

AN ANALYSIS METHOD FOR IH-TYPE LINEAR ACCELERATORS

K.Satoh, S.Kamohara, T.Hattori and E.Arai
 Research Laboratory for Nuclear Reactors, Tokyo Institute of Technology
 O-okayama, Meguro-ku, Tokyo 152, Japan

ABSTRACT

Equivalent circuit analyses have been performed for interdigital H (IH)-type accelerating structures to predict important design parameters such as resonance frequencies, accelerating-field and magnetic-field distributions. We have determined parameters of an equivalent circuit by using numerical calculation methods and analytical formulations for three-dimensional configurations. Good agreements with experimental results have been obtained.

INTRODUCTION

The IH structure is well known for its high shunt impedance at low beta values ($\beta < 0.1$). Good results have been reported from the laboratories which constructed IH structures for accelerating heavy ion beams¹⁻³. The fundamental parameters such as resonance frequency and accelerating-field distribution depend very sensitively on other parameters including the electric capacity of the drift tubes. Control devices of accelerating-field distribution are indispensable. Design studies have been done through scale model experiments which require a large amount of labor^{4,5}. Two dimensional numerical calculation codes such as "SUPER FISH" cannot be applied for the design work because the IH structure has no axial symmetry.

Usually, the lumped equivalent circuit analysis has been applied to analyze cavities which have no axial symmetry⁴⁻⁶. This method is based on one-dimensional approximation. However, the parameters for the equivalent circuit must be calculated in considering three-dimensional geometries. Although three-dimensional calculation codes have recently become available⁷⁻⁹ owing to the progress of computer technology, it is still difficult to solve Maxwell's equation in a complicated structure such as IH cavities. The main bottleneck is the limitation of memory size.

We have applied the three-dimensional codes^{8,9} to determine the parameters for the lumped equivalent circuit. Resonance frequencies, accelerating-field distributions and shunt impedances of IH structures have been evaluated by means of the four-terminal network method. To estimate the distribution of the RF power loss in IH structures, we have calculated the magnetic-field distribution by using the three-dimensional finite element method⁸. The structure of the cavity has been simplified, as will be mentioned later, to clear the limitation of computer-memory size. In this paper the procedure of the analysis is described and the results are compared with model experiments.

EQUIVALENT CIRCUIT FOR IH STRUCTURES

Figure 1a shows a schematic view of the IH structure studied. The drift tubes are connected to the top of the stems which are mounted on the ridges. Characteristics are "end spaces" and "magnetic flux inducers"⁵ at the both end regions of the structure³. The magnetic flux is enhanced in this region to equalize the accelerating field throughout the structure.

One accelerating cell is represented by an equivalent circuit as shown in Fig. 1b. The ohmic resistance is neglected. In addition to the capacity between two drift tubes C_d , we take account of the stray capacities C_s and C_r . The symbol C_s corresponds to the capacity between the drift tube and the upper plate of the opposite ridge, and C_r denotes the capacity between two opposite upper plates of the ridges. The sign of L_t stands for the shunt inductance which reacts against the azimuthal current through the one-turn loop which consists of cavity wall and the two side plates of the ridges. The conduction current on the stems generates magnetic energy which is stored in the equivalent self

