Construction and Calculation of High Gradient Rare Earth Permanent Magnet

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Abstract

Rare Earth Permanent quadrupole magnets with very high magnetic field gradient up to 250 T/m were constructed. Theory to calculate three dimensional magnetic field of a permanent magnet is developed and presented.

1. Introduction

Recent investigations of a high electric field gradient linac triggered a renewed interest in rare earth permanent quadrupole magnet. Samarium cobalt and Neodymium iron are attractive material among others. It is said, a theoretical limit of a maximum energy product, $(BH)_{max}$, is 64 MGOe for the latter. In addition, it was shown that segmented type multipole magnet invented by K. Halbach[1] could exhibit a strong and uniform multipole field. Segment multipole magnet is a magnet composed of multiple number of anisotropically magnetized sector blocks.

We have developed rare earth cobalt (REC) and rare earth iron (REI) alloys quadrupole magnets at KEK. Our primary objective was to create a high magnetic field gradient magnet to maximize an acceptance of the high electric field linac. And a design goal, which was a world highest permanent quadrupole magnet, was achieved and reported[2].

There was discrepancy of a strength of a field gradient between the measurement and a theory of two dimension. The inner and outer diameter of our model magnet, Mark IV, was 13.0 mm and 62.0 mm and its length is 20 mm. Is the length enough to treat the central field in two dimension or not ? To clarify the issue, we have developed a three dimensional theory for an arbitral shaped magnet for magnetic field computation[3]. This theory is applied to the segment magnet.

2. Construction of a high gradient rare earth permanent quadrupole magnet

Four different types of rare earth permanent magnet were developed as model magnets for the focussing system of the 100 MeV Alvarez linac of the GEMINI synchrotron at KEK. The injection energy, RF frequency, accelerating gradient and drift tube radius are 1 MeV, 400 MHz, 3.5 MV/m and 5 mm, respectively. Two of the REC magnets, Mark I and Mark III, are with iron poles. Mark II is a segment REC magnet without iron (16 segments). Mark IV is a segment REI magnet. The length is all 20 mm except Mark III, where it is 18 mm. The linac has its maximum acceptance when the maximum field gradient in the first drift tube is around 235 T/m in the present design.

We have attained the field gradient and the pole tip field of 254 T/m and 1.65 T with Mark IV. Field gradient, effective length material, and pole tip field B_{pole} are tabulated below.

	g_0 (T/m)	ℓ_G (mm)	$\int g(s)ds$ (T)	Material	ℓ_{G_0} (mm)	B_{pole} (T)
Mark I	128	26.3	3.37	H-22A	20	0.77
II	161	22.4	3.48 ∿ 3.6	H-30	20	1.21
III	76	24.7	1.7 ∿ 1.87	H-30	18	0.46
IV	254	22.4	5.69	Nd-Fe-B(34)	20	1.65

Table 1 Measured data for four model magnets

According to K. Halbach's 2D formula, the magnetic field at (virtual) pole, B_{pole}, for 16 segments quadrupole magnet,

$$B_{pole} \simeq 1.87\, B_{rem} \left(1 - \frac{r_1}{r_2}\right)$$

For Mark IV we insert B_{rem} = 1.187 T, r_1 = 6.5 mm, r_2 = 31.0 mm, then we have B_{pole} = 1.75 T. The disrepancy between 2D theory and the measurement is 6 %.

A reduction in the strength of field gradient could be explained quantitatively by the fringe field effect which is dominant for short magnet. Up to now, a very little information on the fringe field is available in a literature for Rare Earth segment magnet.

Three dimensional analysis might be possible by using the computer code like the one developed at Univ. of Okayama[4], or by JMAG[5]. (Both codes employ Finite Element Method) It turned out in these code, an implimentation of the Input data is very tedious and discouraging for layman. In addition, memory size and CPU time seems enormous for 3D field computation to achieve an accuracy level better than 1 %.

As an alternative approach to 3D computation we have found an Integral Equation Method is extremely powerful both in CPU time and memory size especially for the computation of the permanent magnet. In the next section, an outline of the theory is presented. A detail of the theory and the result of the calculation shall be published elsewhere in near future.

3. Three dimentional calculation of the segment quadrupole permanent magnet

Assume our segment quadrupole magnet is composed of n trapezoidal prisms. The magnetic field B inside a bore is expressed as,

$$\vec{B} = -\frac{\mu_0}{4\pi}\int \frac{(\mathrm{rot}\,\vec{M})\times\vec{r}}{r^3}\,dv \qquad (1)$$

A volume integral should be performed over the sum of n trapezoidal prisms.

To simplify the discussion, we evaluate the volume integral for a particular trapezoidal prism whose vertices are located at special location:

$$A(x_1, -y_1, z_1),\quad B(x_1, y_1, z_1),\quad C(x_1, y_0, z_1),\quad D(x_1, -y_0, z_0)$$

$$A'(-x_1, -y_1, z_1),\quad B'(-x_1, y_1, z_1),\quad C'(-x_1, y_0, z_1),\quad D'(-x_1, -y_1, z_0)$$

We consider a conventional segment permanent quadrupole: an axial component of the Magnetization vector is zero. We take cartesian coordinates such that charged particles travel in x direction and a betatron oscillation is on x-y plane. Other trapezoidal prisms are treated same way as the particular trapezoidal prism only by rotating the z-y axis, and the same as the resultant field components.

Rare Earth materials have an important property that a permeability is close to that of vacuum. This property results in a constant magnetization inside the rare earth material and greatly reduce the difficulty of the integral; a tripple integral in eq. (1) could be reduced to a double integral.

