# STUDY OF THE CENTRAL REGION OF AN AVF CYCLOTRON

H. Inoue, S. Seikaku*, R. Nishi**, T. Sugitate and Y. Yoshizawa
Department of Physics, Faculty of Science, Hiroshima University
Higashisenda-cho, Nakaku, Hiroshima, 730, Japan


## ABSTRACT

Orbits of accelerated particles in the central region of an AVF cyclotron were calculated based on the measured magnetic field obtained from a model magnet and an assumed dee configuration. The aim of calculations is to find proper conditions for the axial injection. Results on stable acceleration orbits are presented.


## INTRODUCTION

Recently,several kinds of AVF cyclotrons are commercially available. However, as for ion sources most of those machines equip only an internal one. Then investigation of an injection system is necessary for them to equip external sources for polarized ions and so on. The purpose of this study is to design an axial injection system for a commercial AVF cyclotron. Using a model magnet we studied the magnetic field and the first several turns of acceleration orbits in the central region. In this paper, we present suitable injection parameters for acceleration.

## MODEL MAGNET

The magnet used for the field measurement is an 1/3 scale model of an existing AVF cyclotron which accelerates protons up to 70 MeV at the extraction radius of 750 mm. The horizontal and vertical sections of the model magnet are shown in Figs. 1A and 1B. The pole diameter and gap between hills are 600 mm and 44.3 mm, respectively. Maximum field strength of 7.5 kG could be produced at maximum current of 120 A. Five pairs of replaceable top metals of center plugs which change the gap at the center in 4 mm step were prepared to investigate the variation of the field in the central region.

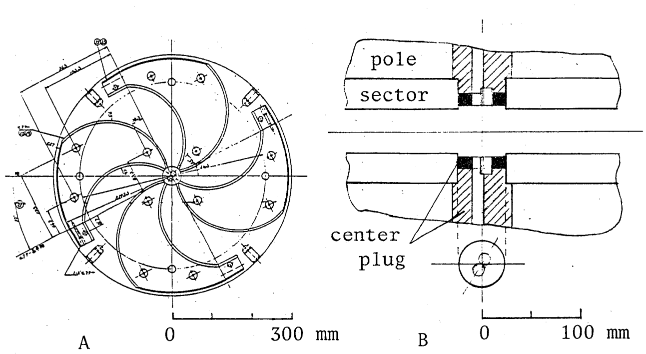


Fig. 1A Horizontal section of the model magnet. Only the pole and sectors are shown.
Fig. 1B Vertical section of the central region. The replaceable parts of center plugs are indicated by black.

## MAGNETIC FIELD

The magnetic field was measured with a Hall probe mounted on a X-Y driving table controlled by a personal computer. Field maps of 3 mm step on the median plane were collected for the area of 180mm by 180mm of the central region for a total of 15 different conditions of five center plugs and exciting current of 15,30 and 45 A. An example of the maps is shown in Fig 2.

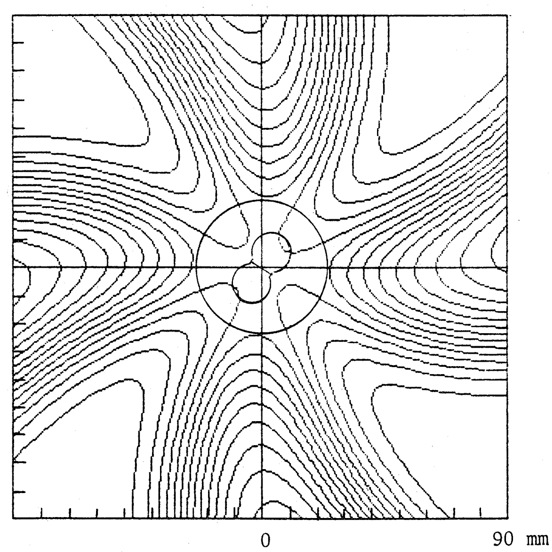


Fig. 2 Field map of the central region for center plug #3 with exciting current of 45 A.