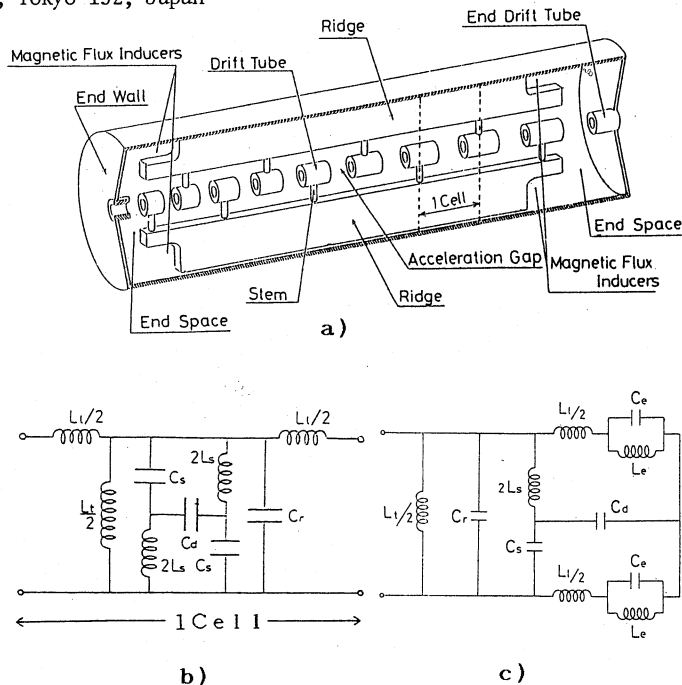


Fig. 1 a) A schematic view of the IH structure studied. b) An equivalent circuit for one accelerating cell, c) for the end cell.

inductance L_s . The series inductance L_l reacts against the longitudinal current on the ridge upper plates. The end cell is approximated as shown in Fig. 1c. The characteristic of the end region is described by the equivalent capacity C_e and inductance L_e . An accelerating structure with N identical cells is represented by a series of N circuits.

DETERMINATION OF LUMPED CIRCUIT ELEMENTS

In calculating the parameters for the equivalent circuit, one cell is assumed to be a part of a constant velocity structure of infinite length.

The capacities C_d , C_s and C_r are calculated by using the triangular surface charge method³ which is suitable for solving three-dimensional problems. In this method the surfaces of all objects are subdivided into a number of triangular elements with a linearly changing charge density. The boundary condition is the potential at each node point, where the charge density is an unknown variable. The formula of this method is similar to that of the boundary element method. The detailed description is presented elsewhere³. Figure 2a shows an example of the configurations subdivided into triangular elements. The number of the node points is 242. To determine C_d and C_s we set the boundary condition as follows: one of the drift tubes has a potential of $1V$ and the other objects are at the ground potential. The capacity C_d is given by the total charge of the drift tube at 0 potential, and C_s corresponds to the total charge of one of the ridge upper plates. The capacity C_r can be determined by giving a potential of $1V$ to one of the ridge upper plates. On the boundary to the next cell the electric field component perpendicular to the boundary surface is assumed to be 0 .

The electric field distribution is also obtained in the gap region. Examples are presented in ref.¹⁰

The tank inductance L_t is regarded as the inductance

of a wide single-turn coil and is calculated by the equation $L_t = \mu_0 S / \ell_c$, where μ_0 and S denote the permeability of vacuum and a half of the free space cross section of the cavity, respectively. The symbol ℓ_c stands for the cell length. On each ridge upper plate the longitudinal current flows in opposite directions. The series inductance L_1 is approximated by the inductance of a pair of long plates:

$$L_1 = \frac{\mu_0 \ell_c}{2\pi w} \left\{ w \ln \left(\frac{w^2 + d^2}{w^2} \right) + 2d \arctan \left(\frac{w}{d} \right) \right\},$$

where w and d are the half width of the ridge and the distance between both ridges, respectively. To determine the equivalent self inductance L_s of one stem we calculate the magnetic flux across the hatched section in Fig. 2b. The magnetic flux in this region is generated by the conduction current on the infinite series of the stems connected to the ridge at the same side. The calculation is numerically performed by summing up the contributions from all stems. To determine the contribution from each stem we use the well known formula for the magnetic field distribution outside a long line on which the current flows homogeneously.

It is not simple to deduce the equivalent values of C_e and L_e of the end region because the electric field as well as magnetic field is complicated. The procedure is as follows: We calculate at first resonance frequencies of TE_{11p} modes ($p=1,2,3$) for cylindrical resonators with ridges, end spaces and magnetic flux inducer. To simplify the problem the stems and the drift tubes are removed from the structure, and both end regions have the same geometry. We assume that the equivalent values of C_e and L_e do not change whether the stems and the drift tubes exist or not in the cavity. We use the three-dimensional numerical program based on the finite element method⁸. This program was developed by Hara, Wada and Kikuchi et al. A detailed explanation of this program is reported in ref⁸. The calculated frequencies agree with experiments within 3% for higher modes as well as the fundamental mode. These calculations are carried out by a super computer. The required number of the node points and CPU-time are typically about 1700 and 60 sec, respectively. An equivalent circuit for the simplified cavity is shown in Fig. 3. The parameters C_r , L_t and L_1 for the homogeneous main region are distributed-constants and expressed in terms of unit length. In this circuit the following relation is realized at the resonance frequencies.

