NONLINEAR EFFECTS IN CYCLOTRON

T. Yamazaki, K. Hosono, M. Inoue and I. Miura
Research Center for Nuclear Physics, Osaka University
Ibaraki, Osaka 567, Japan

ABSTRACT

By using static phase plots the nonlinear effects of particle motion and stability limits are investigated for the RCNP AVF cyclotron and a proposed ring cyclotron as a new facility of RCNP. For the ring cyclotron the effects of trimming coil shapes by calculating isochronous magnetic fields with circular-shape approximation and orbital-shape approximation are compared.

INTRODUCTION

For the design study of RCNP ring cyclotron, the betatron frequencies, eigen ellipses, phase plots and the stability limit have been studied for various types of sector magnets.[1,2] The iron fields used for these calculations were the measured field of a model magnet and the artificial fields calculated with computer code FIGER.[3]

At extraction radius 375 cm, protons can be accelerated up to 300 MeV. The proton beams are accelerated through $\nu_\gamma = 4/3$ resonance. The phase ellipse is slightly deformed in crossing this resonance, and largely deformed near $\nu_\gamma = 8/4$ stop band.

The protons also cross the $\nu_\gamma = 8/5$, 12/8 and other higher harmonic resonances. These higher harmonic resonances don't have large disturbance to the accelerated orbits, but may change the fine structure of beam profiles and beam qualities required to high resolution studies on the nuclear structure. It is interest to study the behavior of higher harmonic resonances.

RING CYCLOTRON

The proposed RCNP accelerator system consists of two ring cyclotrons and an injector cyclotron. A first ring cyclotron redesigned in the end of 1982 can accelerate protons up to 300 MeV. The proton beams are accelerated through $\nu_\gamma = 4/3$ resonance. The phase plots across the $\nu_\gamma = 4/3$ resonance are shown in Fig. 1. The phase plots show triangular shape around the equilibrium orbit and show the existence of $\nu_\gamma = 4/3$ resonance.

For the ring cyclotron we have studied the effects of trimming coil shapes by calculating isochronous magnetic fields with circular-shape approximation and orbital-shape approximation. The isochronous fields are calculated by iteration method. Figs. 2 and 3 show the phase plots of the stable region for protons at large radius for a spiral-sector ring cyclotron with four magnets. Outer eight islands are related to the stable fixed points of $\nu_\gamma = 12/8$ and inner five islands are related to the stable fixed points of $\nu_\gamma = 8/5$. Many small islands are related to higher order fixed points. These islands are surrounded by random plots in higher energy and outer region. The size of this stochastic region is different between two types of isochronization. But for both types of isochronization the sizes of stable regions are large enough to accelerate particles.

Fig. 4 shows the phase plots above extraction energy (E_{ex} = 300 MeV). The size of the stochastic region becomes larger above 330 MeV, and the stable region disappears at 354.3 MeV. In figures FR means the radial betatron frequency ν_γ and FZ means the vertical betatron frequencies ν_z , and FR and FZ values near 90 show imaginary values. In 341.7 MeV case the radial betatron frequency ν_γ becomes 1.90 and close to the stop band of $\nu = 8/4$. In the figure there are two stable fixed points of $\nu = 8/4$. Fig. 4 shows an orbit behavior at stop band region. It may be difficult to accelerate particles up to this energy, and the extraction energy is designed at the betatron frequency $\nu_\gamma \simeq 1.7$.

In some phase plots the large islands are surrounded by small islands and these small islands are

