Coherent Beam-beam instability in collision with a large crossing angle

K. Ohmi, N. Kuroo, K. Oide (KEK)

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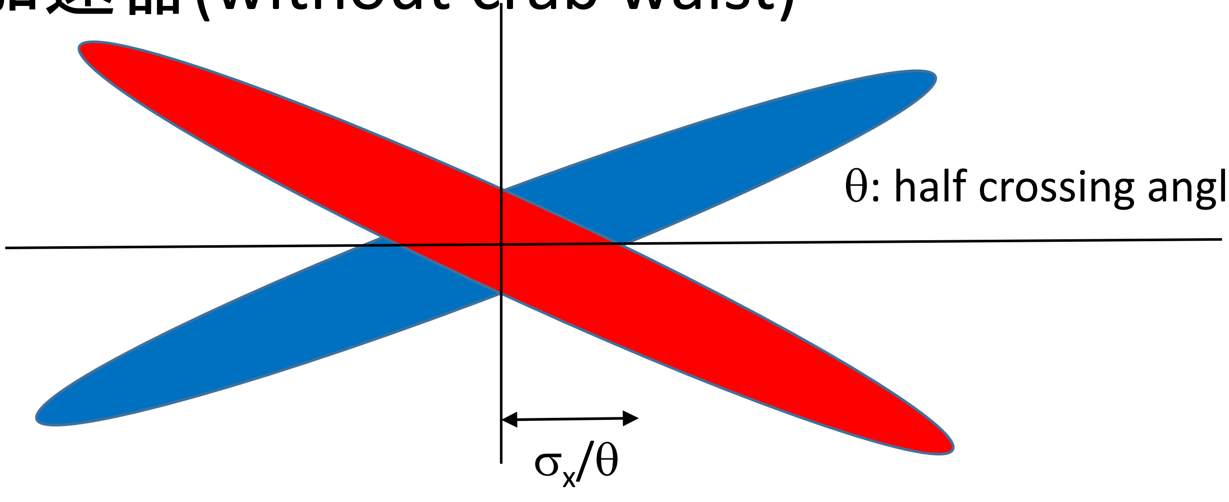
Thanks to D. Shatilov, D. Zhou, F. Zimmermann

Collision with a large crossing angle

- クラブウェストと組み合わせて、最近の円形電子陽電子衝突加速器の設計に広く使われるようになった。(P. Raimondi)
- 特徴づける量、Piwinski角$\sigma_z\theta/\sigma_x$. 衝突領域に対するバンチ長
- 実証実験DAFNA $\sigma_z\theta/\sigma_x=2$, with crab waist
- SuperKEKB $\sigma_z\theta/\sigma_x=20$、この方式での初めての本格的な加速器(without crab waist)



この衝突方式に死角はないか？

- DAFNEの実験はPiwinski角が2と小さい。KEKBは1。SuperKEKB $\sigma_z\theta/\sigma_x$=20
- Beam-beam simulationによる検討がされてきたが、ほとんどはweak-strong simulationだった。片(強)ビーム固定、他(弱)のインコヒーレント効果を調べることができる。
- コヒーレント振動の研究にはstrong-strong simulationが必要。
- バンチを進行方向にスライスするがその数が大きくなる。$N_{sl}=10\sigma_z\theta/\sigma_x$
- SuperKEKBのstrong-strong simulationは衝突当たり N_{sl}^2=200x200=40,000回のポテンシャル計算。
- クラブウェストと組み合わせると、weak-strong simulationではビームビームパラメータ、ξ=0.1は簡単に越えられる。
- これは本当か

Beam-beam limit

- Luminosity

$$L = \frac{N^2 f_{rep}}{4\pi\sigma_x\sigma_y} R\left(\frac{\sigma_z}{\beta_y} \text{ or } \frac{\sigma_x}{\theta_c\beta_y}, \frac{\theta_c\sigma_z}{\sigma_x}\right)$$

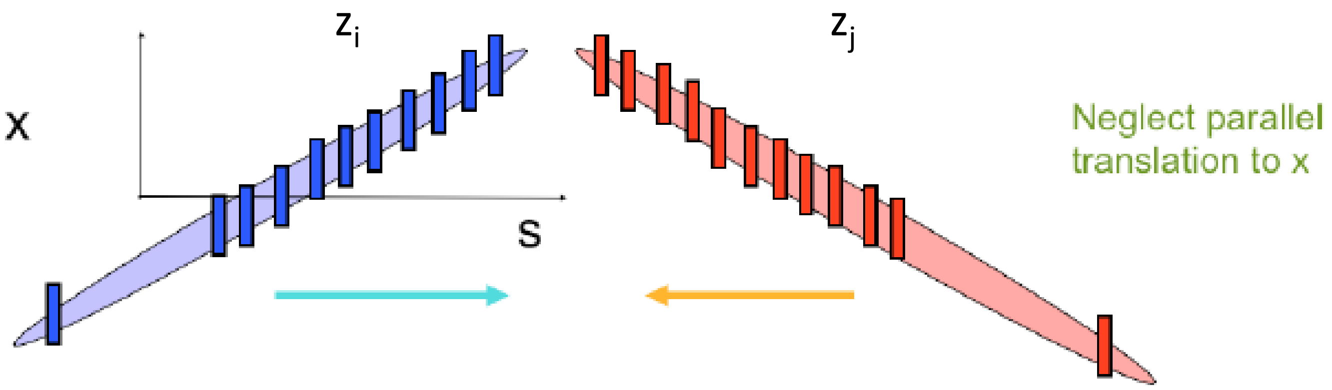
$N=N_+=N_-$: bunch population
f_{rep}: collision freq.
θ_c: half crossing angle

- $\frac{\sigma_z}{\beta_y}$ or $\frac{\sigma_x}{\theta_c\beta_y}$: hourglass (衝突領域と$\beta_y$の比), $\frac{\theta_c\sigma_z}{\sigma_x}$: normalized crossing angle (Piwinski angle)
- Tune shift $$\xi_y = \Delta\nu_y = \frac{Nr_e}{2\pi\gamma}\frac{\beta_y}{\sigma_y(\sigma_x+\sigma_y)} R\left(\frac{\sigma_z}{\beta_y} \text{ or } \frac{\sigma_x}{\theta_c\beta_y}, \frac{\theta_c\sigma_z}{\sigma_x}\right)$$
- Nを増やすとビームサイズ特にyが大きくなりtune shiftは飽和し、ルミノシティはN^2で増えなくなる。この状態をBeam-beam limit.

$$L = \frac{N\gamma f_{rep}}{2r_e\beta_y}\xi_y \qquad \sigma_x \gg \sigma_y$$

- この式はまたアワーグラスが効かなければ、β_yが小さいほどルミノシティが大きくできることを示す。大衝突角
- SuperKEKBはcrab waistを使わない。IR非線形が強すぎて、crab waist sextupoleの非線形がIRでキャンセルできず、DAが小さくなってしまう。

Strong-strong simulation for Large crossing angle



- Two colliding bunches are divided into many slices, $N_{sl} \sim 10 \times \sigma_z \theta / \sigma_x$.
- Sort slices with their positons $z_i + z_j$, collision order.
- Each slice contains >10,000 macro-particles
- Solve potential slice-by-slice collision, or Gaussian approx.

Coherent beam-beam instability

- A coherent beam-beam instability in head-tail mode was found to start beam-beam studies using strong-strong simulation.
- In Strong-strong simulation, both beams which are represented by macro-particles, interacts with each other in their classical EM field.
- The instability is cross-checked by D. Shatilov using quasi-strong-strong simulation.

