BUNCH DIFFUSION IN A STORAGE RING

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1. Introduction

In the proton bunch storage experiment at the CERN SPS machine, two typical longitudinal phenomena have been observed. One is the turbulent coherent oscillation, mainly of quadrupole mode, in the early period of the storage. This oscillation cause the emittance blow-up. The other is observed after the turbulent oscillation has stabilized itself and the bucket is more fully occupied. One sees gradual decrease of bunch intensity with the lost particles still coasting in the ring. Therefore, it is called a diffusion process. Soon it was recognized that the diffusion is closely related with the noise in the RF system; the reduction of the band width of the radial position feedback loop has brought about the remarkable improvement of beam life. In this report the basic procedures of the computer simulation of the bunch diffusion process are described.

2. Principle of Simulation

In the stationary condition, the equation of phase oscillation is written in $(\varphi, \Delta\beta\gamma)$ coordinate as

$$\frac{d\varphi}{dt} = \frac{h\Omega\eta}{\beta\gamma}(\Delta\beta\gamma) \tag{1}$$

$$\frac{d(\Delta\beta\gamma)}{dt} = \frac{\Omega e V}{2\pi\beta E_0}\sin\varphi \tag{2}$$

We will consider here phase noise. It is represented by small disturbance of φ in (2) as

$$\varphi \longrightarrow \varphi + \Delta(t)$$

where $\Delta(t)$ is a small random time variable. We can introduce white noise by taking normal distribution for $\Delta(t)$.

In order to see how the RF noise gives rise to bunch diffusion, we take many test particles ($\gtrsim 200$) in the RF bucket. Then we trace the phase motion of those particles and observe how the distribution varies from the initial one. However, even if we assume a very localized distribution at first, it is soon elongated due to the nonlinearity of phase oscillation. We can eliminate such apparent diffusion by locating the particles on a matched trajectory. All the particles remain on the trajectory as long as there is no noise and the noise effect is demonstrated by the deviations of particles from the trajectory.

From (1) and (2) we have the following constant of motion in the stationary condition without noise;

$$D^2 = \sin^2\frac{\varphi}{2} + \frac{\pi h\eta E_0}{2\gamma e V}(\Delta\beta\gamma)^2 \tag{3}$$

This means that D^2 is constant on the trajectory and the trajectory is transformed into a circle in the normalized coordinate system (X,Y) of Fig.1, where

$$X = \sin\frac{\varphi}{2}, \quad Y = \sqrt{\frac{\pi h\eta E_0}{2\gamma e V}}(\Delta\beta\gamma) \tag{4}$$

In this coordinate system, D is the distance of the particle from the origin and the bucket boundary is transformed into a unit circle. Therefore, the diffusion by noise can be demonstrated by the growth of the variance of D_i of the particles, $V(D_i)$ which is defined as follows;

$$V(D_i) = \frac{1}{N}\sum_{i=1}^{N}(D_i - \overline{D}_i)^2, \quad \overline{D}_i = \frac{1}{N}\sum_{i=1}^{N} D_i \tag{5}$$

3. Some Results

An example of simulation is shown in Fig.2 for large amplitude oscillation. We see linear growth of $V(D_i)$ which does not appear in small amplitude oscillation. This shows the effect of nonlinearity.

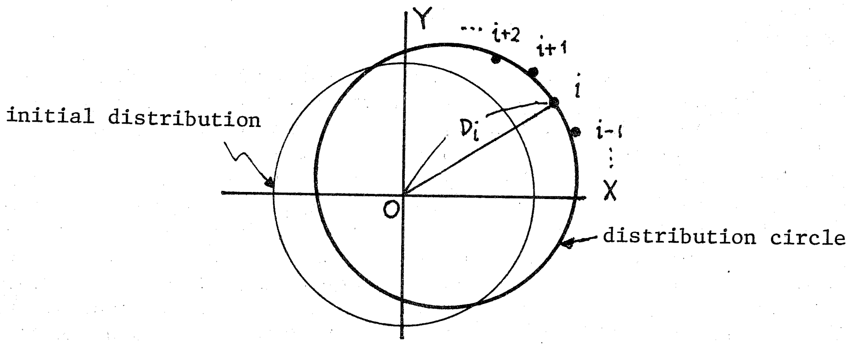

Fig. 1 Distribution of test particles in the normalized coordinate

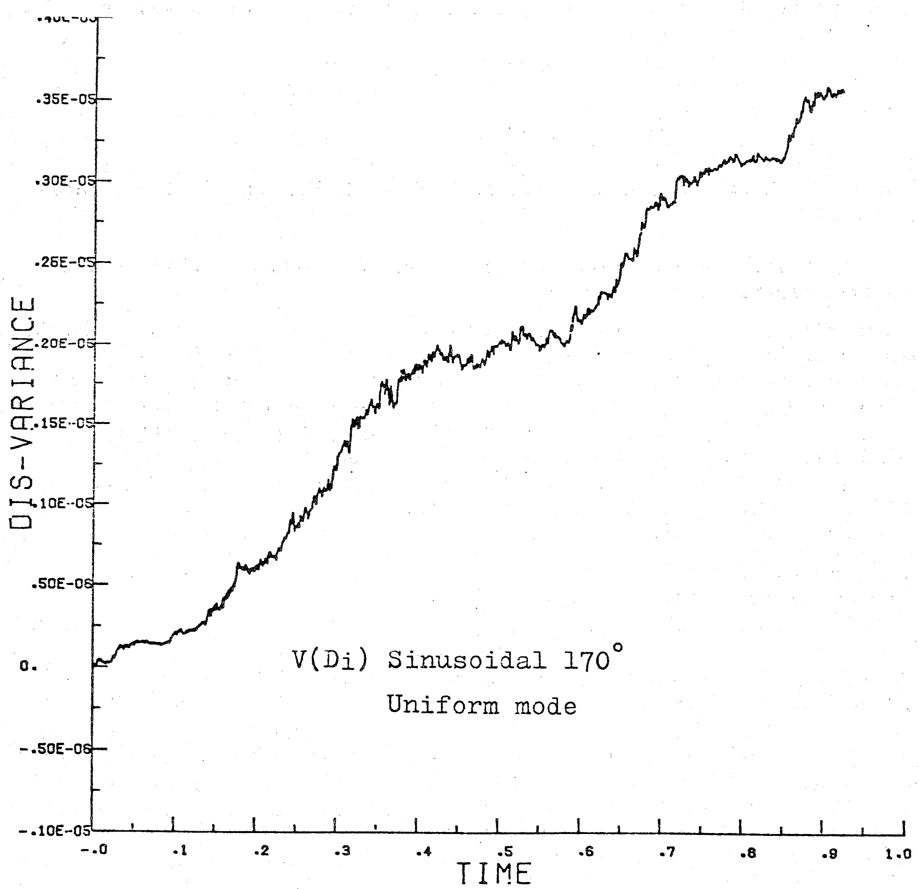

Fig. 2 Growth of particle distribution spread for large amplitude oscillation