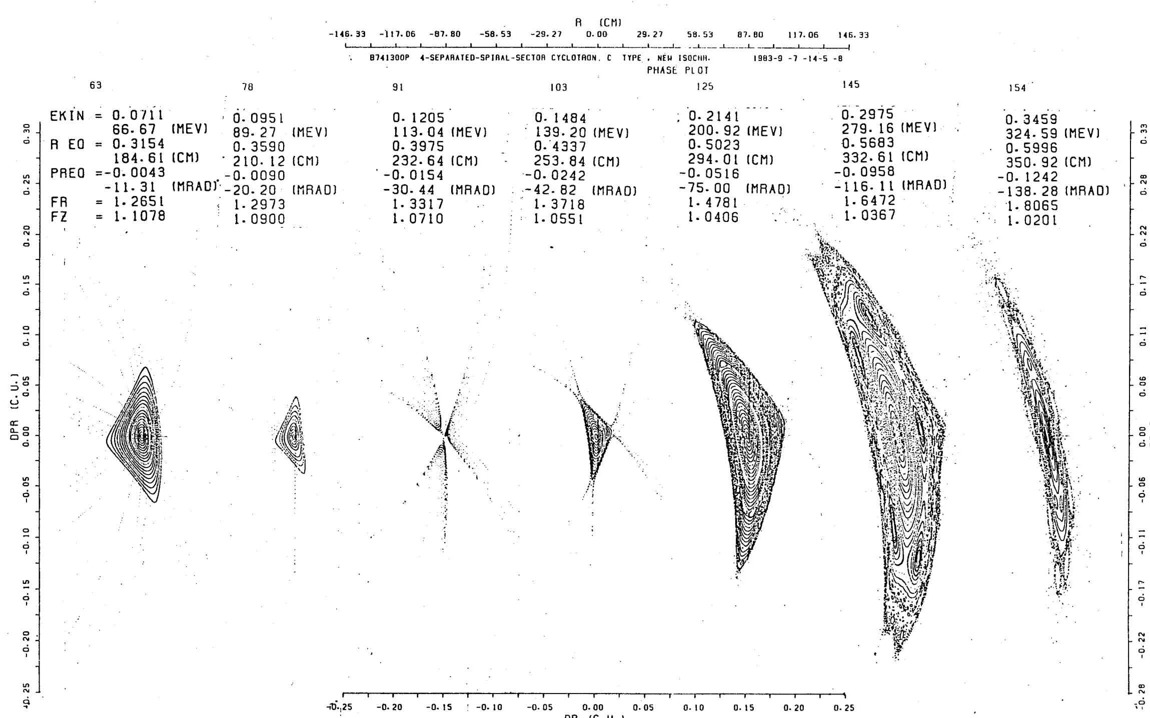

Fig. 1 Phase plots of the proton beams.

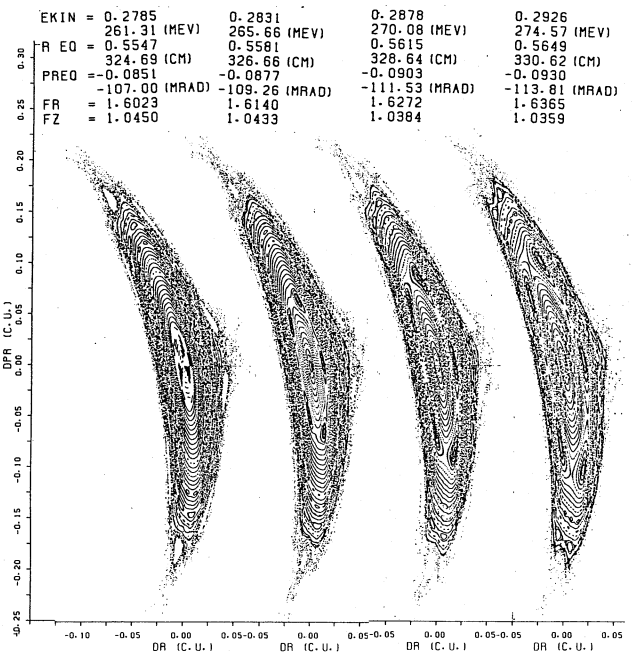

Fig. 2 Phase plots above $\nu_\gamma = 8/5$ resonance. The orbital-shape isochronous field is used for the calculation. Five islands around the equlibrium orbit move outward with energy.

