SPACE CHARGE EFFECT IN PROTON LINAC

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The longitudinal oscillation of a particle near the synchronous phase of a linear accelerator is usually expressed by

$$\frac{d}{dz}\left[\gamma_s^3 \beta_s^3 \frac{d}{dz}(\psi-\psi_s)\right] = \frac{2\pi}{\lambda}\frac{eE_0T}{m_0c^2}(1-S)\sin\psi_s\cdot(\psi-\psi_s) \tag{1}$$

Vlasov gives the space charge term S by

$$S = \frac{3\eta\lambda M_z}{4\pi a_x a_y E_0 T \sin\psi_s}\cdot\frac{1}{(\psi-\psi_s)_{max}} \tag{2}$$

where

$$M_u = \frac{a_x a_y a_z}{2}\int_0^\infty \frac{dt}{(a_u^2+t)\sqrt{(a_x^2+t)(a_y^2+t)(a_z^2+t)}} \tag{3}$$

$\eta_0 = \mu_0 / \varepsilon_0 = 120\ \pi\Omega$, a_u $(u = x,y,z)$ is the half axis of the ellipsoid, and I is the current. Eq. 1 describes the averaged motion in a whole cell. The decreased phase stable region gives the upper limit of the current.

If we use the number of rf cycles dl needed for the synchronous particle to traverse the distance dz and assume the change of β_s in a cell to be small, Eq. 1 becomes

$$\frac{d}{dl}\left(\frac{d}{dl}(\psi-\psi_s)\right) = \frac{2\pi e\lambda E_0 T}{m_0c^2\gamma_s^3\beta_s}(1-S)\sin\psi_s\cdot(\psi-\psi_s) \tag{4}$$

Instead of using a single equation for an averaged field in a cell, we can use two separate equations as follows.

$$\frac{d^2\chi}{dl^2} = -n(l)\chi \tag{5}$$

$$n(l) = \begin{cases} n_1 = (1-S')F & \text{in a gap} \\ n_2 = \quad -S'F & \text{in a drift tube} \end{cases} \tag{6}$$

where

$$F = -\frac{2\pi e\lambda E_m}{m_0c^2\gamma_s^3\beta_s}\sin\psi_s \tag{7}$$

$\chi = \psi - \psi_s$ and $S' = ETS/E_m$.

If a linac consists of gaps with rf cycles l_1 and of drift tubes with rf cycles l_2, we can define the transform matrix M_1 and M_2. The trace of the multiplication of two matrices gives the stable region. Fig.1 shows the stability diagram for $l_1 = 0.3$ and 0.25. If $a_x = a_y = a_z = 5\times10^{-3}$ m, $(\psi-\psi_s)\max = \pi/3$, then we get the maximum current of 0.349 A for conventional proton linac.

The APF structure part of PIGMI project has seven superperiod, each of which consists of four gaps and four drift tubes in four rf cycles. The synchronous phase of the first gap is damped by 2° in every superperiod. The stability region for this FODODOFO structure can be solved in the same way (Fig.2). This shows the remarkable increase for upper S' compared to a simple FODO structure. In the first cell with F = 1.341, S' is 0.062 and the upper current is 367 mA. In the last cell F = 1.192, S' is 0.039 and the current limit is 247 mA.

In the design of transverse dynamics, it is common to use the integral form in a cell.

$$\Delta x = d_n x', \tag{8}$$

$$\Delta x' = \frac{-\pi e V_n T \sin\psi_n}{m_0c^2\beta^3\gamma^3\lambda}x, \tag{9}$$

where Vn is the peak voltage in the n-th gap. However we can make a different approach for transverse dynamics just as the logitudinal one.