The measured field $B_m(r,\vartheta)$ at radius r and azimuthal angle $\vartheta$ is expressed in Fourier series as

$$B_m(r,\vartheta)=\overline{B_m}(r)\left[1+\sum_{k=1}^{15} a_{4k}\cos(4k\vartheta)+\sum_{k=1}^{15} b_{4k}\sin(4k\vartheta)\right]. \quad (1)$$

The lowest order term $\overline{B_m}(r)$ expressing the average field are shown in Fig. 3 for five center plugs. The coefficients $a_{4k}$ and $b_{4k}$ are functions of radius r, and depend scarcely on the exciting current for the same center plugs. The difference between the field expressed in Fourier series and the measured one are at most 0.6 %.

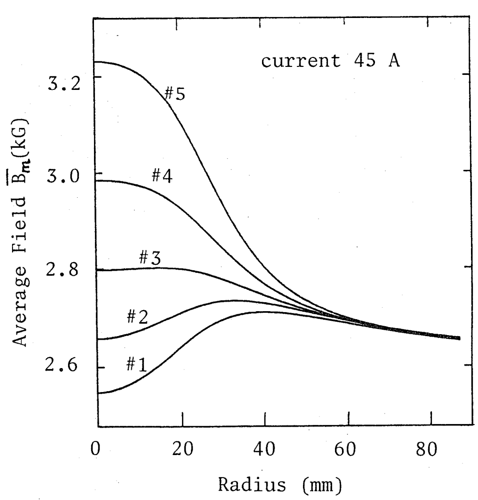


Fig. 3 Average magnetic field for five different center plugs of #1 to #5.

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Present Addresses:
* Ryomei-Giken Co. Ltd, 4-6-22 Kannon-shinmachi, Nishiku, Hiroshima, 730 Japan
** Mitsubishi Electric Corp., 8-1-1 Tsukaguchi-honmachi, Amagasaki, 661 Japan

## EQUILIBRIUM ORBIT

Using the Fourier series expression of measured field, we first calculate equilibrium orbits and betatron oscillation frequencies. Hereafter the length is always described in net scale.

The frequencies $\nu_r$ and $\nu_z$ of the radial and axial betatron oscillations for an equilibrium orbit are approximately given by analytical formulae[1] with expansion coefficients. The results for raw fields of five center plugs without isochronous conditions are shown in Fig. 4. For the radial oscillation, the frequencies are smaller than 1 in all cases for r>130 mm, but these values turn to be larger than 1 under the isochronous condition. For the axial motion, defocusing region appears for plugs #1,#2 and #3 in the central region.

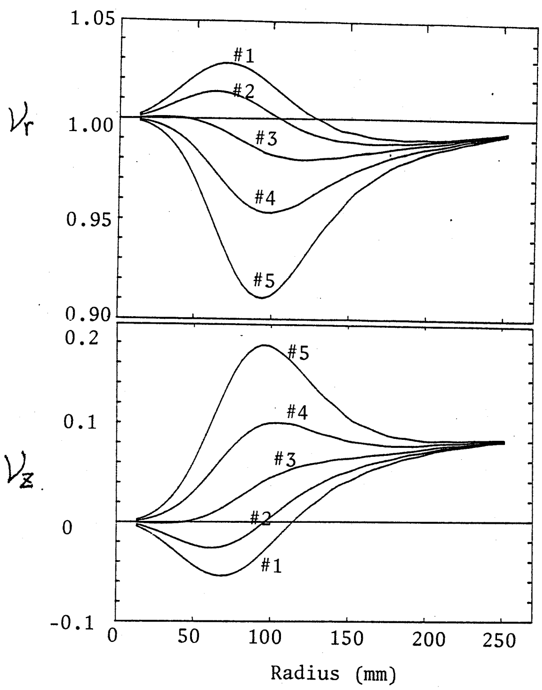


Fig. 4 Betatron oscillation frequencies for five center plugs without any field modification.

