BEAM DYNAMICS EXPERIMENTS USING TRANSVERSE FEEDBACK SYSTEM

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ABSTRACT

The transverse feedback system with a positive feedback method can induce coherent betatron oscillation. The damping rate of betatron oscillation was measured using the method as a function of beam current and chromaticity. The growth rate of the head-tail instability is estimated from the damping rate, since the growth rate is equal to the damping rate at the threshold. The transverse wake function is obtained from the growth rate as a function of bunch length.

INTRODUCTION

The transverse feedback system[1] enhances damping rate of coherent betatron oscillation in addition to that of beam itself. The damping rate is controlled by feedback gain of the system as shown in Fig. 7 of Ref. 1. When the feedback gain is relatively low, however, the damping rate depends on beam current and chromaticity.

If the system is switched to a positive feedback loop and set in suitable gain, stable betatron oscillation is induced without an external force. Exponential decay of the oscillation is observed just after the self-oscillation due to the positive feedback is switched off. The damping rate can be measured from the observation.

When a chromaticity parameter is negative in the AR, the head-tail instability occurs. The transverse feedback system cures the head-tail instability of the lowest mode m = 0. Transverse displacement signals from the instability are detected through pick-up electrodes, amplified and fed back on deflection electrodes with a phase shift as to counteract the betatron motion. The instability occurs, when the growth rate exceeds the damping rate. We can estimate the growth rate of the instability from measuring the damping rate. Transverse wake field is obtained from the growth rate, if the other parameters contributing to the growth rate are known.

BASIC EQUATIONS

The growth rate of the head-tail instability for m = 0 is given[2] as follows:

$$\frac{1}{\tau} = 2\sqrt{\frac{2}{\pi}} \cdot \frac{e \cdot w_0}{m_0 \gamma \cdot \omega_\beta} \cdot \frac{\xi \cdot \sigma}{\alpha} \cdot I \qquad (1)$$

e: electric charge
m_0: rest mass of electron
γ: relativistic factor
ω_β: angular betatron frequency
α: momentum compaction factor
ξ: chromaticity
σ: bunch length r.m.s. value
w_0: transverse wake function in unit azimuthal length
I: beam current of a bunch

The transverse wake function w_0 makes transverse kick propotional to a displacement. The relation to the coupling impedance is expressed as

$$w_0(\sigma) = -\frac{1}{\pi}\int_0^\infty I_m(Z(\omega))e^{-\omega^2\sigma^2}\,d\omega \qquad (2)$$

where $I_m(Z(\omega))$ is an imaginary part of the transverse impedance. The wake function is a function of bunch length.

EXCITATION OF BETATRON OSCILLATION

It is necessary to excite stable coherent betatron oscillation for the experiments of transverse beam dynamics. Positive feedback of the system was testd for this purpose. As we increased the feedback gain, the system began to oscillate with the fractional betatron frequency which was easily read by a frequency counter. The amplitude of the self-oscillation has been controlled by the gain and phase of the feedback loop. The phase for maximum amplitude was an antipole to that for maximum damping rate. The threshold of the oscillation depended on beam current. The self-oscillation has proved to have fluctuation of ± 1 kHz about the betatron frequency, which is not the tune-spread. The origin of the fluctuation is not clear. The amplitude seen in Fig. 1 corresponds to a peak-to-peak value of 0.5 mm. An amplitude modulation has been observed with frequency of several 10 Hz which may be related to synchrotron motion. The modulation appears when the amplitude is relatively large.

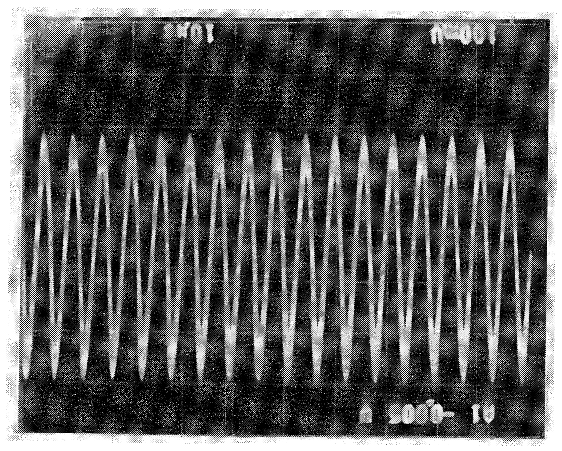

Fig. 1 The self-oscillation. The frequency shows the fractional betatron frequency.