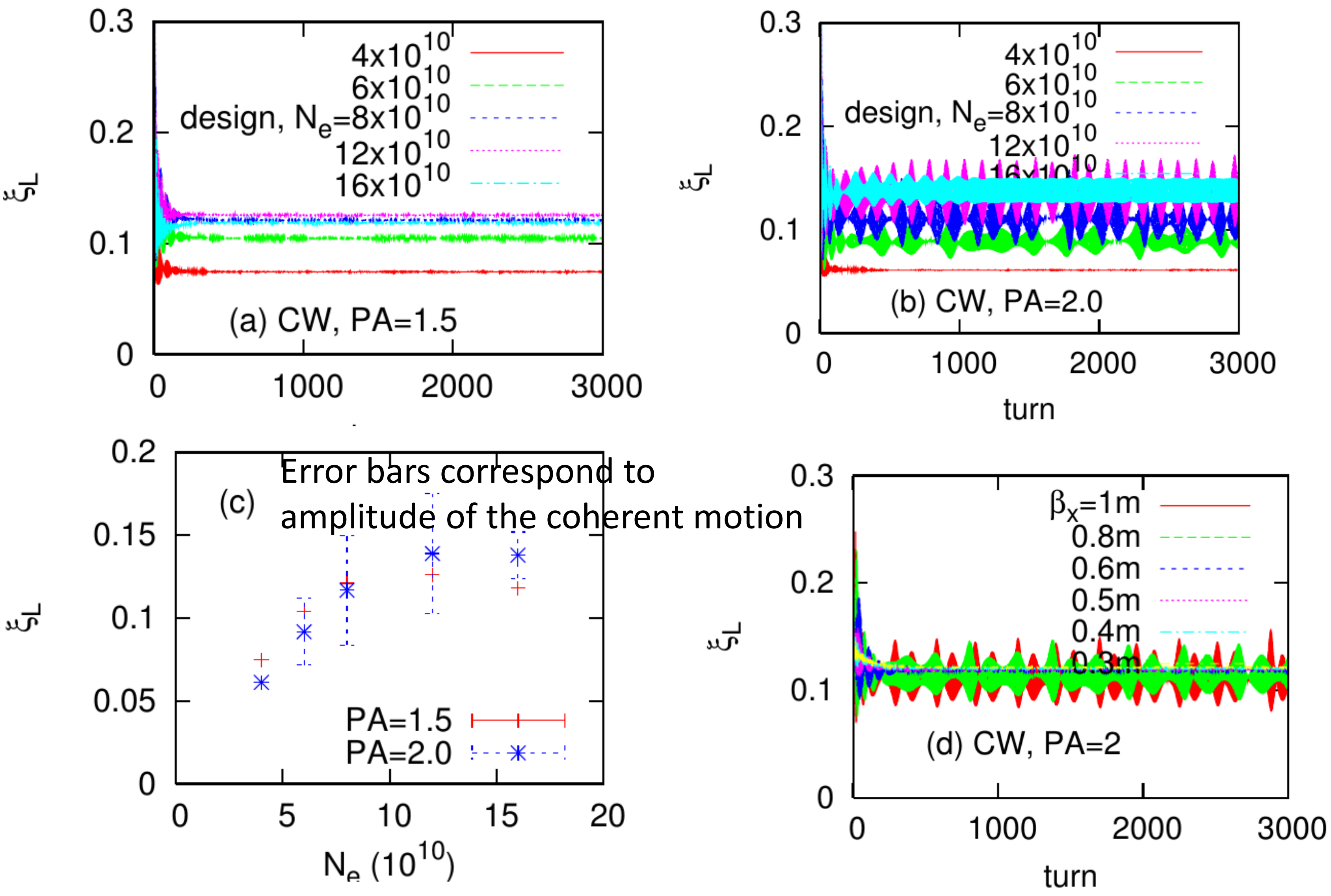
Parameters studied by early 2017

Parameter		SuperKEKB		FCC-ee-Z		H
		design	commissioning	HiLum	base	
Energy	$E_{+/-}$ (GeV)	4/7		45.5	45.5	120
Bunch population	$N_{+/-}(10^{10})$	9/6.5	6.3/5	10	3.3	8
Emittance	$\varepsilon_{x/y}$ (nm/pm)	3.2/8.64	3.2/44	0.2/1	0.09/1	0.61/1.2
Beta at IP	$\beta^*_{x/y}$ (m/mm)	0.032/0.27	0.25/2.2	0.5/1	1/2	1/2
Bunch length	σ_z (mm)	6		6.7	3.8	2.4
Energy spread	σ_δ (%)	0.08		0.22	0.09	0.12
Damping time	τ_x/T_0	4000		3000		150
Synchrotron tune	ν_z	0.025		0.036	0.025	0.056
Luminosity per IP	L (10^{34} cm^{-2}s^{-1})	80	-	207	90	5.1
Beam-beam	$\xi_{x/y}$	0.0028/0.088	-	0.025/0.16	0.05/0.13	0.08/0.14
Piwinski angle	$\sigma_z\theta_c/\sigma_x$	20	8.7	10	6	1.5

Simulation for H

$$\xi_L = \frac{2r_e\beta_y}{N\gamma f_{rep}} L$$

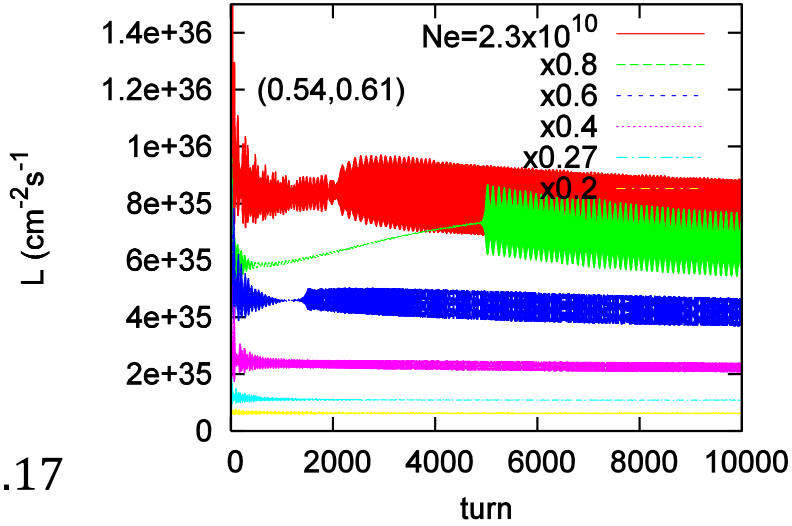
- PA=1.5 in the design. Safe for the instability

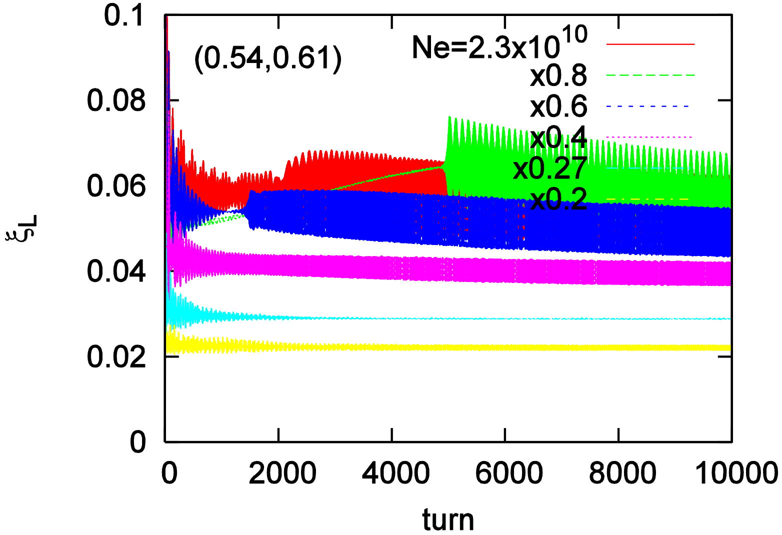


Error bars correspond to amplitude of the coherent motion

Strong-strong simulation for Z factory

$L_{\text{target}} = 2.2 \times 10^{36}\ \text{cm}^{-2}\text{s}^{-1}$

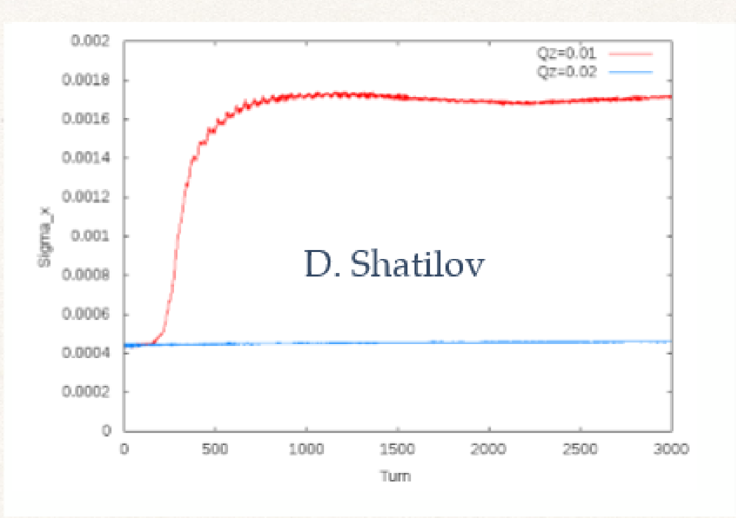


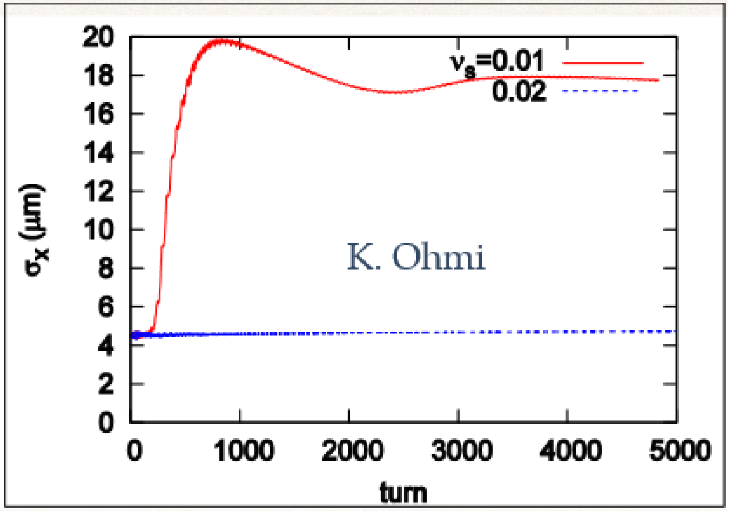


$\xi_{\text{target}} = 0.17$

ξ limit is around 0.06-0.07.