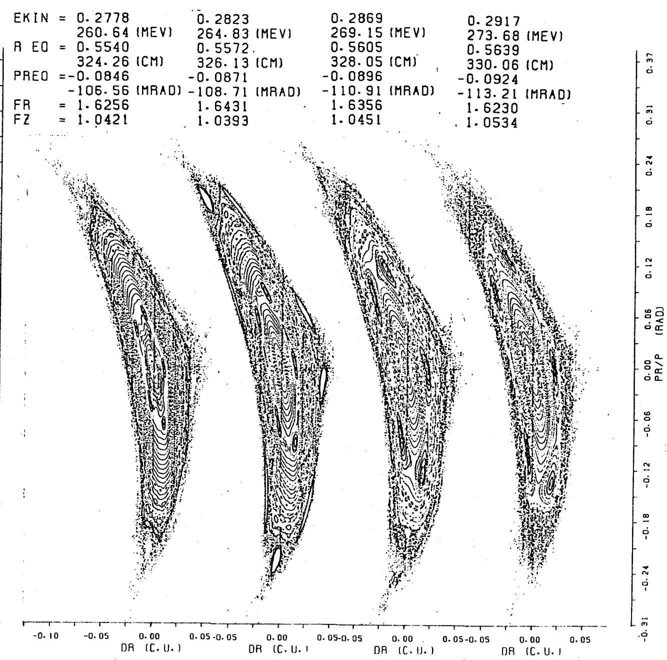

Fig. 3 Phase plots above $\nu_\gamma = 8/5$ resonance. The circular isochronous field is used for the calculation. The size of the stable region is smaller than that for the orbital-shape isochronous field.

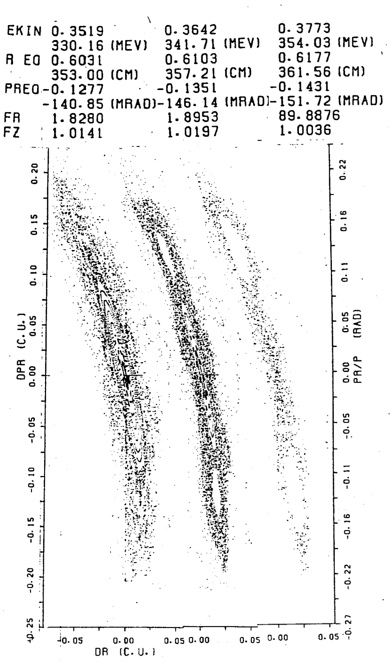

Fig. 4 Phase plots at large radius. If the magnetic field is well isochronized to larger radius, the size of the stochastic region becomes larger and two islands related to the stable fixed points of $\nu = 4/2$ appear.

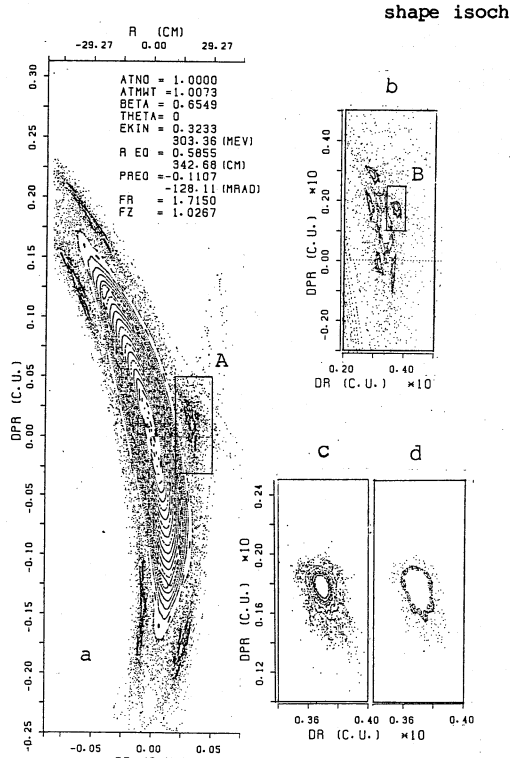

Fig. 5

Complicated phase plots.

a. Outer five islands around the equilibrium orbit are related to the stable fixed points of $\nu = 8/5$.

b. A magnification of the littel box in A. An island is surrounded by five small islands.

c,d. Further magnifications of the small box in B. Figs. c and d show the same phase plots of the small island with different starting conditions. They show very small islands around the small island.