Decomposition of B in eq. (1) into x, y, z components yields:

$$B_x = \frac{\mu_0}{4\pi}(R_{zx} + R_{yx})$$

$$B_y = \frac{\mu_0}{4\pi}(S_{yx} - R_{zy} + S_{yz}) \qquad (2)$$

$$B_z = \frac{\mu_0}{4\pi}(-S_{zy} + R_{yz} - S_{zx})$$

where,

$$R_{zx} = \int \frac{z}{r^3}\frac{\partial M_z}{\partial x}\,dv \qquad R_{yx} = \int \frac{y}{r^3}\frac{\partial M_y}{\partial x}\,dv$$

$$R_{zy} = \int \frac{z}{r^3}\frac{\partial M_z}{\partial y}\,dv \qquad R_{yz} = \int \frac{y}{r^3}\frac{\partial M_y}{\partial z}\,dv \qquad (3)$$

$$S_{yx} = \int \frac{x}{r^3}\frac{\partial M_y}{\partial x}\,dv \qquad S_{yz} = \int \frac{y}{r^3}\frac{\partial M_y}{\partial z}\,dv$$

$$S_{zx} = \int \frac{z}{r^3}\frac{\partial M_z}{\partial x}\,dv \qquad S_{zy} = \int \frac{y}{r^3}\frac{\partial M_z}{\partial y}\,dv$$

with $$r = \{(z-z')^2 + (y-y')^2 + (z-z')^2\}^{\frac{1}{2}}$$

If we assume the magnetized trapezoidal prism keeps its magnetic properties unchanged after implementation into segmented magnet, magnetization inside the magnet is regarded to be constant. In other words, rotM is treated as delta function on the surface of the trapezoidal prism. And its can be shown that the volume integrals given in eqs. (2) and (3) could be reduced to double integrals. We could also show that these double integrals are <u>all</u> integrable and could be expressed with elementary functions, namely, arctangent and logartithmic functions.

Due to a limited space allocated to the article, only the integral S_{yz} among the eight integrals is shown here:

$$
\begin{aligned}
S_{yz} = M_{y0}\{ & g_0(A) - g_0(B) + g_0(C) - g_0(D) - g_0(A') + g_0(B') - g_0(C') + g_0(D') \\
& - bg_1(a;\ -y_1,\ 1+b^2,\ -2(y'+bz')) - ag_1(B;\ y_1,\ 1+a^2,\ -2(y'+az')) \\
& + ag_1(C;\ y_0,\ 1+a^2,\ -2(y'+az')) + bg_1(D;\ -y_0,\ 1+b^2,\ -2(y'+bz')) \\
& + bg_1(A';\ -y_1,\ 1+b^2,\ -2(y'+bz')) + ag_1(b';\ y_1,\ 1+a^2,\ -2(y'+az')) \\
& - ag_1(C';\ y_0, 1+a^2,\ -2(y'+az')) - bg_1(D';\ -y_0,\ 1+b^2,\ -2(y'+bz')) \}
\end{aligned} \tag{4}
$$

with

$$
\begin{aligned}
g_0(B) & \equiv g_0(z_1,\ y_1,\ z_1) \\
& = \frac{z_1}{(z_1-z')} \tan^{-1} \left\{ \frac{(x_1-x')(y_1-y')}{(z_1-z')\{(x_1-x')^2 + (y_1-y')^2 + (z_1-z')^2\}^{1/2}} \right\}
\end{aligned} \tag{5}
$$

and

$$
\begin{aligned}
g_1(B;\ t_0,\alpha,\beta) & = -\frac{1}{\alpha} \log\left|(x_1-x') + (x_1-x')^2 + y'^2 + z'^2 + \alpha(t_0+\beta)t_0\right|^{\frac{1}{2}} \\
& \quad + \frac{\beta}{2}\left(t_0 + \frac{\beta}{2\alpha}\right) I\,(x_1,\ t_0,\alpha,\beta)
\end{aligned} \tag{6}
$$

with

$$
I(x,t_0,\alpha,\beta) = \frac{1}{\sqrt{y'^2 + z'^2 - \frac{\beta^2}{4\alpha}}} \tan^{-1} \frac{(x-x')}{\sqrt{y'^2 + z'^2 - \frac{\beta^2}{4\alpha}}} \left\{ \frac{t_0(\alpha t_0+\beta) + \frac{\beta^2}{4\alpha}}{(x-x')^2 + y'^2 + z'^2 + t_0(\alpha t_0+\beta)} \right\}^{\frac{1}{2}}
$$

$$
\text{for} \quad t_0(\alpha t_0+\beta) > -\frac{\beta^2}{4\alpha}
$$

$$
\frac{1}{\sqrt{y'^2+z'^2 - \frac{\beta^2}{4\alpha}}\sqrt{-\frac{\beta^2}{4\alpha} - t_0(\alpha t_0+\beta)}} \log\left| \frac{x\sqrt{-\frac{\beta^2}{4\alpha} - t_0(\alpha t_0+\beta)} + \sqrt{(y'^2 + z'^2 - \frac{\beta^2}{4\alpha})\{(x-x')^2+y'^2+z'^2+t_0(\alpha t_0+\beta)\}}}{\sqrt{x - \frac{\beta^2}{4\alpha} - t_0(\alpha t_0+\beta)} - \sqrt{(y'^2 + z'^2 - \frac{\beta^2}{4\alpha})\{(x-x')^2+y'^2+z'^2+t_0(\alpha t_0+\beta)\}}} \right|
$$

$$
\text{for} \quad t_0(\alpha t_0+\beta) < -\frac{\beta^2}{4\alpha} \tag{7}
$$

with

$$
a = \frac{y_1 - y_0}{z_1 - z_0} \quad \text{and} \quad b = \frac{-y_1 + y_0}{z_1 - z_0}
$$

It should be emphasized the essential story outlined in this section is applicable to the magnetized iron.

4. Acknowledgement

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References

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