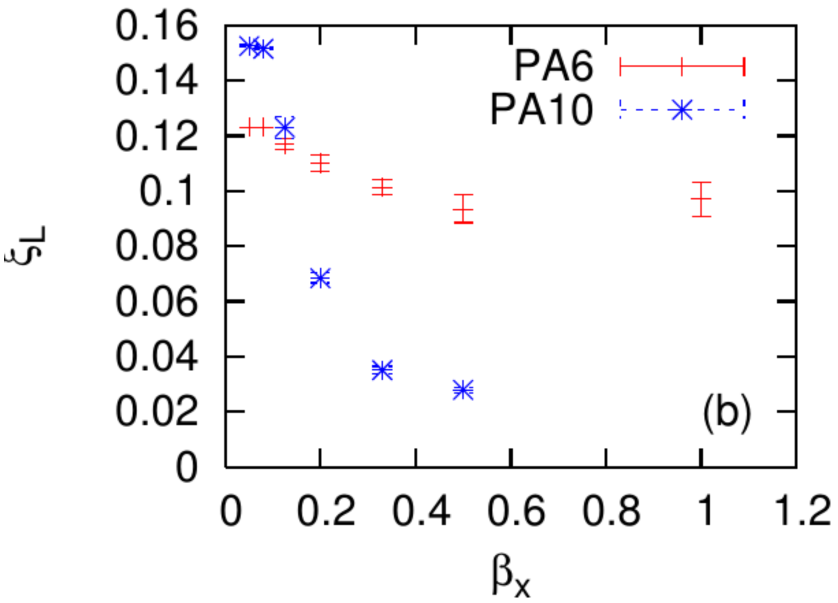
Coherent instability is strong.

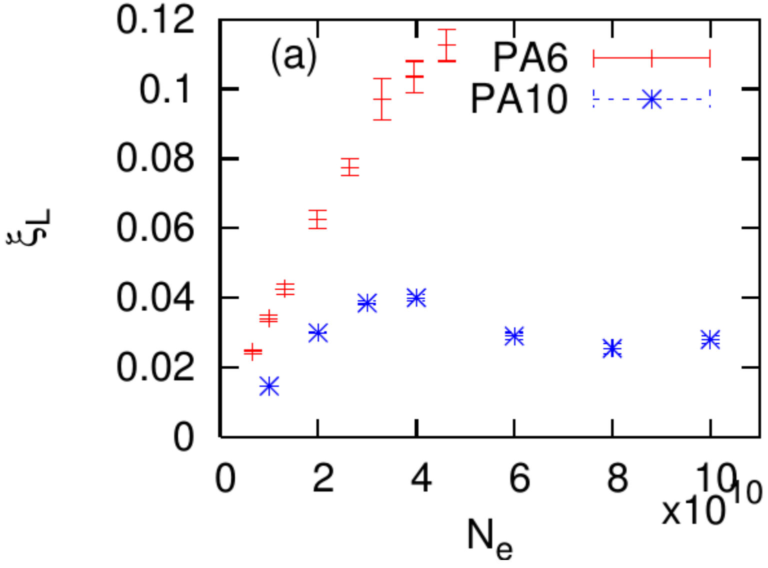


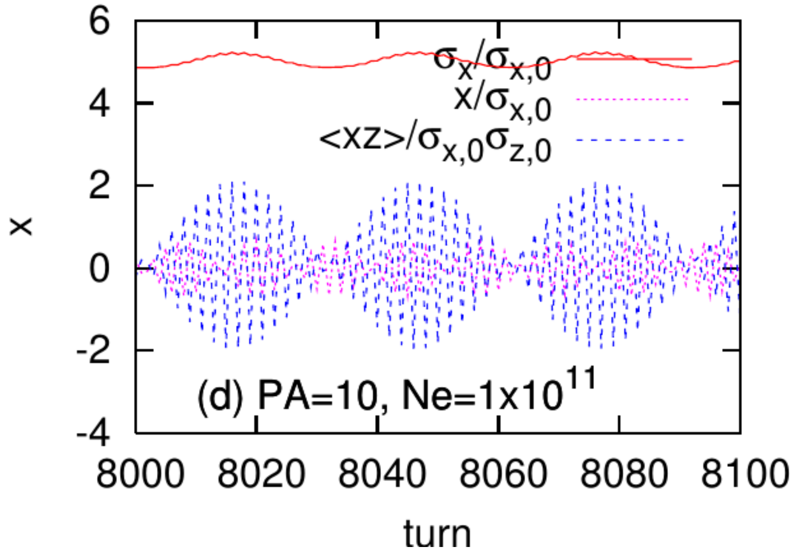


Simulation for Z

- Larger PA is more serious
- σ mode of head-tail motion, in which head-tail phases of two beams are in phase, is seen.

