BEAM DYNAMICS IN HIGH ENERGY LINEAR COLLIDERS

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INTRODUCTION

The progress of particle physics has been pushed up the relevant energy range and it is about to reach tera electron volts in the near future. Two types of accelerators to this end have been investigated intensively, namely large hadron storage rings with superconducting magnets and electron positron linear colliders with high accelerating gradient. For the latter numerous acceleration mechanisms have been proposed. More than 1GeV/m might be obtained by plasma beat wave accelerator or laser-grating accelerator. Other mechanisms such as inverse free electron laser, two beam accelerator and wake field accelerator are claimed to give several hundred MeV/m. It seems, however, much more than a decade of R&D is required in order to give a practical design of these accelerators with the beam energy of 1TeV or so.

If one is satisfied with a beam energy of 1TeV or less, a relatively low accelerating gradient of 100MeV/m is enough to give this energy with the length less than the circumference of the LEP ring. It seems that this gradient is within the range of the conventional acceleration mechanism, i.e. microwave plus accelerating cavities, although an intensive R&D is necessary for the microwave generator. 'Lasertron' is now under investigation as a possible candidate of the generator and will be reported by other people in this conference. Here we discuss the beam dynamics in this electron-positron linear collider to clarify the various problems to get a high luminosity assuming the field gradient of 100MeV/m is realized.

The major problems except the acceleration itself are:
1) positron production.
2) design of the damping ring which cools the positron (and/or electron) beam to match the small emittance required by the high luminosity.
3) beam instability in the linac.
4) design of the final focusing system.
5) beam-beam interaction.

We discuss here 3) and 5) only. For the problem 1) we assume that at least one useful positron can be created from one electron, which looks feasible. The damping ring has no serious problem. A storage ring with the beam energy of 2GeV or so is sufficient for 1TeV linear collider. The problem 4) has not yet been pursued by the author. It is not an easy problem but here we only pick up the problems for future investigations.

BEAM-BEAM INTERACTION

From the physical point of view, our major concern except the beam energy is the luminosity. If the two bunches containing N particles each with r.m.s. transverse dimension σ_x and σ_y collide in a repetition frequency f_{rep}, the luminosity is given by

$$\mathcal{L} = f_{rep} \frac{N^2}{4\pi\sigma_x\sigma_y} P \quad . \tag{1}$$

We assume that initially the bunches have gaussian distribution in three dimensions. If the two bunches are oppositely charged like electron and positron, each bunch tends to focus by the electromagnetic field caused by the other bunch. This is called pinch effect. The factor P in the above formula comes from it. It is unity at the low current limit and usually greater than unity. One can easily see that the equation of motion during the collision can be scaled using two parameters, namely 'disruption parameters'. They are defined as

$$D_{x(y)} = \frac{2N r_e \sigma_z}{\gamma\sigma_{x(y)}(\sigma_x+\sigma_y)} \ , \tag{2}$$

where r_e is the classical electron radius, σ_e the r.m.s. bunch length and γ the beam energy in the unit of the rest mass. The pinch effect factor P is a function of D_x and D_y. Fig.1 shows its behavior at small values of D's, which is estimated by a computer tracking. It is known [1] that for very large values of D's the factor P saturates and then drops, but this region is not important in 1TeV collider because of the beamstrahlung effect described below.

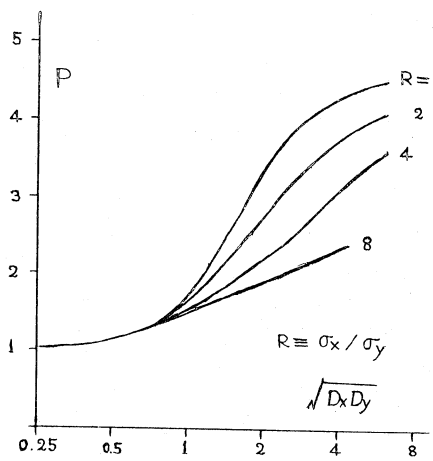

Fig.1

The electromagnetic field in a bunch causes not only the pinch effect but the synchrotron radiation called beamstrahlung. Although the bunch length is very short , of the order of mm, energy loss is very large because the magnetic field in a bunch amounts to several hundred Tesla in a typical case. The energy loss is given by [2]

$$\frac{\Delta E_{bs}}{E} = \frac{2}{3}\frac{r_e^3 N^2 \gamma}{\sigma_x\sigma_y\sigma_z} F(R) \ , \tag{3}$$

where

$$R = D_y/D_x = \sigma_x/\sigma_y$$

and

$$F(R) = \begin{cases} 0.325 & (R=1) \\ 1.36/R & (R\gg 1) \end{cases} \ . \tag{4}$$

We assume R>=1 in the following. In this formula, the pinch effect and the quantum mechanical correction are ignored. The latter is necessary because the critical energy of the radiation becomes a considerable fraction of the initial particle energy in a 1TeV collider.

The beamstrahlung brings about various undesirable effects. The center-of-mass energy of the collision inevitably becomes to have a spread of the order of ΔE_{bs}. It also produces background events on the experiments and moreover it makes difficult to handle the beam after collision.

As an example we give a relatively reasonable parameters of 1TeV x 1TeV collider.