A nonlinear damping effect which depends on amplitude of oscillation is introduced in order to explain the bahavior of the self-oscillation. Assuming that the motion obeys the van der Pol's equation shown in Eq. (A4) in Appendix, the amplitude of the oscillation in stationary state is

$$x_0 = 2\sqrt{\frac{\alpha_{fo} - \alpha_{effo}}{k}} \qquad (3)$$

The derivation of Eq. (3) is seen in Appendix, where α_{fo} and α_{effo} are antidamping rate due to the feedback system and the damping due to beam itself for small amplitude, respectively, and k is a constant describing amplitude-dependent damping as seen in Eq. (A3). The amplitude as a function of feedback gain is shown in Fig. 2. The measured value almost agrees with the calculated one using Eq. (3), where k is 400 $sec^{-1}\cdot mm^{-2}$.

MEASUREMENT OF DAMPING RATE

A decaying oscillation is observed just after the self-oscillation is switched off. The oscillation is detected by the bunch oscillation detector[1]. An output signal from the detector is displayed on an oscilloscope. Time constant of exponential decay is measured from the display. The damping rate is an

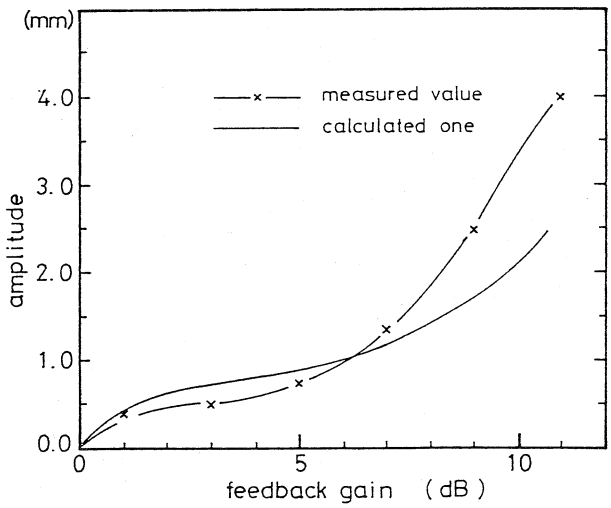

Fig. 2 Amplitude of self-oscillation vs feedback gain. The valve of 0 dB for the feedback gain corresponds to the threshold of the oscillation.

inverse of the time constant.

The damping rate observed is much faster than the radiation damping rate of 23 $(\mathrm{sec})^{-1}$. The horizontal damping rate is measured as a function of the chromaticity parameter defined by $\Delta\nu_x = \xi_x (\Delta p/p)$ and beam current as shown in Fig. 3 and 4. The damping rate is roughly propotional to chromaticity and beam current when bunch length is constant (σ = 1.1 cm).

These phenomena cannot be explained by the damping mechanism due to momentum-dependent tune-spread produced by sextupoles, because each particle has same mean momentum during a synchrotron oscillation. A head-tail damping effect is considered in order to explain the result of the measurement. The head-tail effect is damping if $\xi > 0$ and growth when $\xi < 0$ as expected from Eq. (1). The damping and growth rates are equal when only the sign of chromaticity is changed under the condition that other parameters are same. An effective damping rate α_{eff} is the sum of those of radiation damping α_0 and head-tail damping α_1,

$$\alpha_{eff} = \alpha_0 + \alpha_1 \qquad (4)$$

The value of α_1 is obtained from the horizontal growth rate of the head-tail instability described in the next chapter using the Eq. (1). The value of α_{eff} is shown in solid line in Fig. 3 and 4 to be compared with the measured one. These two values roughly agree with each other. When beam current and chromaticity are relatively high, other second-order damping mechanisms should be considered.

HEAD-TAIL INSTABILITY

An experiment was carried out at the 2.5 GeV injection. When vertical chromaticity is negative, a bunch begins to oscillate in vertical direction at threshold current. The oscillation grows up and causes beam loss. Beam current is limited by the instability. Maximum beam current as the threshold value has been measured as a function of feedback gain of the system. The relation between the feedback gain of the system and the vertical damping rate has been measured under the same condition using the injection kickers. Then, we obtain the relation between the growth rate and the threshold current, since the growth rate is equal to the damping rate at the threshold.