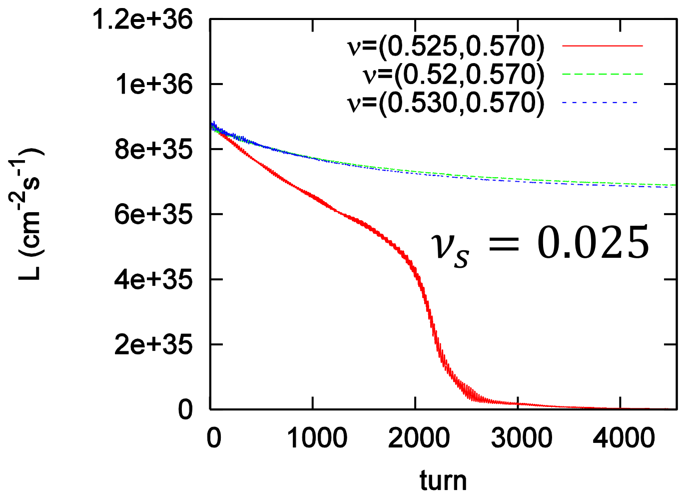


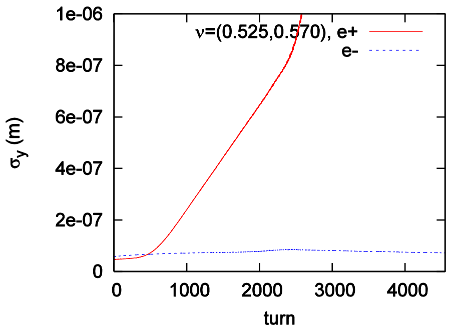


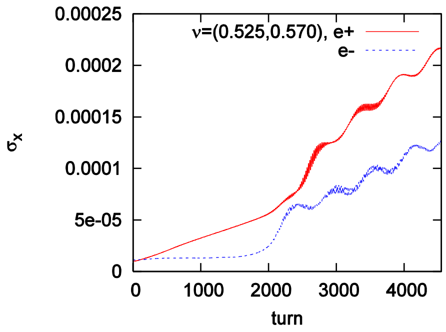


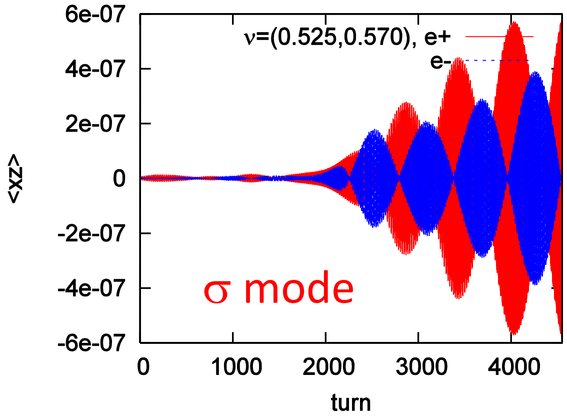
Strong-strong simulation in SuperKEKB

$$\frac{\theta_c \sigma_z}{\sigma_x} = 20 \qquad \xi_{x/y} = 0.0028/0.088$$

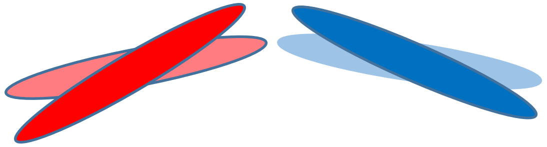








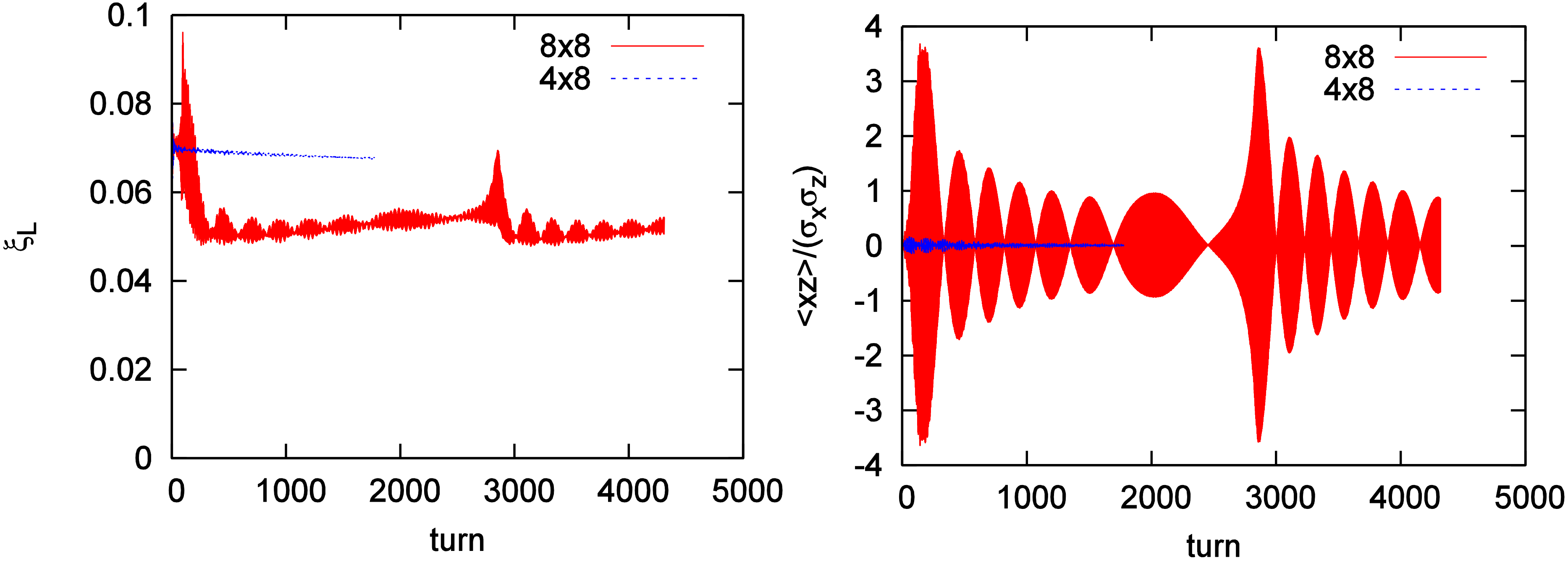
Strong-head-tail instability is seen only in limited tune. The stopband seems narrow.



SuperKEKB Phase 2

β_x=8xβ_{x0}, β_y=8xβ_{y0} and β_x=4xβ_{x0}, β_y=8xβ_{y0}

I_+=1mA, I_-=0.8mA, Crab waist

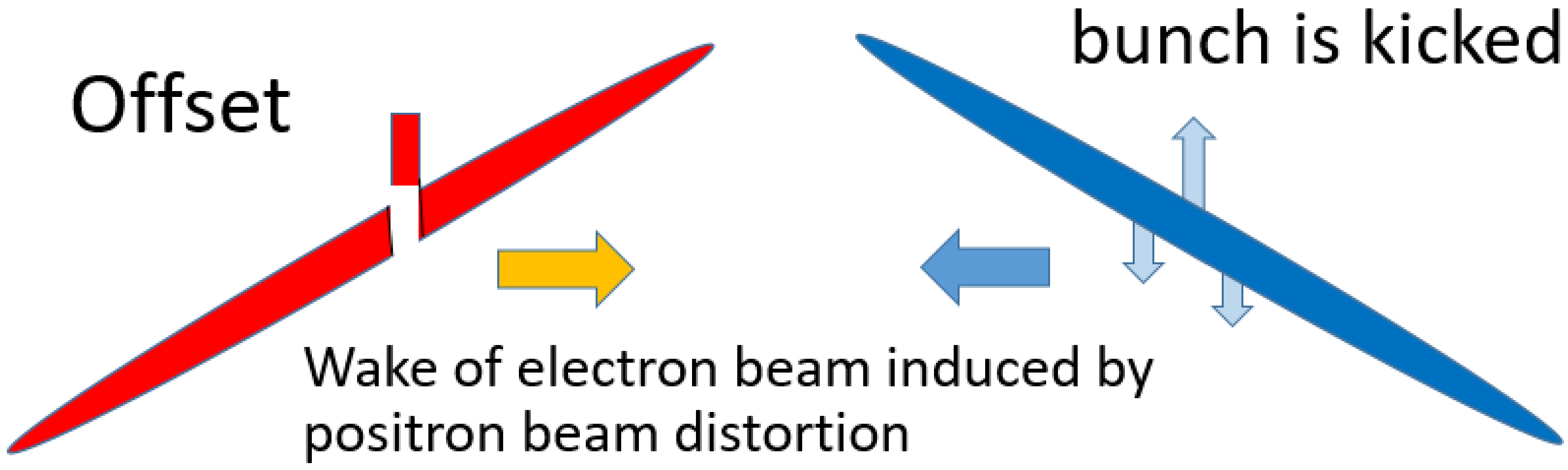


β_{x0}=0.03m, β_{y0}=0.3mm,

This instability can be observed in SuperKEKB Phase II commissioning. Phase II starts from 2018.

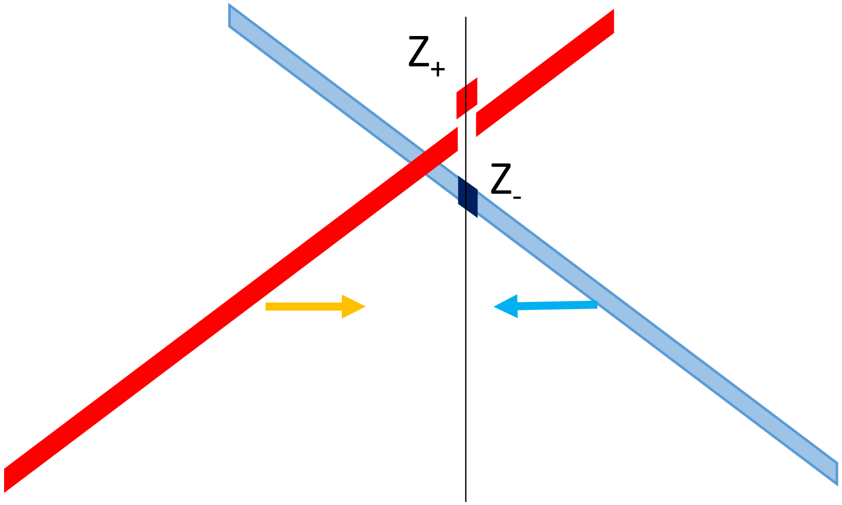
Study of the mechanism of the instability

- Cross-Wake force during collision



Analytic expression of the cross-wake force

- Slice-slice force $\Delta p_x^{(-)} = \frac{N_+ \rho_0(z_+) r_e}{\gamma}(F(x_- - x_+ - \Delta x) - F_x(x_- - x_+))$



$$F(x,y) = F_y + iF_x = \frac{2\sqrt{\pi}}{\Sigma}\left[w\left(\frac{x+iy}{\Sigma}\right)\right.$$

$$- \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\sigma_y x/\sigma_x + i\sigma_x}{\Sigma}\right.$$

$$\Sigma = \sqrt{2(\sigma_x^2 - \sigma_y^2)} \qquad \sigma_{x(y)} = \sqrt{\sigma_{x(y),-}^2 + \sigma_{x(y),+}^2}$$

$$F_x((z_- - z_+)\theta_c - \Delta x, 0) - F_x((z_- - z_+)\theta_c, 0)$$

$$= -\left.\frac{\partial F_x(x,0)}{\partial x}\right|_{x=(z_- - z_+)\theta_c} \Delta x$$

