[08-A07]

SIMULATION RESEARCH ON SASE FEL

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The multi-frequency 3-D SASE FEL simulation code is developed using SDE method. And using this code, the evolution of the radiation spectrum are analyzed and the amplification characteristics of SASE radiation trapping by electron beam are clarified.

INTRODUCTION

In the short wavelength (ultra violet ~ x-ray) or ultra-high power FEL, it is not possible to use the Fabry-Perot type resonator because high-reflection mirror does not exist, or the mirror damage limit the power. Therefore, it is necessary to amplify the radiation in the very long wiggler where the radiation is trapped in the electron beam and to reache saturation. This amplification scheme is called by Self Amplified Spontaneous Emission (SASE).

To analyze this amplification process, we developed 3-D SASE simulation code. In this code, we adopt the Source Dependent Expansion (SDE) method for describe the radiation field[1]. In addition, in SASE FEL, the characteristic of radiation amplification is different from normal FEL because the gain width in spectral region change with the growth of radiation[2]. So we extend this code to include multi-frequency radiation modes.

Using this code, we calculate the evolution of radiation amplitude and spectrum of SASE FEL. In section 2, the model equations are described. In section 3, numerical examples are described and discussed.

MODEL EQUATIONS

The electron beam is assumed to propagate along the z-direction which is parallel to the axis of plane wiggler. The vector potential of wiggler magnetic field normalized by mc^2/e is given by

$$\mathbf{A}_w(z,y) = K(z)\cosh\tilde{y}\sin\tilde{z}\mathbf{e}_x, \tag{1}$$

where $\tilde{y} = k_w y$, $\tilde{z} = k_w z$, $k_w = 2\pi/\lambda_w$, λ_w is the wiggler wavelength.

The vector potential of radiation normalized by mc^2/e is given by

$$\mathbf{A}_s(t,r,z) = (1/2)\{\sum_n a_n(r,z)e^{i(kz-\omega_n t)} + c.c\}\mathbf{e}_x. \tag{2}$$

In SDE, the amplitude of radiation fields are expanded by the wave modes as[1]

$$a_n(r,z) = \sum_m a_{mn}(z)g_{mn}(r,z), \tag{3}$$

$$g_{mn}(r,z) = \mathrm{L}_{mn}(\zeta_n)\exp[-\{1-i\alpha_n(z)\}\zeta_n/2], \tag{4}$$

$$\zeta_n = r^2/r_{sn}^2(z), \tag{5}$$

where $r_{sn}(z)$ is the radius of 0-th mode radiation, $\mathrm{L}_m(\zeta_n)$ is the m-th order Laguerre function that argument varies with z as the function of $r_{sn}(z)$. That is the advantage of SDE to describe the radiation radial profile trapped strongly by the electron beam. While $\alpha_n(z)$ is the variables related with the curvature radii of radiation front $R_n(z)$ as

$$R_n(z) = -\omega_n r_{sn}^2(z)/2c\alpha_n(z). \tag{6}$$

Assuming that the radiation radial profile is almost Gaussian, the equation for $r_{sn}(z)$ and $\alpha_n(z)$ are described by following equations[1],

$$\begin{aligned}\tilde{r}_{sn}'(z) &= 2\alpha_n(z)/\tilde{\omega}_n\tilde{r}_{sn}(z) \\ &\quad -\{2C_n(z)/\tilde{\omega}_n\tilde{r}_{sn}(z)\} < \sin\psi_{ni} >,\end{aligned} \tag{7}$$

$$\begin{aligned}\alpha_n'(z) &= 2\{1+\alpha_n^2(z)\}/\tilde{\omega}_n\tilde{r}_{sn}^2(z) \\ &\quad -\{4C_n(z)\tilde{\omega}_n\tilde{r}_{sn}^2(z)\}(< \cos\psi_{ni} > \\ &\quad +\alpha_n(z) < \sin\psi_{ni} >),\end{aligned} \tag{8}$$

where $\tilde{r}_{sn}(z) = k_w r_{sn}(z)$, $\tilde{\omega}_n = \omega_n/k_w c$ and the prime denote the derivative with respect to $\tilde{z}$.

The wave equation for $a_{mn}(z)$ is given as follows

$$a'_{mn}(z) = -A_{mn}(z)a_{mn}(z) - i\{F_{mn}(z) - mB(z)a_{(m-1)n}(z) - (m-1)B_n^*(z)a_{(m+1)n}(z)\}, \quad (9)$$

where

$$A_{mn}(z) = 2[\alpha_n(z) + i(2m+1) - C_n(z)\{<\sin\psi_{ni}> + i(2m+1)<\cos\psi_{ni}>\}]/\tilde{\omega}_n\tilde{r}_{sn}^2(z), \quad (10)$$

$$B_n(z) = 2[\{\alpha_n^2(z) - \alpha_n(z) - C_n(z)<\cos\psi_{ni}>\} + iC_n(z)<\sin\psi_{ni}>]/\tilde{\omega}_n\tilde{r}_{sn}^2(z), \quad (11)$$

$$C_n(z) = \{2\nu K/\gamma|a_{0n}(z)|\}\tilde{r}_{sn}^2(z) \times\{\tilde{r}_{sn}^2(z) - \tilde{r}_b^2(z)\}/\{\tilde{r}_{sn}^2(z) + \tilde{r}_b^2(z)\}, \quad (12)$$