The result is shown in Fig. 5. The growth rate is propotional to threshold current as is predicted by the

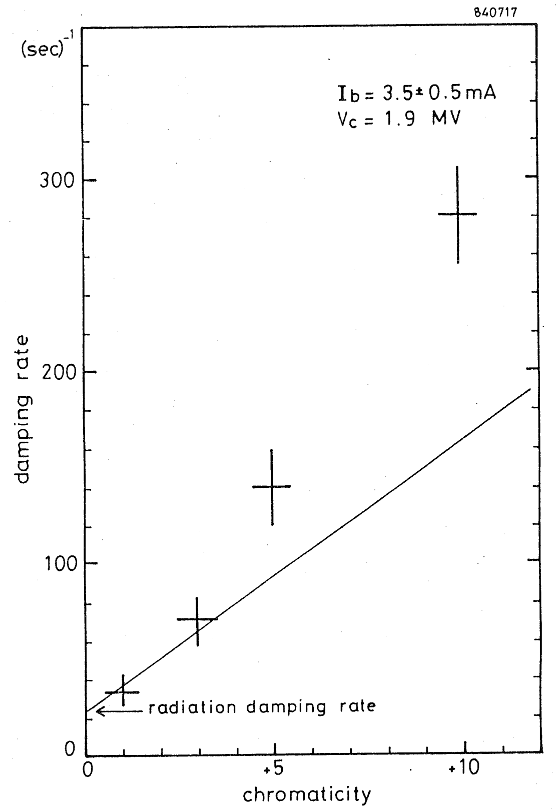

Fig. 3 Damping rate vs chromaticity. The solid line is calculated valve using eq. (4).

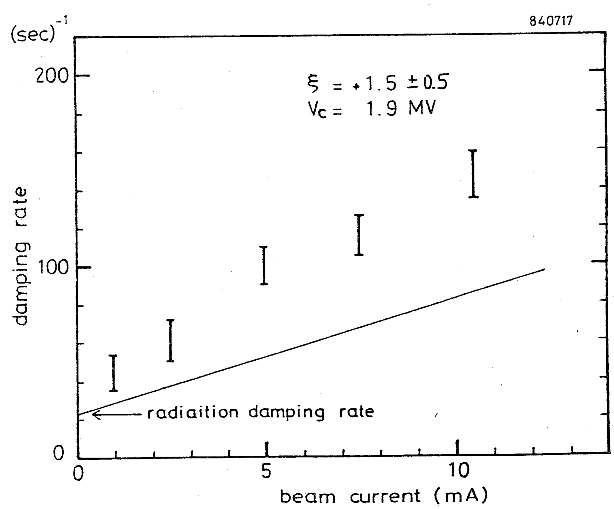

Fig. 4 Damping rate vs beam current. The solid line is calculated valve using eq. (4).

theory. Bunch length is inverse to a square root of cavity voltage, before bunch lengthening starts. As cavity voltage is increased or bunch length shortens, the slope seen in Fig. 5 becomes steeper, which shows an increase of the wake function defined in Eq. (2). The wake function is shown in Fig. 6 as a function of bunch length together with a calculated value reported by Y. Chin[3]. He calculated the transverse wake function of bellows and cavities installed in the AR. The measured value involves total contribution from accelerator components. We should take count of a contribution from other vacuum components than the bellows and the cavities at short bunch length of 1 cm.

CONCLUSION

Damping and exciting of betatron oscillation are surveyed in the AR using the transverse feedback sys-

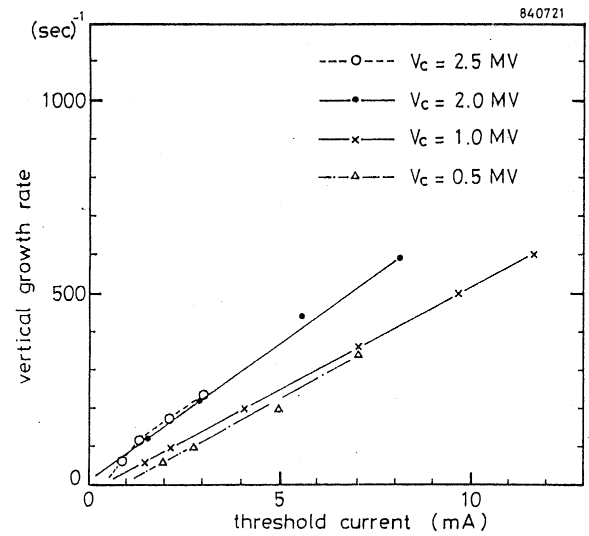

Fig. 5 Vertical growth rate vs threshold current with a parameter of cavity voltage. Vertical chromaticity is $\xi_y = -9.6$.

