EVALUATION OF THE FIELD HOMOGENEITY OF THE MAGNETS BY PHASE SPACE MAPPING

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1. Introduction

Criterion of the field error of the magnets has been one of the major concerns of the accelerator designers. The simplest criteria may be given by the displaced closed orbit by random dipole errors, the betatron tune shifts and tune spread (stop band) due to quadrupole errors, which are given by E. D. Courant and H. S. Snyder in 1958. The effect of the higher harmonics are much more extensively investigated by R. Hagedorn (1957), A. Schoch (1957), P. A. Sturrock (1958) and more recently by G. Guignard (1978). Hagedorn and Guignard adopted a Hamiltonian formalism and obtained the invariants of motion in addition to the total energy. In their work, the stability of the particle orbit is investigated from the property of these invariants. Some criteria such as the sign of the harmonics and the stop band are explicitly given. Sturrock[1] obtained the invariants by a variational method. He plotted these invariant contours in the phase space for typical cases. The resulting curves are of wider generality and help us to understand the effect of the nonlinear error field visually.

On the other hand, due to recent innovations in the technology of the Large Scale Integrated Circuit, a personal computer with a full colour graphic display is now easily available. A very simple but efficient computer code to draw a two-dimensional contour has been developed by the author to perform the phase space mapping. The effect of any higher harmonics on the stability of the particle orbit is now clearly seen in this mapping. In this paper invariants given by Guignard rather than that of Sturrock are chosen simply because the former is derived from the Hamiltonian which is rather familiar to accelerator designers. Due to the limited space of this paper, the range of the mapping is limited to the typical one-dimensional oscillations without coupling.

2. Invariant of the motion

We start from one of the invariant derived by G. Guignard[2]. For one degree of freedom without coupling, this is written as

$$C = \Sigma k_{2(\overline{N}/2)} r^{2(\overline{N}/2)} + k_{2N-1} r^N \cos N\phi \tag{1}$$

where r and ϕ denote the amplitude, phase in the polar coordinate, $\overline{N}/2$ is the smallest integer $\geq N/2$, and N is the order of resonance. The coefficients $k_{2(\overline{N}/2)}$ and k_{2N-1} may be called stabilizing coefficient and excitation coefficient respectively. They are written as

$$k_{2(\overline{N}/2)} = e - \int_0^{2\pi} \beta_x{}^q \frac{R^2}{|B\rho|} \cdot \frac{\partial^{(2q-1)} B_z}{\partial x^{(2q-1)}} \cdot d\theta / (2\pi (2R)^q q!^2) \tag{2}$$

and

$$k_{2N-1} = \frac{-2N}{2\pi(2R)^{N/2}N!} \cdot \int_0^{|N|/2} d\theta\beta_x^{|N|/2} \cdot \exp[i(N\mu-(NQ-p)\theta)]$$

$$\cdot \frac{R^2}{|B\rho|} \cdot \frac{\partial^{(N-1)} B_z}{\partial x^{(N-1)}} \tag{3}$$

where e stands for the distance from the resonance line, p and q are integers, R is a average machine radius, and $B\rho$ is a magnetic rigidity.

Putting the invariant C as a parameter, zones where C lies in a specified range are drawn. We call this the phase space mapping. Figure 1 is a typical example of the phase space mapping of order 3. The parameter in this case is k_4/k_7 where k_4 is given by the fundamental harmonic of the octupole component by eq. (2) and k_7 is given by 3rd harmonic of the sextupole by eq. (3). The stabilization effect of the octupole is clearly seen in this figure. It is possible to obtain the criterion from this figure. Many other examples will be presented in the talk of the conference or in the KEK report.

3. Summary

We have given an example that the criterion of the field homogeneity could be given by evaluating the Guignard's invariant by phase space mapping. Extention to the case where the system has two degrees of freedom may be also possible by surface of section (Poincaré type) mapping. It should be noted, however, that the invariant given is just "approximate" and it is not very clear to what extent the invariant is constant. Indeed, the strong nonlinearity with one degree of freedom or even rather weak nonlinearity coupled with slow synchrotron oscillation could admit the stochastic random behaviour. The stochasticity behaviour is, however, out of scope of this paper and shall be discussed elswhere.

References

1) P. A. Sturrock, "Nonlinear Effects in Alternating Gradient Synchrotrons", Annals of Physics, 3, (1958) 113.
2) G. Guignard, "A General Treatment of Resonances in Accelerators", CERN 78-11, 1978.

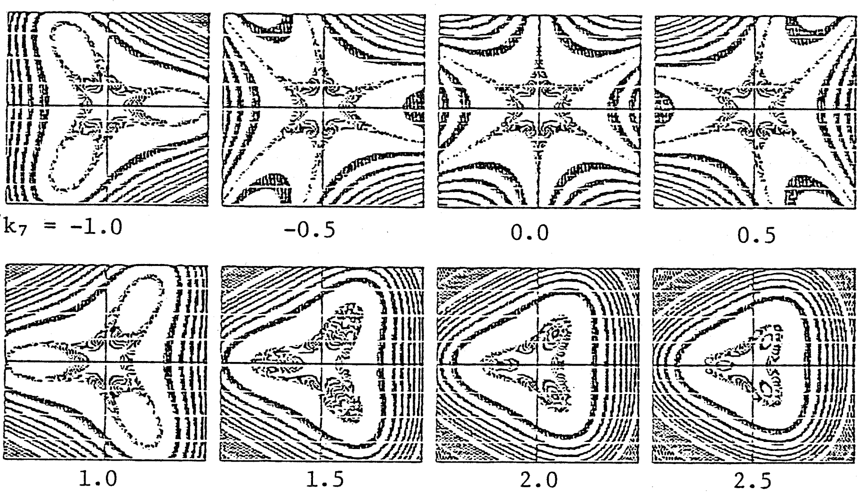

Fig. 1 Stabilization by octupole of the third order resonance