$$F_{mn}(z) = -\{4\nu K a_{0n}(z)/\tilde{\omega}_n|a_{0n}(z)|\} < e^{-i\psi_{ni}}/\gamma > \times\{\tilde{r}_{sn}^2(z) - \tilde{r}_b^2(z)\}^m/\{\tilde{r}_{sn}^2(z) + \tilde{r}_b^2(z)\}^{m+1}, \quad (13)$$

where <...> represents a time average over N periods of the fundamental wave, namely $2\pi N/\omega_0$, $\nu = (\tilde{\omega}_p\tilde{r}_{b0}/2)^2$ is the Budker parameter and $\tilde{r}_{b0} = k_w r_b(0)$, $\tilde{r}_b(z) = k_w r_b(z)$.

The equations of motion for each particles are given by[3]

$$\gamma'_i = (iKf_B/4\beta_{zi}\gamma_i)\sum_m\sum_n \tilde{\omega}_n a_{mn} g_{mn} e^{i\psi_{ni}} + c.c., \quad (14)$$

$$\psi'_{ni} = 1/\beta_{zi} - (\tilde{\omega}_n/2\beta_{zi}\gamma_i^2) \times\{1 + (K^2/2)\cosh^2\tilde{y}_i + \gamma_i^2(\tilde{y}'_i)^2\}, \quad (15)$$

$$\tilde{y}''_i = -(K^2/2\beta_{zi}\gamma_i^2)\tilde{y}_i, \quad (16)$$

where $\tilde{y}_i(z)$ is the radial position of i-th electron and $\psi_{ni} = (\omega_n/c + k_w)z_i - \omega_n t$ is the ponderomotive phases of i-th electron for n-th frequency mode and $f_B = J_0(\xi) - J_1(\xi)$, $\xi = K^2/2(1+K^2)$, $J(\xi)$ is Bessel function. Both the linear and nonlinear evolution of FEL amplification can be investigated by Eqs.(7)-(9) with Eqs.(14)-(16) for the orbits of an ensemble of electron phases $-N\pi < \psi_{ni}(0) < N\pi$ [4]. Where $N = \tau_s c/\lambda_s$, τ_s is the FWHM of the Gaussian radiation pulse and λ_s is the wavelength of radiation .

Table 1 Simulation parameters

Electron beam			
energy	E_{b0}	=	150MeV
current	I_b	=	200A
radius	r_{b0}	=	100μm
Radiation			
wavelength	λ_{s0}	=	65nm
input power	P_0	=	1W
radius	r_{s0}	=	120μm
mode number	m	=	5
Wiggler			
period	λ_w	=	1cm
amplitude	B_w	=	0.535T
number	N_w	=	1000

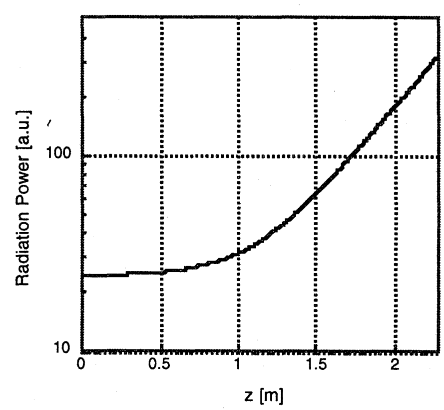

Figure 1 The evolution of total radiation power along the z-axis.

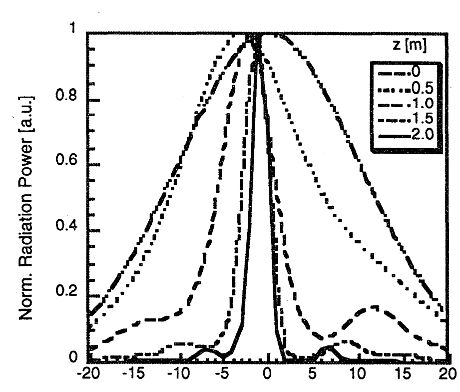

Figure 2 The frequency spectra of normalized radiation power for z=0m, 0.5m, 1.0m, 1.5m, 2.0m.

NUMERICAL EXAMPLES

In this section, we show the numerical examples using the parameters that correspond to the SASE FEL experiment at FELI. The parameters are listed in Table 1.

We analyze the evolution of radiation spectrum by multi-frequency simulation. In this case, we put that the fundamental radiation wavelength is λ_{s0} = 65 nm, frequency resolution is $\Delta\omega/\omega_0$ = 1/1000 and peak power of Gaussian input pulse is 1W.

Fig. 1 shows the evolution of total radiation power from $z = 0$ to 2.3m. In this figure, we can see that the input radiation is building up from spontaneous emission regime to exponential gain regime after $z \sim 1$ m.

Fig. 2 shows the normalized frequency spectra of radiation power. From this figure, we can see that the resonant frequency shifts to lower direction and spectrum narrowing occur in the exponential gain regime, i.e. FWHM of radiation spectra are 1/Nw at z= 0.5 and 1.0m and $\Delta\omega/\omega_0 \sim (\rho/N_w)^{1/2}$ at z=1.5 and 2.0m, where $\rho = 1.53 \times 10^{-3}$ is FEL parameter and $N_w(z)$ is wiggler number.

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