(s)\right] \tag{7}
\end{align*}
$$

for the flat bump and

$$
\begin{align*}
& \frac{\Delta x(s)}{\sqrt{\beta_{x}(s)}}=\frac{\Delta y_{0}{ }^{\prime}}{2 \sin \pi v_{x}} \times \\
& \quad\left[\left(K_{1} \sqrt{\beta_{x 0}}\right) \cos \psi_{x}(s)+\left(K_{2} / \sqrt{\beta_{x 0}}\right) \sin \psi_{x}(s)\right] \tag{8}
\end{align*}
$$

for the tilt bump. Here $\beta_{x}(0)=\beta_{x 0}$ and $\psi_{x}(0)=-\pi \nu_{x}$, and we have assumed that the horizontal Twiss parameter $\alpha_{x}(0)$ is zero. Equations (7) and (8) show that an analysis of the two horizontal CODs will give $K_{0}, K_{1}$ and $K_{2}$.


Fig. 1: Two types of vertical local bump orbit at NewSUBARU. The ring has 18 beam position monitors (BPMs), and LU is located between BPM-12 and BPM13. When the vertical displacements at BPM-12 and BPM-13 are +2 mm , the displacement in LU is $\Delta y_{0}=1.65$ mm . When the vertical displacements at BPM-12 and BPM-13 are +2 mm and -2 mm , respectively, the orbit angle in LU is $\Delta y_{0}{ }^{\prime}=-0.22 \mathrm{mrad}$.

## MEASUREMENTS

## Measurement of the Gap-Dependent SkewQuadrupole Components

First, the COD was corrected using the ring steering magnets so that the rms value was less than 0.01 mm in both the horizontal and the vertical direction. Next, the desired vertical local bump was produced using the vertical steering magnets while the settings of the horizontal steering magnets remained the same, and then the change of the horizontal COD was measured. Third, a similar bump but with the opposite sign was produced, and the change of the horizontal COD was again measured.
The process was performed for two types of vertical local bump and for several values of the undulator gap. The change in the horizontal COD included contributions from skew-quadrupole fields in the area of the local bump other than in LU and from rotation of the vertical steering
magnets. However, these contributions were cancelled out by subtracting the COD change obtained with the undulator gap open. In this way, we obtained the dependence on the gap.

By fitting the change of the horizontal COD with the functions given by Eqs. (7) and (8), we obtained $K_{0} \sqrt{\beta_{x 0}}$, $K_{1} / \sqrt{\beta_{x 0}}, K_{1} \sqrt{\beta_{x 0}}$ and $K_{2} / \sqrt{\beta_{x 0}}$. The values of $K_{0}, K_{1}$ and $K_{2}$ calculated from the results of the fit are shown in Fig. 2. The values of $K_{1}$ obtained from the flat bump and from the tilt bump agreed with each other.


Fig. 2: Gap dependence of the moments of the skewquadrupole field: (a) the 0th moment $K_{0}$, (b) the first moment $K_{1}$ and (c) the second moment $K_{2}$. The values at a gap width of 119 mm (gap open) are defined to be 0 . The solid, broken and dotted lines represent values before correction, after the shimming correction and after the coil correction, respectively. The open and shaded symbols in (b) represent values of $K_{1}$ obtained from flat and tilt bumps, respectively.

## Correction of Skew-Quadrupole Components

The first step was to use a shimming technique to minimize the active correction by the magnets. $K_{1}$, which was the largest component, was corrected by putting shim plates near the entrance and the exit of LU. The correction was successful, as shown in Fig. 2(b). However, the gap dependences of $K_{0}$ and $K_{2}$ (Fig. 2(a) and (c)) were different from that of the field strength of the undulator, and it was almost impossible to cancel them by adding shim plates. Therefore, the gap dependence of $K_{0}$ was
corrected by a pair of skew-quadrupole coils at both ends of LU. This correction of $K_{0}$ made $K_{2}$ smaller but not zero. $K_{0}$ and $K_{2}$ cannot be cancelled at the same time with the present system. For the correction of $K_{2}$, one more skewquadrupole coil is necessary at the center of LU.

## Vertical COD Produced by Horizontal Displacement

Three moments, $K_{0}, K_{1}$ and $K_{2}$ were also obtained by measuring vertical COD produced by the horizontal local bumps. This alternative method confirmed the results. When horizontal local bump was produced in LU, dispersion is zero at the location of LU but dispersion is not zero at the location of horizontal steering magnets. Therefore, it was necessary to make an additional local bump with opposite sign at a symmetric section to cancel the shift of the orbit length. The results of the measurements for the gap of 60 mm and 30 mm are listed in Table 3.

The three moments obtained by two methods were consistent with each other except the result for $K_{2}$ on the gap of 60 mm . It was possible that the orbit drift with time produced this error.

## SUMMARY

The errors in the skew-quadrupole field along the length of an undulator were expressed by use of three moments, $K_{0}, K_{1}$ and $K_{2}$. The gap dependence of $K_{1}$ was corrected by shimming technique. The gap dependence of $K_{0}$ was corrected by a pair of skew-quadrupole coils. The accuracy of the correction was confirmed by a beambased measurement, using the local bump method.

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Table 3: $K_{0}, K_{1}$ and $K_{2}$ obtained the horizontal local bump and the vertical local bump. The number in the brackets shows the error.

|  | $K_{0}$ |  | $K_{1}$ |  | $K_{1}$ |  | $K_{2}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | H Bump | V Bump | H Bump | V Bump | H Bump | V Bump | H Bump | V Bump |
| $\Delta(60-119)$ | -0.81 | -0.89 | -1.81 | -2.77 | -2.57 | -2.84 | -16.04 | -8.16 |
|  | $(22.5 \%)$ | $(22.5 \%)$ | $(48.9 \%)$ | $(21.2 \%)$ | $(25.6 \%)$ | $(26.2 \%)$ | $(30.7 \%)$ | $(23.7 \%)$ |
| $\Delta(30-119)$ | 1.19 | 1.05 | -31.23 | -22.27 | -26.16 | -32.05 | 55.38 | 54.44 |
|  | $(23.2 \%)$ | $(25.8 \%)$ | $(20.5 \%)$ | $(20.1 \%)$ | $(20.5 \%)$ | $(20.5 \%)$ | $(28.3 \%)$ | $(20.8 \%)$ |


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