$$x_\pm \approx z_\pm \theta_c$$

θc: half crossing angle

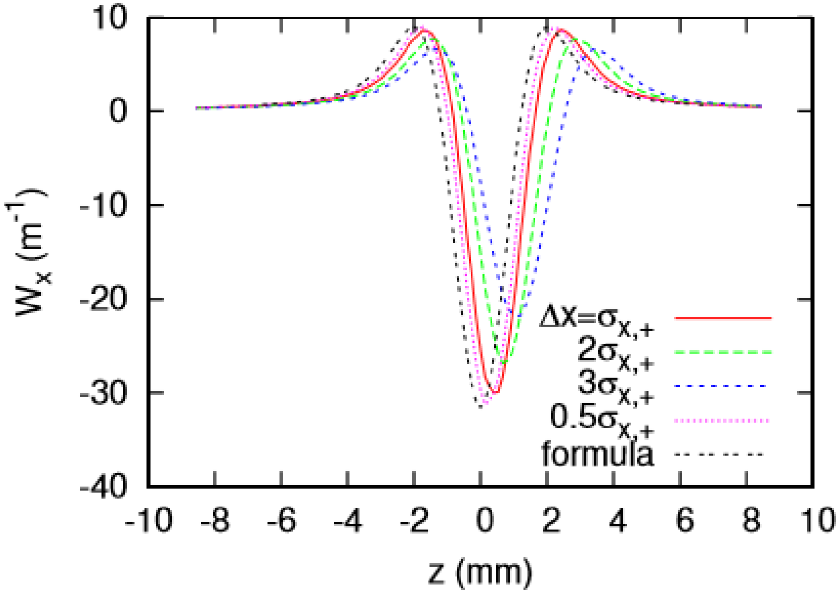
Cross-Wake force due to beam-beam collision

$$\Delta p_{x,\pm}(z_\pm) = -\int_{-l}^{l} W_x(z_\pm - z'_\mp)\rho_x(z'_\mp)dz'_\mp \qquad l \sim 3\sigma_z$$

$$\rho_x(z_+) = \rho_0(z_+)\delta(z'_+ - z_+)\Delta x$$

$$\Delta p_x^{(-)} = -W_x(z_- - z_+)\rho_0(z_+)\Delta x.$$

$$W_x(z_- - z_+) = \frac{N_+ r_e}{\gamma}\left.\frac{\partial F_x(x,0)}{\partial x}\right|_{x=(z_- - z_+)\theta_c}$$

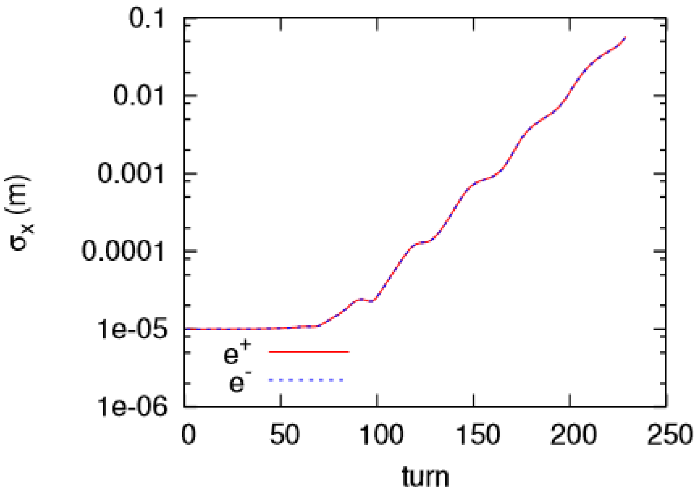


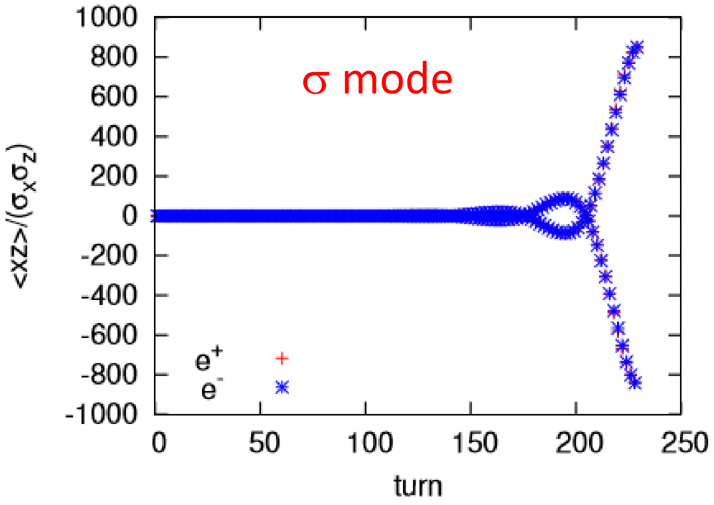
Minimum $W_x(0) = \frac{N_+ r_e}{\gamma}\frac{2}{\sigma_x(\sigma_x + \sigma_y)}$ $\qquad \sigma_{x(y)} = \sqrt{\sigma^2_{x(y),-} + \sigma^2_{x(y),+}}$

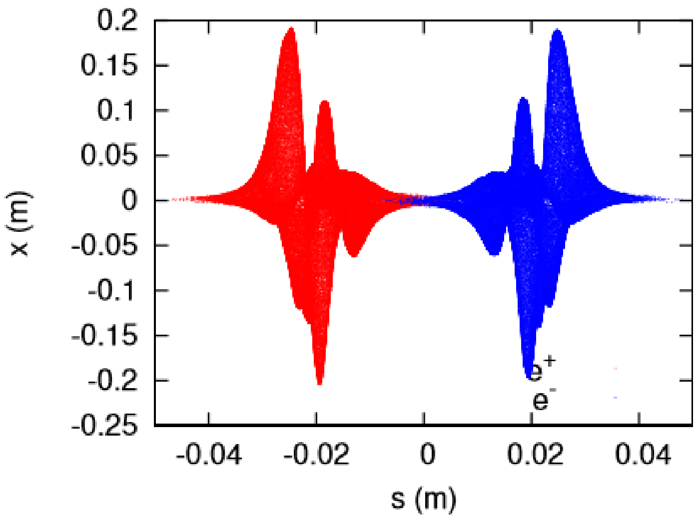
$W(z) = 0$ at $z \approx \pm 1.3\sigma_x/\theta_c$

Maximum W ≈ $0.28|W_x(0)|$ at $z \approx \pm 2.2\sigma_x/\theta_c$

Simulation result using the cross-wake







Correlated wake simulation, not beam-beam simulation.

Both beams have the same distribution. σ mode oscillation.

Instability theory

- Two beams had the same (identical) distribution in the simulation, σ mode head-tail.
- The two beam cross-wake force is treated as a single beam wake force for σ mode.

$l \sim 3\sigma_z$

$$\Delta p_{x,\pm}(z_\pm) = -\int_{-l}^{l} W_x(z_\pm - z'_\pm)\rho_x(z'_\pm)dz'_\pm$$

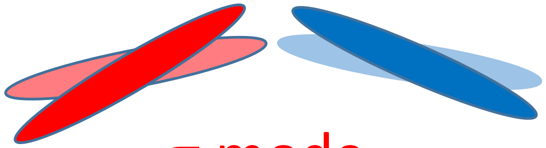
σ mode

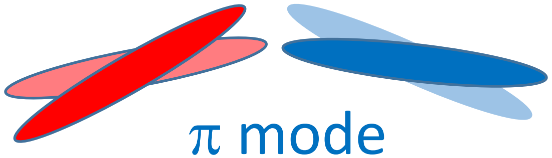
- For π mode, the sign of wake is inversed.

$$\Delta p_{x,\pm}(z_\pm) = \boxed{+}\int_{-l}^{l} W_x(z_\pm - z'_\pm)\rho_x(z'_\pm)dz'_\pm$$

π mode

- Conventional instability theory can be applicable.