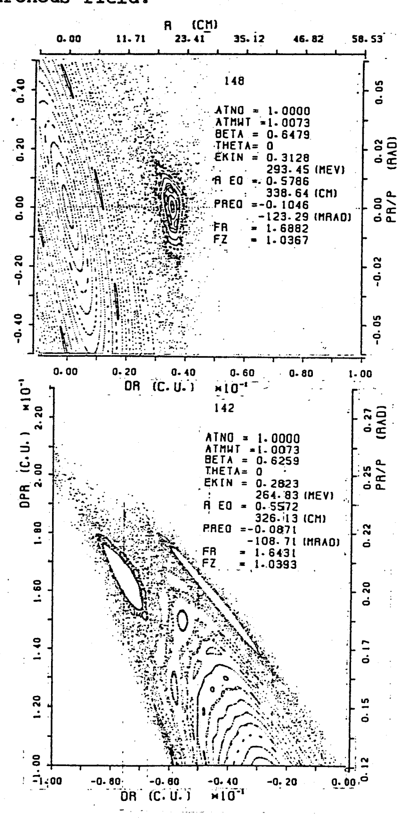

Fig. 6

The large islands are surrounded by small islands. The behavior of small islands changes remarkably with small change of particle energy.

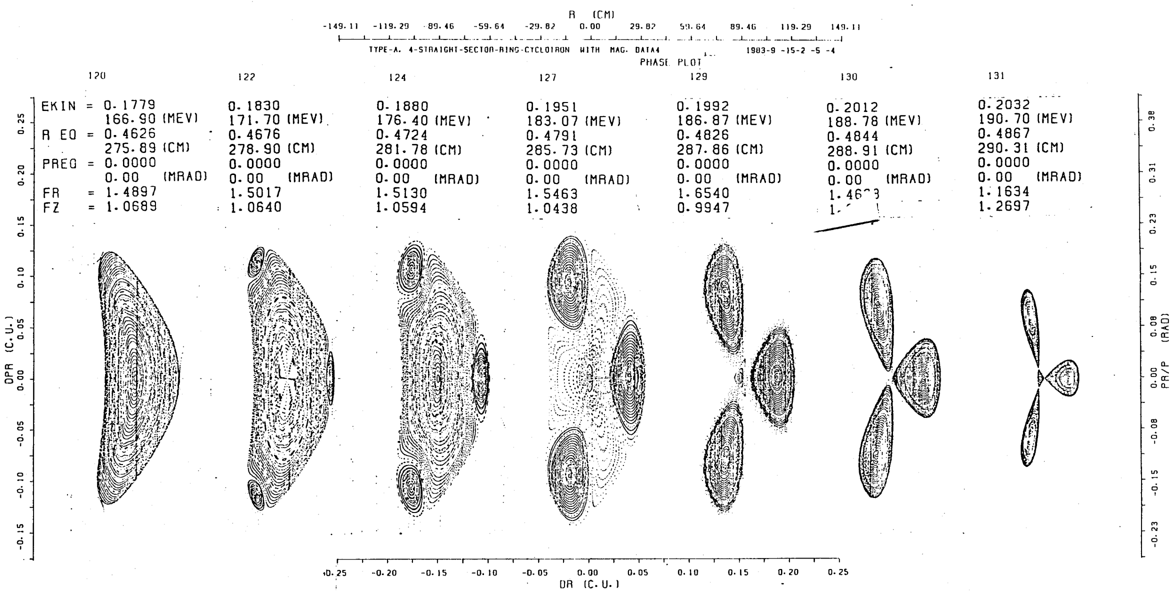

Fig. 7 Phase plots near extraction region. If the magnetic field is slightly lower than isochronism, three islands related to the stable fixed points of $\nu = 4/3$ appear and radial betatron frequency (FR) decreases with energy.

surrounded by very small islands. Fig. 5 shows a typical example of the complicated phase plot near extraction energy. Fig. 6 show other examples of the complicated phase plots below extraction energy.