$$\frac{2L_e}{1 - \omega_n^2 L_e C_e} = (-1)^{n+1} \left[\tan \left(\frac{\ell_r}{2} \omega_n \sqrt{L_1 \left(C_r - \frac{2}{\omega_n^2 L_t} \right)} \right) \right] (-1)^n \sqrt{\frac{L_t L_1}{\omega_n^2 L_t C_r - 2}},$$

where ω_n and ℓ_r are the angular frequency of n -th resonance mode and the length of the ridge, respectively. We can deduce C_e and L_e from this relation using the calculated resonance frequencies. Some examples of the calculated parameters are presented in ref.¹¹

CALCULATION OF RESONANCE FREQUENCIES AND ACCELERATING-FIELD DISTRIBUTIONS

To analyze an original IH cavity we use the four-terminal network method. At first we have analyzed a constant velocity model with four types of unit cell length ℓ_c . This model was constructed at INS to study the fundamental properties of the IH structure⁵. The end-space length ℓ_e can be varied by moving the tank end plates. The magnetic flux inducer cannot be equipped in this model. The distance between both ridges d is also variable. The dimension of this model is summarized in Table 1. Figure 4 shows examples of the calculated resonance frequencies for various configurations as well as experimental data. The deviation of the measured values from the calculated ones is within 5% for the lowest mode.

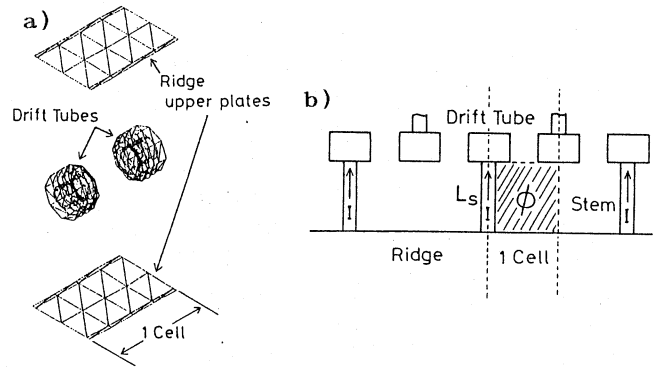


Fig. 2 a) An example of the subdivided configuration. b) The magnetic flux across the hatched section has been calculated to determine L_s .

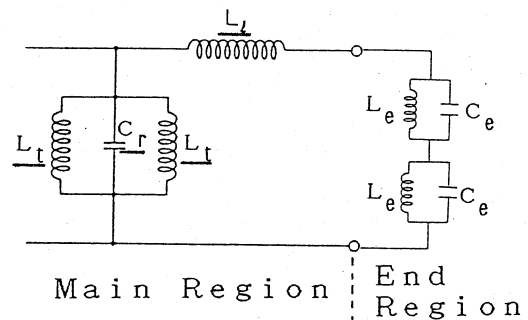


Fig. 3 An equivalent circuit for the simplified cavity (stems and drift tubes are removed).

Table 1
The physical dimensions of the models

	Constant Velocity Model	Accelerating Model	
Cavity Length	80, 84, 88, 92, 96	150	cm
Cavity Diameter	40	37.5	cm
Cell Length (ℓ_c)	3, 4, 5, 8	5.66 - 6.65	cm
Drift Tube Diameter	3.33	2.5	cm
Bore Diameter	0.58	1.25	cm
Gap to Cell Length Ratio	1/3	1/2	
Stem Diameter	1.0	1.5	cm
Ridge Width	3.0	4.0	cm
Ridge Distance (d)	8, 16, 24, 30.4	15	cm
End Space Length (ℓ_e)	2, 4, 6, 8	11	cm
Inducer Length (ℓ_i)	0	0 - 15	cm