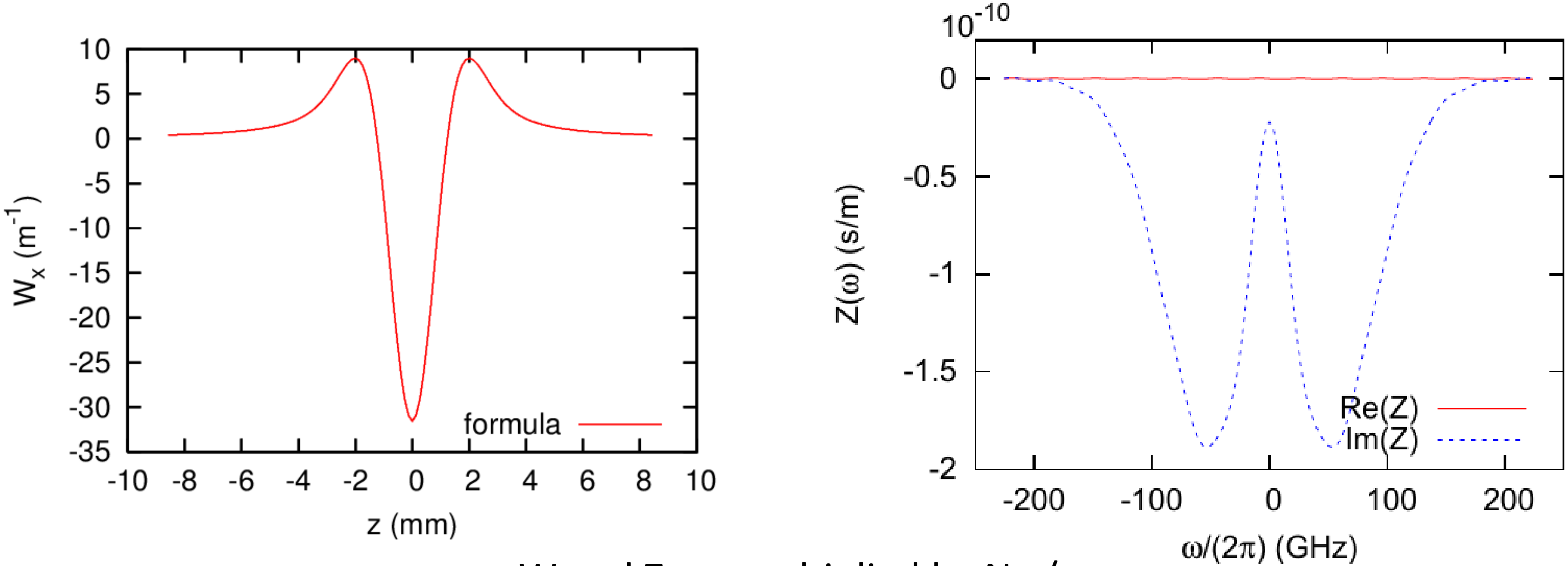

σ mode


π mode

Impedance

$$Z_x(\omega) = i\int_{-\infty}^{\infty} W_x(z)e^{-i\omega z/c}\frac{dz}{c}$$

- The wake is symmetric for z.
- The impedance is pure imaginary and symmetric for ω.

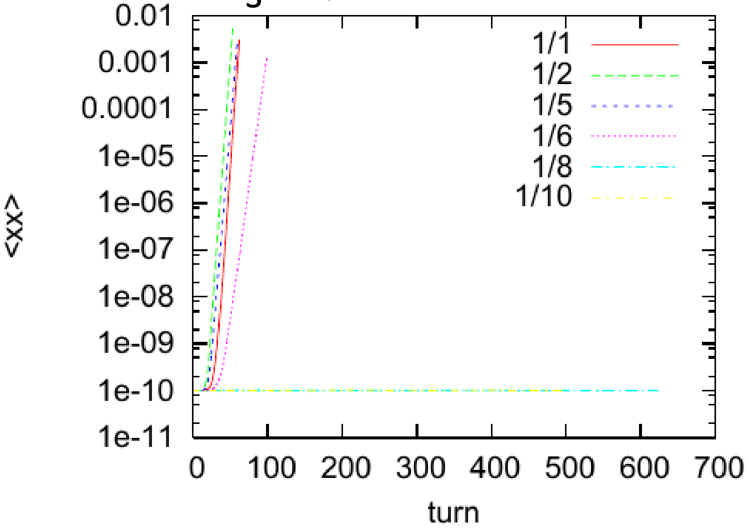


W and Z are multiplied by Nr_e/γ.

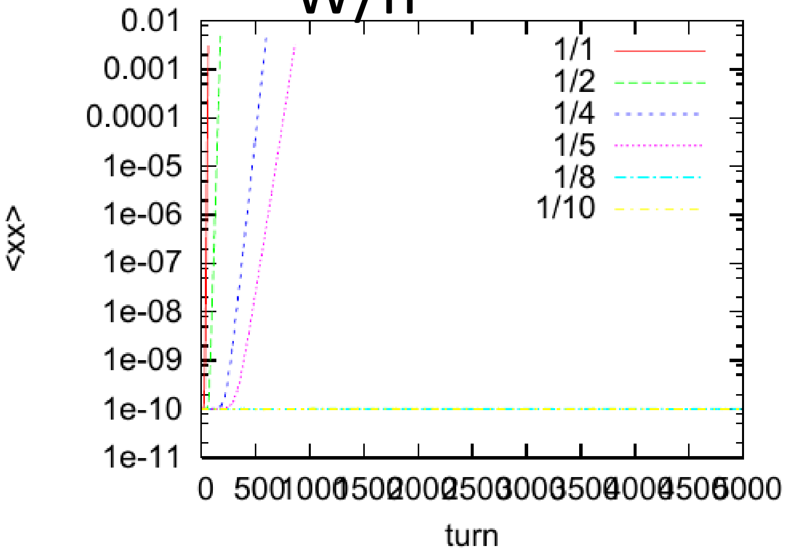
Simulation of single beam instability using the wake force

The growth disappears ν_s/n, W/n, n->∞

ν_s/n, W/n



W/n



- W/8 is stable independent of ν_s. Strength of the localized W is essential.
- The wake with opposite sign is stable. π mode head-tail is stable.

Theory for instability due to a localized wake force

based on the bunch lengthening theory by
K. Oide, Part.Accel. 51, 43 (1995)

- Synchrotron 位相空間 (J,ϕ)上でのダイポールモーメント.

$$x_{ij} = x(J_i, \phi_j) \qquad p_{ij} = p(J_i, \phi_j) \qquad \psi_i = \psi(J_i)$$

$$J_i = i\Delta J \qquad \phi_j = 2\pi\nu_s j \qquad z_{ij} = \sqrt{2\beta_z J_i}\cos\phi_j$$

Synchrotron motion

$j \rightarrow j+1$。 $1/\nu_s = n_s$

- Revolution of the dipole moments

$$\begin{pmatrix} x_{ij} \\ p_{ij} \end{pmatrix} = \sum_{j'=1}^{n_s} M_{ij,ij'} \begin{pmatrix} x_{ij'} \\ p_{ij'} \end{pmatrix} = \sum_{j'=1}^{n_s} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix} \delta_{j-1,j'} \begin{pmatrix} x_{ij'} \\ p_{ij'} \end{pmatrix}$$

- Wake force

$$\begin{pmatrix} x_{ij} \\ p_{ij} \end{pmatrix} = \sum_{i'j'} W_{ij,ij'} \begin{pmatrix} x_{i'j'} \\ p_{i'j'} \end{pmatrix} = \sum_{i'j'=1} \begin{pmatrix} 1 & 0 \\ -W(z_{ij} - z_{i'j'})\psi_{i'} & 1 \end{pmatrix} \begin{pmatrix} x_{i'j'} \\ p_{i'j'} \end{pmatrix}$$

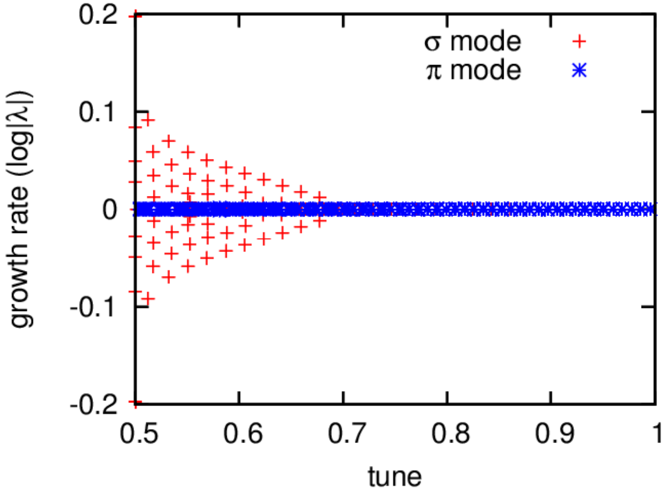
- Solve eigenvalue problem

$$M_W = \begin{pmatrix} \delta_{i,i'}\delta_{j,j'+1} & 0 \\ -\beta_x W(z_{i,j} - z_{i',j'+1})\psi_{i'}\Delta J\Delta\phi & \delta_{i.i'}\delta_{j,j'+1} \end{pmatrix} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix}$$