$$\beta \frac{d}{dz}\left(\gamma\beta \frac{dr}{dz}\right) = \frac{e}{m_0 c^2}\left(F_r - c\beta B_\theta\right) \tag{10}$$

discribes the accurate transverse equation. By neglecing the magnetic term at low beta region, approximating for $\partial E_z/\partial z$ and ρ by the value on the axis, and assuming a constant beta value, Eq. 10 reads to

$$\frac{d^2 r}{d\ell^2} = -n_t(\ell)\, r \tag{11}$$

and

$$n_t(\ell) = (G_t - S_t')\, F_t \quad , \tag{12}$$

where F_t, G_t, and S_t' are difined by

$$F_t = \frac{e}{2 m_0 c^2}\,\frac{\beta_s^2 \lambda^2}{\gamma \beta^2}\left(\overline{\frac{\partial E_z}{\partial z}\cos\psi}\right), \tag{13}$$

$$G_t = \left(\overline{\frac{\partial E_z}{\partial z}\cos\psi}\right) \Big/ \left(\overline{\frac{\partial E_z}{\partial z}\cos\psi}\right), \tag{14}$$

and

$$S_t' = \frac{\rho}{\varepsilon} \Big/ \left(\overline{\frac{\partial E_z}{\partial z}\cos\psi}\right), \tag{15}$$

The suffix 1 difines the values of the first gap. In this expression the field gradient along z is replaced by the average value with appropriate effective length at the exit and the entrance of the drift tubes. Fig.3 shows the phase stable region for a simple APF structure, in which a superperiod consists of two gaps in two rf length.

The above arguments and cited papers should be referenced to LASL reports (LA-8287-MS ∿ LA-8289-MS (1980)). The auther thanks Drs. R. Gluckstern and K. Crandall for the discussion.

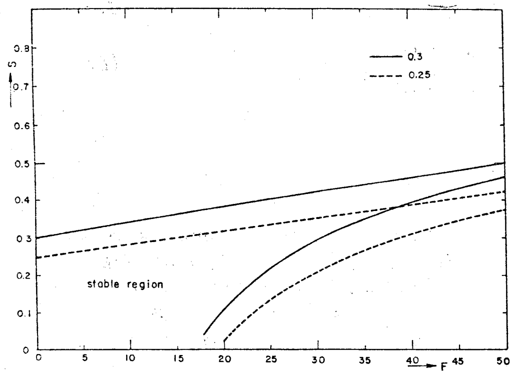

Fig.1 Stability diagram for ℓ_1 = 0.3 or 0.25.

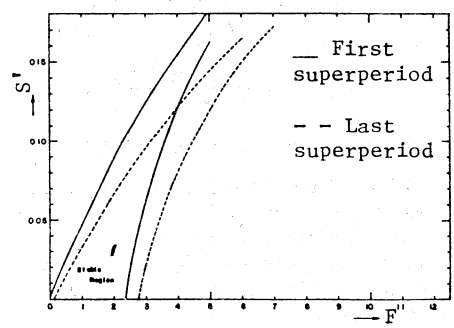

Fig.2 Stable region for FODODOFO structure.

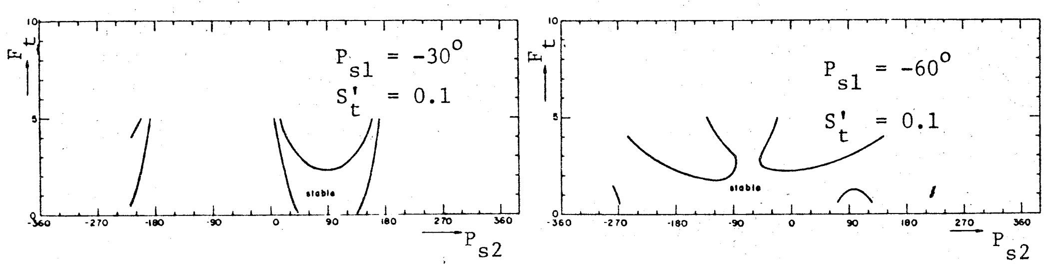

Fig.3 Stable phase region for S_t' = 0.1.