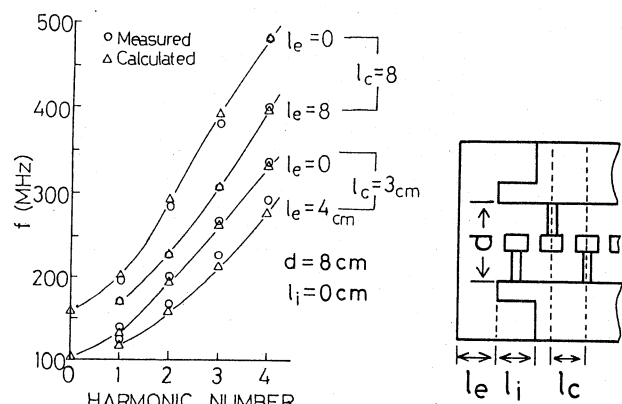


Fig. 4 Examples of the calculated resonance frequencies. Measurements are also presented.

As a next step we have analyzed an accelerating model. This model is a 1/2-scale model of a linac constructed at T.I.T.³ This linac is designed as a beam energy booster to accelerate heavy ion beams from the injection energy of 2.4 MeV/u to 3.4 MeV/u. The ratio of the final cell length to the first cell length is relatively small (1.18). The physical dimension of the model is listed in Table 1. Figure 5 shows resonance frequencies and accelerating-field distributions for various configurations of the end region. The calculated results agree well with the experimental data. To flatten the accelerating-field distribution we have adjusted the length of the magnetic flux inducers at the low energy side (denoted by l_1) as well as the high energy side (l_2). A combination of $l_1=105$ mm and $l_2=130$ mm gives an almost flat accelerating-field distribution which is adopted by the original cavity.

CALCULATION OF THE SHUNT IMPEDANCE

We can calculate the current and the potential at each point of the equivalent circuit. For the determination of the shunt impedance we have introduced surface resistances to the equivalent circuit. The shunt impedance Z_s is given as follows: $Z_s = \Sigma V d^2 / (P_t \cdot l_t)$, where Vd denotes the RF voltage across the accelerating gap, P_t stands for the total power consumption in the cavity and l_t is the cavity length. The surface of the cavity wall and the contact between components are assumed to be ideal. The shunt impedance of the beam energy booster is calculated to be 250 Ω/m . This value is higher than the measured value of 168 Ω/m by a factor of 1.5. This discrepancy can be explained partly by the surface roughness and the contact resistance.

THE FIELD DISTRIBUTION IN THE CAVITY

One of the serious problems with the IH type linac is the concentration of the RF power loss around the magnetic flux inducer³. It is useful for the heat engineering design to estimate the RF current distribution in the cavity. We can calculate the RF-field distribution in the simplified IH structure. Figure 6a shows an RF magnetic field in the section indicated in the figure. The magnetic flux is induced into the magnetic flux inducer. Figure 6b represents the amplitude distribution of the magnetic field near the cavity wall. This distribution must be similar to the RF current distribution. Strong peaks are observed in the calculated result as well as measurements at the boundaries of the magnetic flux inducers. A measurement of the original cavity which has the stems and the drift tubes on the ridges is also given. The magnetic field concentrates on the low energy side due to the shorter cell length in this region. The sharp peaks, however, reappear at the same position.

CONCLUSION

The parameters of the lumped equivalent circuit are determined by using numerical calculation codes and analytical formulas for three-dimensional configurations. Using these parameters the four-terminal network analysis has well reproduced the experimental data such as the resonance frequency, the accelerating-field distribution and the shunt impedance. The RF power loss distribution in the original cavity can be roughly predicted by calculating the magnetic-field distribution in a simplified cavity where the stems and the drift tubes have been removed.

The method discussed in this paper is a powerful supplement to scale model experiments for the design studies. We are now testing the applicability of this method for various structures including a high energy gain machine ($E_{out}/E_{in}=10$) like the main linac³ at the T.I.T.