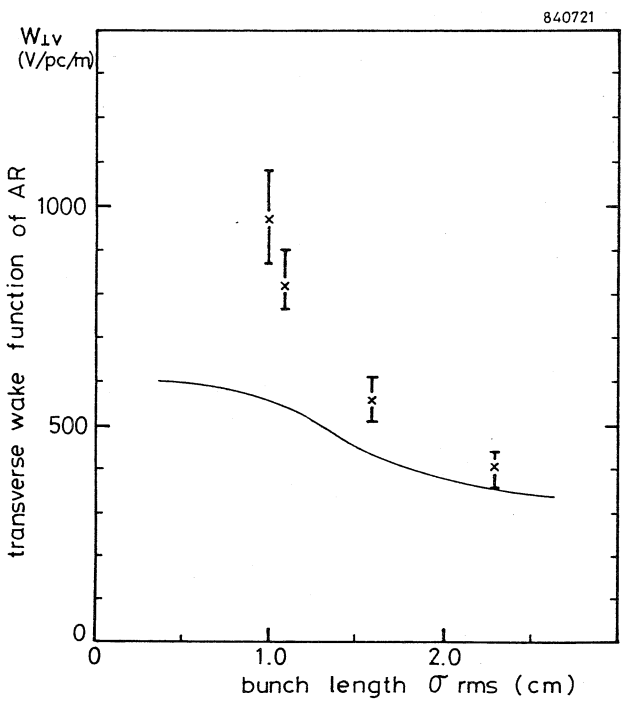

Fig. 6 Transverse wake function vs bunch length. The solid line shows calculated value for bellows and cavities.

tem. The following is remarked.

1) The damping rate of a single bunch depends on beam current and chromaticity. The effective damping rate is considered, which is the sum of radiation damping and head-tail damping rates.
2) The self-oscillation of betatron is carried out by positive feedback of the system. The oscillation has the amplitude determined by Eq. (3). This technique is used for the measurement of the effective damping rate.
3) The transverse wake function of the AR is measured as a function of bunch length from the growth rate of the head-tail instability. A contribution from other vacuum components than the bellows and the cavities should be taken into account in order to explain the difference between the measured and the calculated values.

REFERENCES

1. T. Ieiri, Y. Mizumachi and J.-L. Pellegrin, "The Transverse Feedback System for the TRISTAN AR", contributed to this conference.
2. A.W. Chao, "Coherent Instabilities of a Relativistic Bunched Beam", SLAC-PUB-2946, June 1982.
3. Y. Chin, "Transverse Impedance of Bellows in the AR", (in Japanese) TN-84-009, 1984.

Appendix

An equation of motion without the transverse feedback is

$$\frac{d^2x}{dt^2} + 2\alpha_{eff}\frac{dx}{dt} + (\nu\omega_0)^2x = 0 \qquad \text{(A1)}$$

α_{eff}: effective damping rate
ν: fractional part of the tune
ω_0: angular revolution frequency

We take count of the feedback effect. Negative feedback increases damping and positive feedback produces antidamping. Then, the equation is

$$\frac{d^2x}{dt^2} + 2(\alpha_{eff} \pm \alpha_f)\frac{dx}{dt} + (\nu\omega_0)^2x = 0 \qquad \text{(A2)}$$

The + sign in the bracket in Eq. (A2) is damping and the - sign is antidamping. Assuming that the damping rate depends on an amplitude due to nonlinear fields, we put

$$\alpha_f - \alpha_{eff} = \alpha_{fo} - \alpha_{effo} - kx^2 \qquad \text{(A3)}$$

where α_{fo} and α_{effo} are damping rates for small amplitude and k is a constant coefficiency. The equation of motion for positive feedback is

$$\frac{d^2x}{dt^2} - 2\{(\alpha_{fo} - \alpha_{effo}) - kx^2\}\frac{dx}{dt} + (\nu\omega_0)^2 = 0 \qquad \text{(A4)}$$

The Eq. (A4) is known as the van der Pol's equation. An amplitude in stationary state is obtained from Eq. (A5).

$$\int_{\text{one revolution}} \{(\alpha_{fo} - \alpha_{effo}) - kx^2\}\frac{dx}{dt} \cdot dx = 0 \qquad \text{(A5)}$$

We put $x = x_0\sin(\nu\omega_0 t)$, since the oscillating term in Eq. (A4) is much larger than the damping term. The equation (A5) becomes

$$\int_0^{2\pi}\{(\alpha_{fo} - \alpha_{effo}) - kx_0^2\sin^2(\nu\omega_0 t)\}\cos^2(\nu\omega_0 t)dt = 0$$

From the above equation we obtain

$$x_0 = 2\sqrt{\frac{\alpha_{fo} - \alpha_{effo}}{k}} \qquad \text{(A6)}$$