ON THE APPLICATION OF INVARIANT IMBEDDING TO RADIATION DOSIMETRY

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The effectiveness of radiation therapy depends critically upon the quantitative knowledge of the dosage delivered at the affected sites of abnormal growth. We pursue this aspect of theoretical radiation dosimetry employing the new theory of invariant imbedding to analyse radiation transport in human tissue in realistic situations. The same methods enable us to tackle the inverse problem of learning about internal growth which acts as internal sources of radiation by selective uptake of radioactive isotopes injected into the blood stream. Such different mathematical techniques as the uncollided approximation, the straight ahead approximation, iteration method, Monte Carlo method, and others are developed with a view to provide algorithms for practicing radiologist to control the depth dose distribution with great accuracy. The customary method is to solve the Boltzmann equation governing the transport of radiation in human tissue under various approximations and thus obtain internal intensities. We adopt a different approach using imbedding principles to derive initial value problems to get internal radiation intensities. This procedure has great advantages both computationally and analytically as will be described below.

In this study (See References by Bellman, Ueno, and Vasudevan) it is proposed to use a much more powerful approach than heretofore adopted. This appeals to the principles of invariant imbedding and suitably modified method of radiative transfer theory in which the groups at RAND Corporation and University of Southern California have developed considerable expertise over the past fifteen years. These principles enable one to replace the local formulation of the problem under consideration by a global formulation in terms of the reflection and transmission functions S and T respectively of the medium. The internal intensities can be subsequently expressed in terms of these two functions.

The advantage of this procedure lies in the fact that we are bypassing the task of solving the Boltzmann equation directly, a two-point boundary value problem in the above formulation. The equations for these reflection and transmission functions are written down using imbedding pricdiple. These constitute a set of initial value problems which can be readily handled by a digital computer. Furthremore we are not simply taking over the S- and T-functions of radiative transfer theory since they lead to non-linear equations. Instead we inject a novelty into the radiative transfer methodology by concentrating on those functions corresponding to the beam that has suffered scattering a finite number of times in the medium. These equations are essentially linear and can be readily successively solved. If we are concerned only with a few orders of scatterings, the equations can even be integrated analytically. In the case of gammarays of low energy traversing a low Z scatterer for a small distance computing these functions up to a small order may be adequate. Even when large thicknesses are involved we have methods which can considerably lighten our labor, but still yield sufficient accuracy of the results.

In realistic situation such as the human body with muscle, bone, air cavities, and low density lung tissues, each patient presents a different configurations. It is necessary to develop an engineering approach based on certain key calculations and experiments. Sufficiently realistic schematization can be handled by the imbedding method. In such a case the source and patient geometries are not simple ones as slabs considered in radiative transfer theory. They are more complicated as the sample of a real life situation. Hence we develop techniques to treat with different types of sources and irregular regions that may by irradiated. The approach stated above is to be applied in the near future to the case of a medium in spherical and cylindrical geometries. Furthermore, we plan to use high linear energy transfer radiations like fast neutrons, high energy alpha particles, protons, negative pions etc. which effectively diminish the oxygen effect, because low radiation therapy requires large fields and long exposures with unpleasant side effects.

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