Fig. 7 shows the phase plots of protons at large radius for a straight-sector ring cyclotron with four sector magnets. The magnetic field at larger radius is slightly lower than isochronism. Three islands appear on the boundary of stable region, they become larger and the central stable region around the equilibrium orbit becomes smaller with particle energy, and there is $\nu_\gamma = 4/3$ resonance near 188 MeV. This behavior of phase plots at $\nu_\gamma = 4/3$ is similar to that of $\nu_\gamma = 3/3$ resonance for three-sector AVF cyclotron near extraction radius.

As seen in Fig. 1, protons cross $\nu_\gamma = 4/3$ resonance at 113 MeV and the size of stable region increases. The stable region has a triangular shape. Three unstable fixed points are on the vertex of this triangle. That is, these unstable fixed points with $\nu = 4/3$ are on the boundary of stable region. Three stable fixed point orbits are far apart from the equilibrium orbit. With increasing energy many fixed points appear at the center and move outward. Each island in the phase plots has stable fixed point in it. Islands are accumulated at the boundary of the stable region with energy. With increasing energy the betatron frequency go away from $\nu = 4/3$ and get nearer to $\nu = 8/4$, and the shape of the stable region changes from triangle to ellipse. Moreover the accumulation of the fixed points increases the size of stochastic region near the boundary of the stable region. The phase plots in Fig. 1 are for four spiral-sector magnets, but these behaviors are similar for four straight-sector magnets.

Fig. 1 shows that three unstable fixed points of $\nu = 4/3$ are on the vertex of the triangle. However as seen in Fig. 7 three stable fixed points of $\nu = 4/3$ appear on the vertex of the triangular stable region, and a new triangle appears around the equilibrium point. There is an apparent discontinuance between the triangle of low energy $\nu_\gamma = 4/3$ region and high energy $\nu_\gamma = 4/3$ region. The situation is different for the $\nu_\gamma = 3/3$ resonance of three sector AVF cyclotron. In this case there is a continuation between the triangle of the central $\nu_\gamma = 3/3$ region and high energy $\nu_\gamma = 3/3$ region near extraction. There is the same triangular stable region and no drastic change of stable region as seen in Fig. 7.

RCNP AVF CYCLOTRON

The RCNP 230-cm AVF cyclotron is three sector machine with K=120 MeV. Radial betatron frequency ν_γ for 65 MeV proton acceleration is from 0.99 to 1.13. Particles cross only high order (radial) resonances except $\nu_\gamma = 3/3$. Fig. 8 shows the phase plots of the stable region at large radius. By using the measured magnetic field at the main magnet current of 600 A an isochronous field for 65 MeV protons was obtained, and the particle trajectories were traced about 10000 revolutions in all. Many islands are seen in these phase plots, and they are related to higher order resonances. Some examples are as follows: 23 islands at 49.45 MeV are related to $\nu = 24/23$ stable fixed points, 11 islands at 57.40 and 62.96 MeV are related to $\nu = 12/11$ stable fixed points and 8 islands at 65.72 MeV are related to $\nu = 9/8$ stable fixed points.

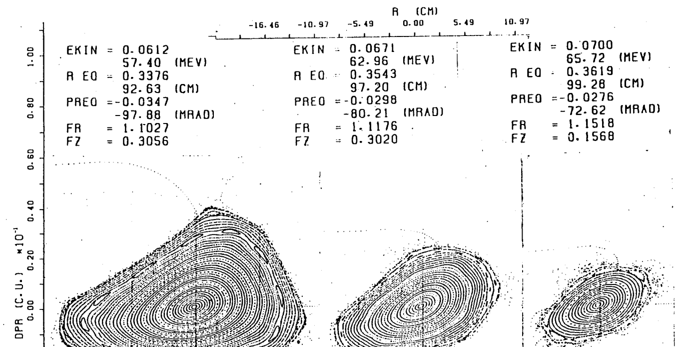

Fig. 8 Phase plots of three sector AVF cyclotron.

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