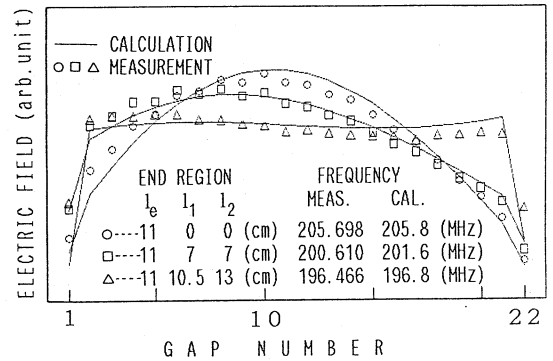


Fig. 5 Resonance frequencies and accelerating-field distributions of the accelerating model.

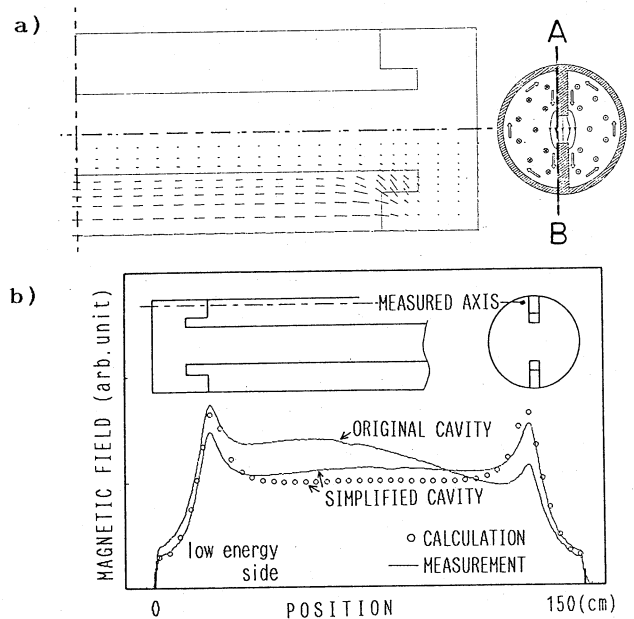


Fig. 6 A calculated RF magnetic field in the simplified cavity, ($l_e=11, l_1=l_2=7$ cm). a) The field component parallel with the section A-B. b) The amplitude distribution of the field. A measurement of the original cavity (with stems and drift tubes) is also given.

REFERENCES

- 1) E.Nolte, G.Geschonke, K.Berdermann, R.Oberschmid, R.Zierl, M.Feil, A.Jahnke, M.Kress and H.Morinaga, Nucl. Instr. and Meth. 158(1979)311.
- 2) K.Furuno, T.Kimura and H.Maeoka, Tsukuba University Annual Report, UTTAC-45(1982)4.
- 3) K.Satoh, T.Mitsumoto and E.Arai, Proc. 7th Tandem Conf. (April 6-10, 1987) Berlin, Germany, to be published.
- 4) Y.Oguri, E.Arai and T.Hattori, Nucl. Instr. and Meth. A235(1985)7.
- 5) S.Yamada, T.Hattori, T.Fujino, T.Fukushima, T.Murakami, E.Tojo and K.Yoshida, INS-NUMA-57 (April, 1987).
- 6) T.Weis, H.Klein and A.Schempp, Proc. 1984 Linear Accelerator Conf. (May 7-11, 1984) GSI-84-11 p.417.
- 7) T.Weiland, Particle Accelerators 15(1984)245.
- 8) M.Hara, T.Wada, K.Mitomori and F.Kikuchi, Proc. 11th Int. Conf. on Cyclotrons and their Applications (October 13-17, 1986) Tokyo, Japan, to be published.
- 9) T.Takuma and T.Kawamoto, "Calculation of Three-dimensional Electric Fields with Triangular Surface Charge Method (in Japanese)" CRIEPI-Report No.180029, 1980-12.
- 10) S.Kamohara, Y.Takahashi, K.Satoh, T.Hattori and E.Arai, Proc. 12th Linear Accelerator Meeting in Japan (Aug. 24-26, 1987) Tokai, Japan, p.98.
- 11) K.Satoh, S.Kamohara, T.Hattori and E.Arai, Proc. 12th Linear Accelerator Meeting in Japan (Aug. 24-26, 1987) Tokai, Japan, p.95.