$N=1\times10^{11}$, $f_{rep}=400\mathrm{Hz}$

$\sigma_x=1.4\ \mu\mathrm{m}$, $\sigma_y=0.18\ \mu\mathrm{m}$, $\sigma_z=1\mathrm{mm}$

$R=8$, $D_x=1.02$, $D_y=0.13$

$L=1.7\times10^{32}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}$

$\Delta E_{bs}/E=0.085$

average number of photons	2.6/electron
average photon energy	33GeV
critical energy	~200GeV

$\langle s\rangle=3.65\ \mathrm{TeV}^2$ (s=c.m. energy square)

$\Delta s_{rms}=0.46\ \mathrm{TeV}^2$

These are estimated through a computer tracking. The luminosity includes the pinch factor P=1.34.

The energy deposited by beamstrahlung is as much as 3kJ per collision which is four orders of magnitude larger than that in SLC. By comparing Eqs.(1),(3) and (4) one finds that, in order to reduce the beamstrahlung without lowering the luminosity, it is necessary to increase the bunch length and/or to make the beam flat (large R) and/or to increase the repetition rate. However, all of these have severe limitations. The bunch length should be kept short because it causes an energy spread in the linac and makes it difficult to design an achromatic final focusing system. Too flat beam imposes a tight allowance of the error in the beam position control. High repetition rate results in an enormous electricity power consumption. Therefore, smaller c.m. energy spread can only be got at the expense of a lower luminosity, though a considerable improvement from the preliminary parameters above may be expected by a careful design of the linac and the final focusing system.

As for the final focusing system we only count up the problems to be investigated.
1) To what extent can the focusing system be made achromatic ?
2) Is it possible to design the strong quadrupole magnets near the colliding point ?
3) What should be done to reduce the background events on the experiments ?
4) Can the degraded beam after collision be used for other purposes such as positron production, fixed target experiments and synchrotron radiation research ?

BEAM INSTABILITY IN THE LINAC

We have not yet pursued which type of accelerating cavity to adopt. Here we make a crude estimation of the influence of the wake field assuming the impedance of the SLAC type cavity.

A particle in the tail of the bunch is decelerated by the longitudinal wake field produced by the head of the bunch and at the exit of the linac the tail has an energy lower than that of the head. This energy difference between the head and the tail must be less than the energy acceptance of the final focusing system, say, ±0.5% or ±1%. If we accelerate 10^{11} particles with r.m.s. bunch length of 1mm to 1TeV through 1km cavity structures, the maximum deceleration is about 15GeV or 1.5%. However, the greater part of this energy spread can be eliminated by accelerating the bunch slightly in front of the crest of the r.f. wave and the residual energy spread may be ±0.3%. This value is well within the range of the chromaticity correction in the final focusing system. Hence, as long as the longitudinal wake field is concerned, the number of particles in a bunch can be increased a little.

The effect of the transverse wake field is more serious. When the bunch is dislocated from the axis of the accelerating cavity, the tail of the bunch feels a transverse force due to the head. The source of the deviation from the axis may be the injection error or the misalignment of the cavities and the quadrupole magnets.

If the bunch is injected at a distance of x_0 from the axis, the oscillation amplitude x_f of the tail of the bunch after acceleration is given by[3)]

$$\frac{x_f}{x_i} \approx \frac{1}{\sqrt{\gamma_f/\gamma_i}}\ \frac{0.23}{\eta^{1/6}}\ e^{1.3\,\eta^{1/3}} \tag{5}$$

$$\eta \equiv 0.10\ \frac{\lambda_\beta L r_e N W_0}{\gamma_f}\ \log\frac{\gamma_f}{\gamma_i}$$

Here, γ_i and γ_f are initial and final beam energy in the unit of the rest mass, L the total length of the accelerating structure, λ_β the betatron wave length, W_0 the slope of the transverse wake function at the origin; i.e.

$$W_0=[\,dW_T(z)/dz\,]_{z=0}$$

If we make λ_β=240m by installing quadrupole magnets at every 30m, then we get η~120 and x_f/x_0~3. The required r.m.s. beam size at the exit of the linac is about 90 μm (horizontal) and 30 μm (vertical). Final oscillation amplitude x_f and y_f given by Eq.5 must be less than these values, which give the tolerance of the injection error of 30 μm (horizontal) and 10 μm (vertical). This is a very tight condition but still looks feasible. The major cause of the error of injection position may be the jittering of the ejection kicker magnets in the damping ring. Hence, if we extract the bunch from the ring in the horizontal plane, the vertical position error will be much smaller than the horizontal one.

The effect of the cavity misalignment can be estimated according to ref.3). Our parameters give the tolerances of the horizontal and vertical misalignment of 200 μm and 80 μm (r.m.s.), respectively.

One sees that the transverse wake field imposes a very strict upper bound of the number of particles to be accelerated. A possible method to relax its effects has been proposed by the members at Novosivirsk[4]. The transverse blow-up is essentialy a forced oscillation of the tail by the head. If the bunch has a large energy difference between the head and the tail, the growth of the oscillation amplitude of the tail may be suppressed owing to the chromatic difference in the betatron oscillation frequency. The blow-up is more serious when the beam energy is lower. Therefore, the relative energy difference required decreases gradually from the initial value, +10% in their case, as the beam energy becomes higher. At the exit of the linac it may be less than the energy acceptance of the final focusing system.

The discussion in this section are still immature but it may be a guide to design the accelerating cavities for our machine.

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