- Real matrix, $2 \mathrm{x} n_J \mathrm{x} n_s$

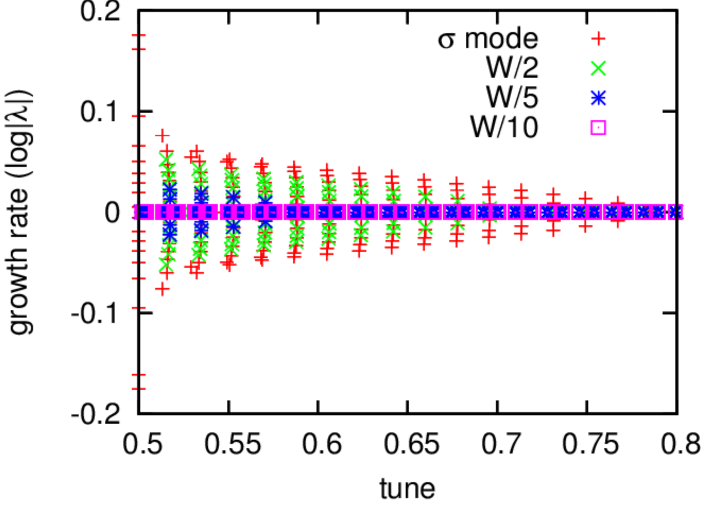
Eigenvalues and eigenvectors

σ/π modes, all π modes are stable



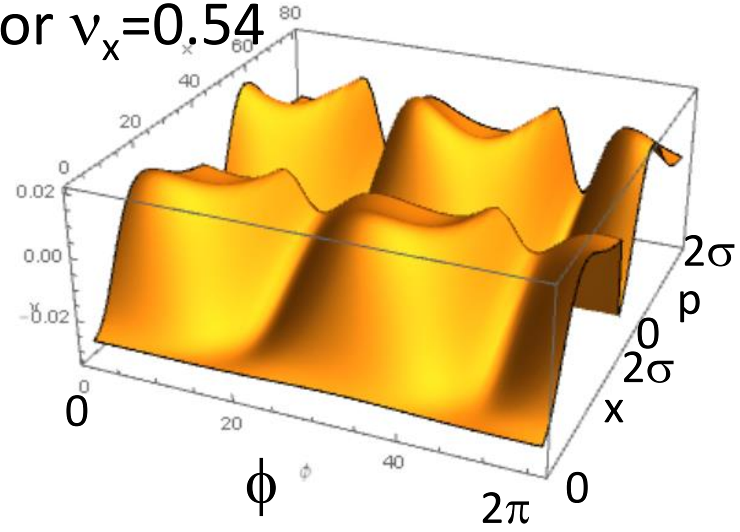
Wake strength scan,

all modes are stable at W/10



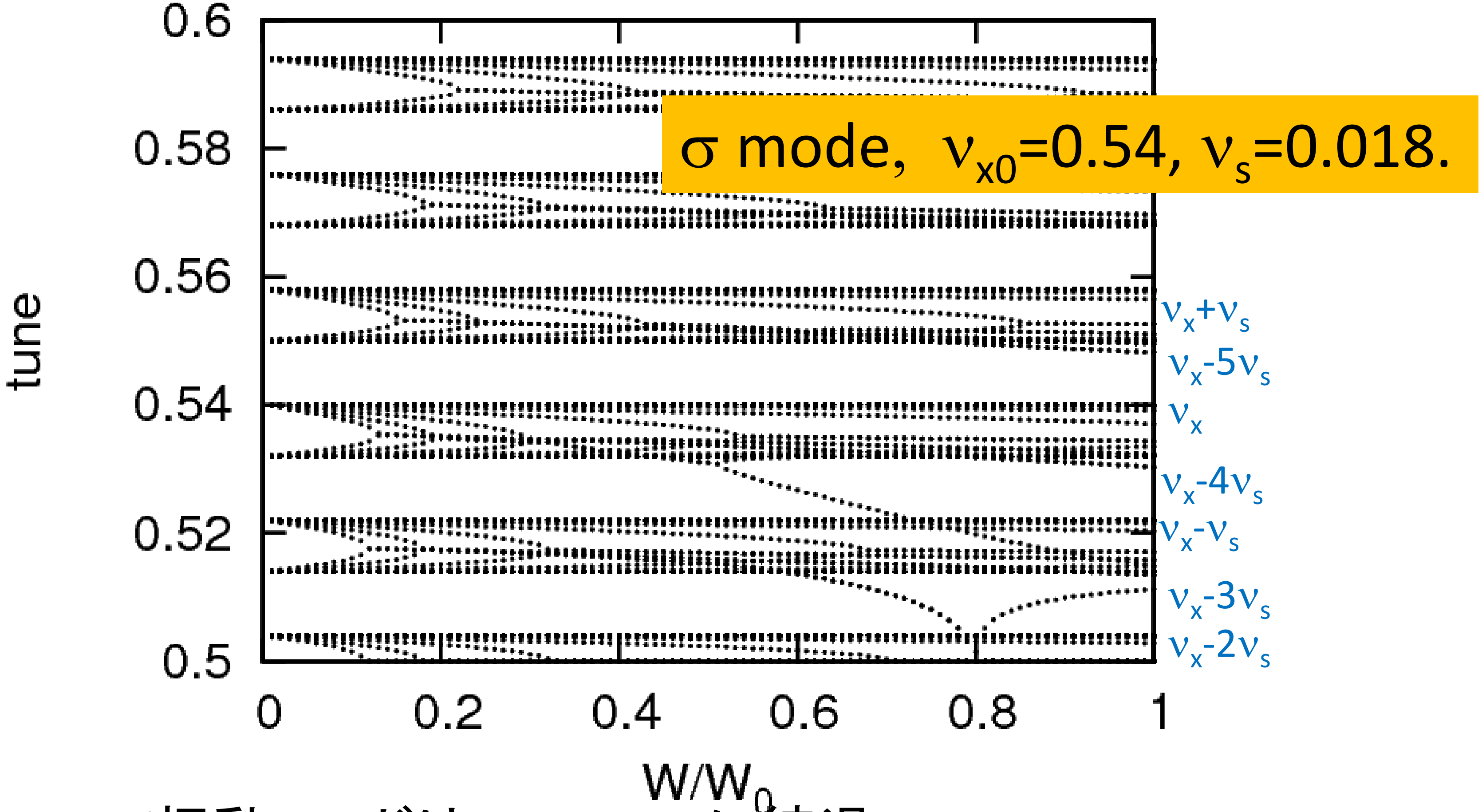
Eigenvector with largest growth

for ν_x=0.54



- σ modes are unstable at $\nu=0.5+m\nu_s$.
- All π modes are stable.
- Threshold exists for strength of the wake.
- Everything is consistent with the single beam simulation
- π modes are unstable in pp collision.

Scan for wake strength

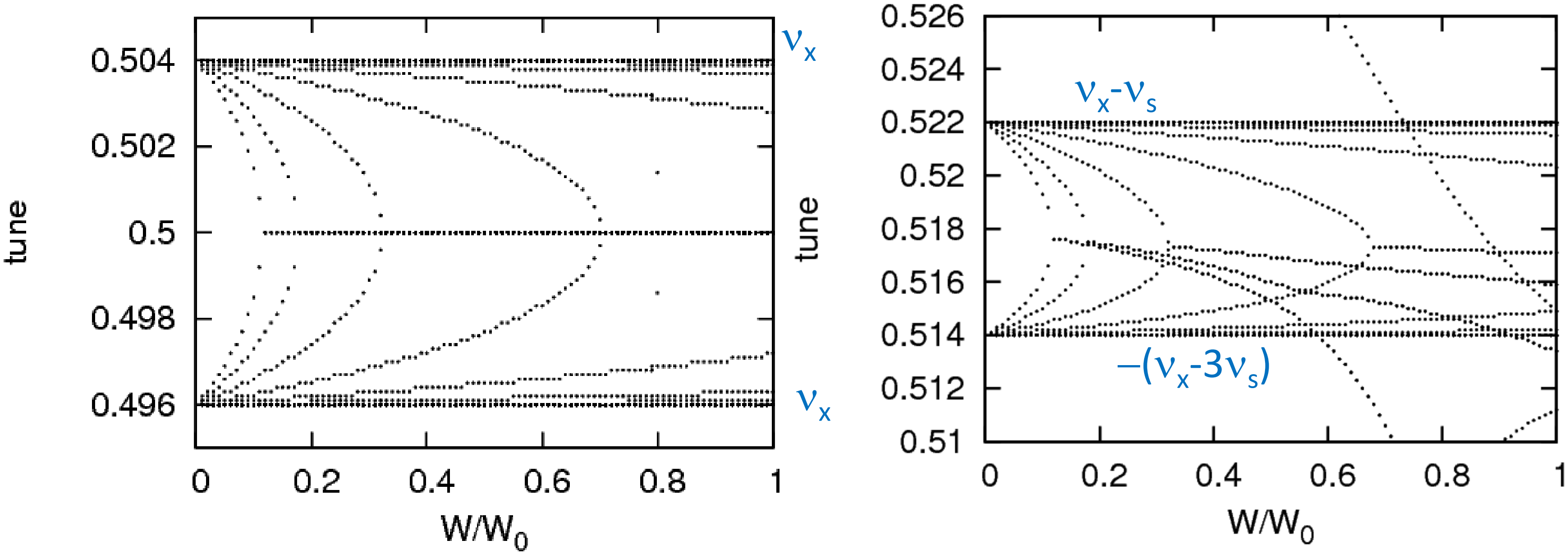


σ mode, ν_{x0}=0.54, ν_s=0.018.

