THREE DIMENSIONAL ANALYSIS OF RF ELECTROMAGNETIC FIELD BY FINITE ELEMENT METHOD

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Abstract

We have developed a program which computes the resonant frequencies and fields in an arbitrary shaped three dimensional cavity by finite element method. We take electric field $E = (E_x, E_y, E_z)$ as unknown variables and solve the eigenvalue problem for the Maxwell equations imposing boundary conditions. Divergence-free condition is treated approximately by penalty method. Satisfactory results are obtained for rectangular cavities, cylindrical cavities and those with smoothly deformed parts.

1. Introduction

With the progress of computer, numerical calculation techniques such as finite element method have been developed for solving partial differential equations. In the case of two dimensional problem such as a waveguide with arbitrary cross sectional shape or axisymmetric electromagnetic resonators, many works have been done with satisfactory results[1]. But actually we have many problems that are hard to treat with two dimensional calculations, and true three dimensional calculations are strongly required. In general three dimensional case, only a few papers have been published[2]. It is no more useful to use only one unknown variable such as a scaler potential. It is convenient to take electric field $E = (E_x, E_y, E_z)$ as unknown variables because of the easy treatment of boundary conditions and easy understanding of physical meaning. In this case, we must take care of the condition div $E = 0$.

2. Basic Equations

We deal with the time-harmonic propagation of electromagnetic waves in vacuum of bounded space Ω surrounded by perfect conductor. Eliminating H, Maxwell's equations are

$$\text{rot rot}E - \lambda E = 0 \ , \ \text{div}E = 0 \ (\ in\ \Omega \ ), \qquad (1)$$

where $\lambda = \omega^2 \varepsilon \mu$, ε and μ are dielectric constant and magnetic permeability in vacuum. Boundary conditions are

$$\mathbf{n} \times E = 0 \ (\ on\ \Gamma \ ), \qquad (2)$$

where $\mathbf{n}$ denotes the outward normal on the boundary Γ. This is an eigenvalue problem for three components of electric field E. Restriction of div $E = 0$ is to be imposed in order to exclude undesired solutions. Penalty method is a convenient way to impose such a restriction[2]. We take a weak formulation.

$$\int_\Omega (\text{rot}E, \text{rot}\delta E)dV + s\int_\Omega (\text{div}E, \text{div}\delta E)dV = \lambda \int_\Omega (E, \delta E)dV, \qquad (3)$$

where s is a penalty parameter. Euler's equation is obtained from eqn.(3) as

$$\text{rot rot}E - s\ \text{grad div}E = \lambda\ E. \qquad (4)$$

In this case, div $E = 0$ and rot $E = 0$ are satisfied when $\lambda = 0$. For positive λ, the solutions of eqn.(1) are always those of eqn.(4). Equations (4) may have other solutions than those of eqn.(1). But we are concerned with the solutions of several small eigenvalues. So far as the parameter s is set large in some degree, such a mixture can be avoided. Divergence-free condition can be checked by evaluating the following equations:

$$\text{div} \equiv \frac{s\int_\Omega (\text{div}E)^2 dV}{\int_\Omega (\text{rot}E)^2 dV + s\int_\Omega (\text{div}E)^2 dV} \qquad (5)$$

3. Calculations

In the present study, four noded tetrahederal elements are used and E_x, E_y and E_z are approximated with the first-order test functions. In order to impose boundary conditions, Euler's angles are used, if necessary, to determine the normal direction and to rotate the axes at each surface point. Jennings method is used to solve the generalized eigenvalue problem $Ax = \lambda Bx$ for real symmetric band matrices[3]. First we made calculations for a recutangular cavity resonator. Parameter s is set to 1 and the smallest five eigenvalues are calculated. In this case, unphysical solutions are not found below the fifth mode. Next we made calculations for a cylinder cavity and the example is shown in fig. 1. In this case, unphysical solutions are mixed for certain region of s values. It can be seen that these solutions do not satisfy the condition of div E = 0. Parameter s-dependence of eigenvalues are shown in fig.2. We can see that the s-dependence is small for physical solutions.

Furthermore, we made some calculations in the case of deforming these rectangular and cylinder cavities slightly and smoothly. Obtained results are consistent with the perturbation theory as far as the deformation is small. When the deformation is large, however, the condition div E = 0 does not become fully satisfied because of the roughness of mesh size and the slow convergence of solution. In the present calculations, number of the nodal points is limited to less than 1000 because of the memory size of the computer, which means about ten nodes in each direction. For very complicated boundary shape, errors become large because mesh size is coarse and therefore the condition div E = 0 is not fully satisfied. But this difficulty will be overcome with increasing available memory size.

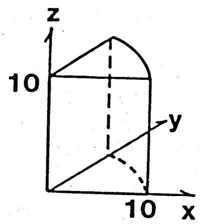

Fig.1-a

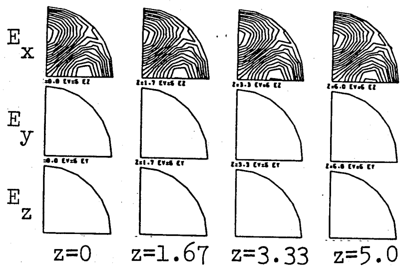

Fig.1-b

Fig. 1 Calculations for 1/4 cylinder of 10 in height and 10 in radial length. At two planes of x = 0 and y = 0, mirror symmetry boundary conditions are imposed. s = 1. Results are shown by contour plots of E_x, E_y and E_z at z = constant planes.
(a) Schematic view of 1/4 cylindrical cavity.
(b) Contour plot for 5th mode (TM_{210} mode).

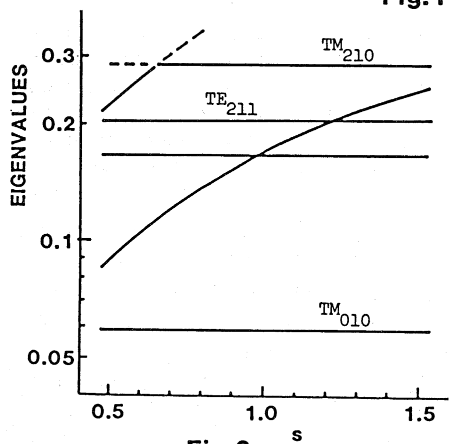

Fig.2

Fig. 2 Penalty parameter dependence of eigenvalues for 1/4 cylindrical cavity. Varying s-parameter from o.5 to 1.5, we calculate five modes for each s. Among these solutions, physical ly allowed ones are three modes (TM_{010}, TE_{211} and TM_{210}). Physically allowed modes have little s-dependence. In order to distinguish physical ones, not only s-dependence but also the condition div E = 0 must be investigated.

References

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