- W=0で振動モードは$\nu_L=\nu_{x0}+L\nu_s$に縮退
- 基本的にモードのチューンシフトは負。$\nu_L<0.5$のモードは$1-\nu_L>0.5$に現れ、正のチューンシフト。
- 偶奇の同じモードが$\nu_L=0.5+m\nu_s$近くにおいてモード結合し不安定になる。

Detailed behavior

σ mode, ν_{x0}=0.54, ν_s=0.018.

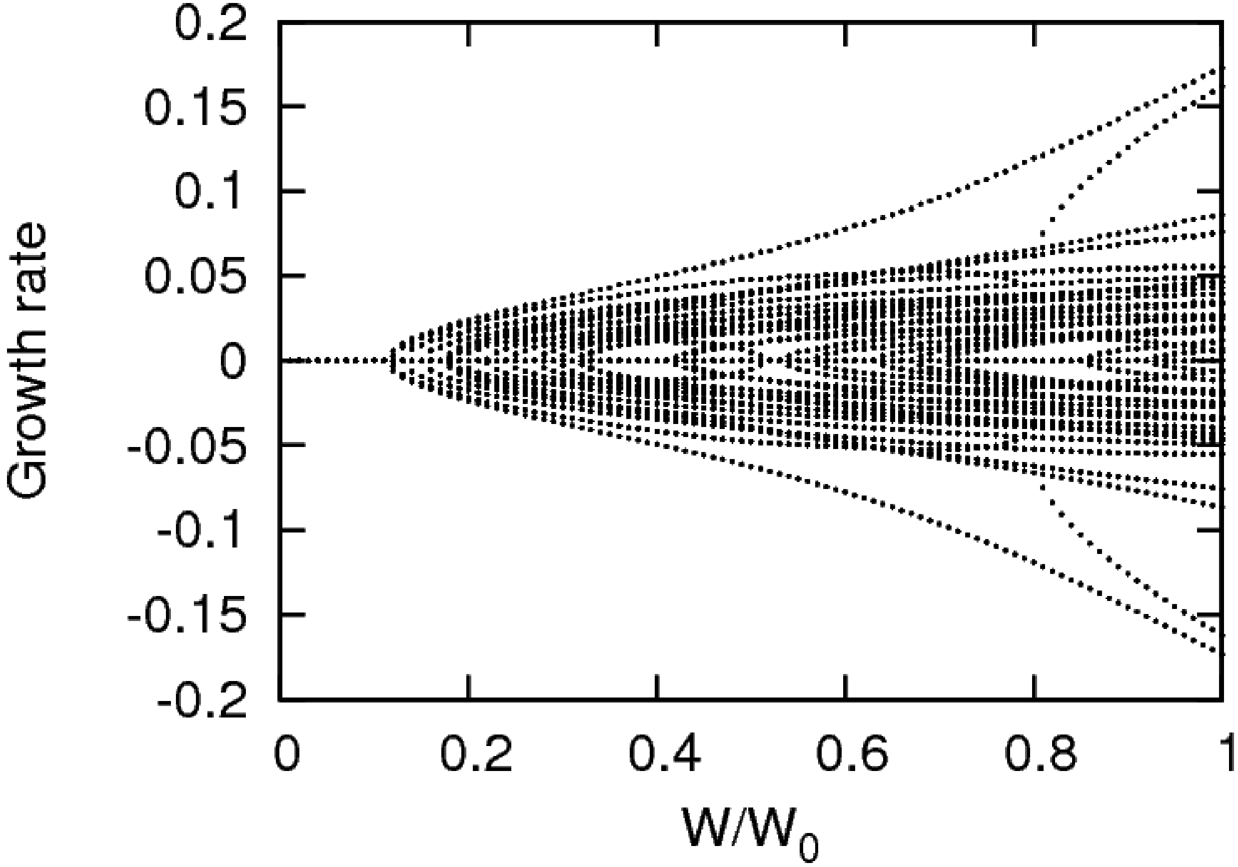


- 偶奇の同じモードが結合。Impedanceがωに対し対称的のため。
- Modes merge at W=0.1W$_0$.
- Radial modeによってチューンシフトが異なる。

Imaginary part- Growth rate

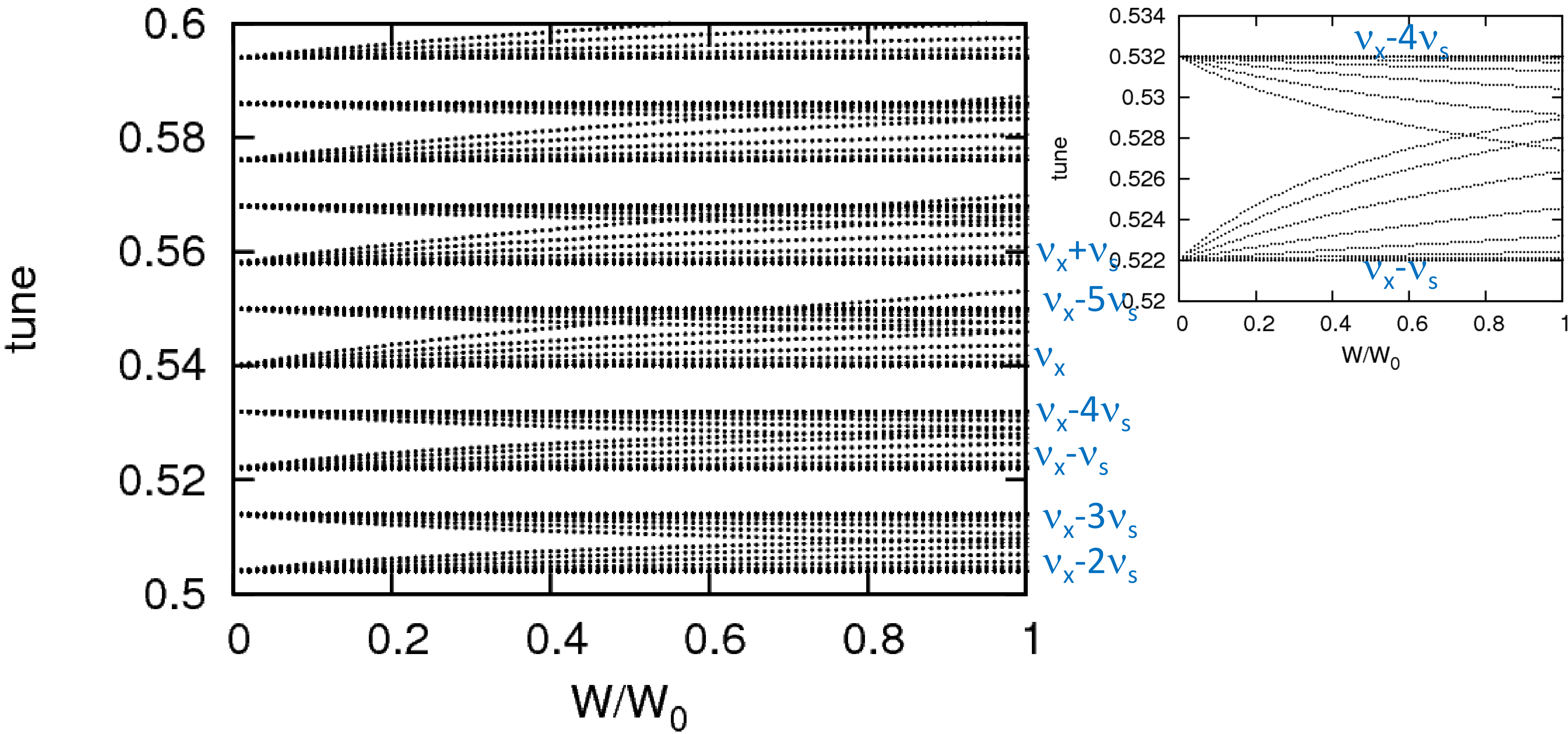
σ mode, ν_{x0}=0.54, ν_s=0.018.

- Threshold W=0.1W_0



π mode, ν_x=0.54, ν_s=0.018

- $W=-aW_0$, $0<a<1$. Wakeの符号を変える. チューンシフトは正になる。
- No coupling between different parity modes.



Summary

- Strong-strong beam-beam simulationは大衝突角での衝突に際し強いコヒーレント head-tail不安定性を示した。
- このビームビーム不安定性は将来の大衝突角、crab waistを基本とした衝突加速器のパラメータを左右する。
- FCCパラメータはこの不安定性を避けるべくパラメータ決定がされている。それでもchallengingなパラメータではこの不安定性が問題となる。
- この不安定性は2ビームの間のcross-wake forceによって説明できる。
- またこのcross-wakeはIPのみで働くことが理論上重要である。
- モード解析を使った理論はこの不安定性をよく説明する。

Thank you for your attention