For further calculations the average field $\overline{B}(r)$ was adjusted to meet the acceleration condition. Here we discuss the case of accelerating protons up to 70 MeV at the extraction radius of 750 mm. An isochronous average field $\overline{B}_i(r)$ is uniquely determined if the coefficients $a_{4k}$ and $b_{4k}$ are assumed not to depend on $\overline{B}_m(r)$. The correction due to the deviation of orbit from a circle is included.

The field $B(r,\theta)$ adopted for orbit calculations was chosen as follows:

$$B(r,\theta)=\overline{B}(r)\left[1+\sum_{k=1}^{15} a_{4k}\cos(4k\theta)+\sum_{k=1}^{15} b_{4k}\sin(4k\theta)\right] \quad (2)$$

$$\overline{B}(r)=\begin{cases} c\overline{B}_m(r) & r<a \quad (3a) \\ [(b-r)c\overline{B}_m(r)+(r-a)\overline{B}_i(r)]/(b-a) & a<r<b \quad (3b) \\ \overline{B}_i(r) & b<r \quad (3c) \end{cases}$$

The boundaries a and b which define the central region are empirically determined so as to realize stable acceleration orbit. The values of a=81 mm, b=90 mm and c=5.5 with the center plug #3 are adopted in the following calculations. The betatron oscillation frequencies for this case are shown in Figs. 5A and 5B. Both frequencies increase in the isochronous region. The axial frequency $\nu_z$ is imaginary for r<50 mm. The electric focusing is effective in this region.

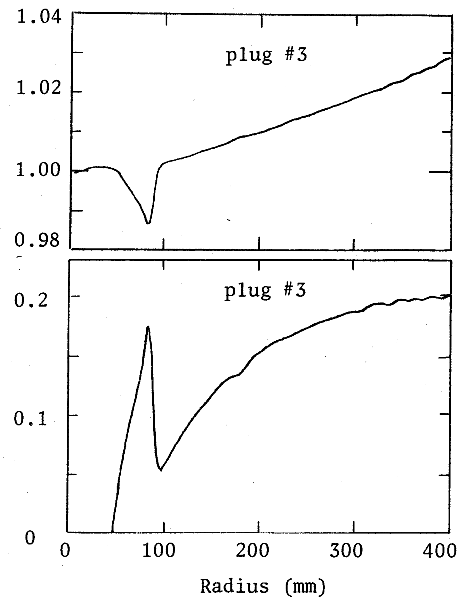


Fig. 5 Betatron oscillation freqencies for the adjusted field with the center plug #3.

## ACCELERATION ORBIT

The orbits of accelerated particles were calculated by integrating the equation of motion with the Runge-Kutta method. Fig. 6 shows the dee configuration adopted for the calculation.

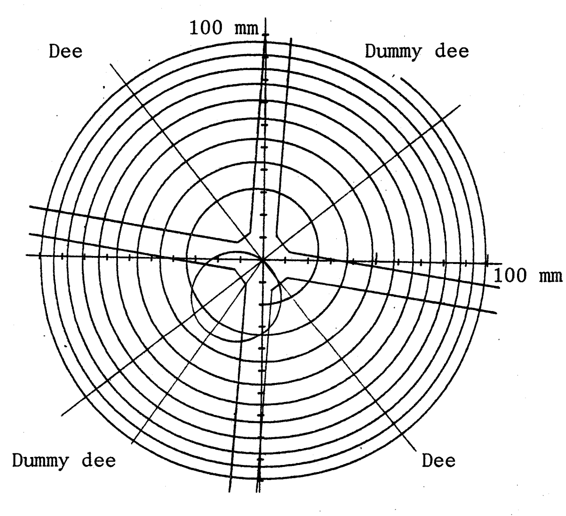


Fig. 6 Dee configuration and orbit of an accelerated particle. A circle near the center indicates the hole for ion sources. Parameters for the acceleration are listed in Table 1.

The electric field $E_y$ on the median plane for the dee voltage V can be approximated with the equations[2]

$$E_y=V/(\sqrt{2\pi}\sigma)\exp[-(y/\sigma)^2/2]\sin(\omega t+\varphi) \quad (4)$$

$$\sigma=0.4W+0.2H \quad (5)$$

where W and H are the gap width and the height of dee, for our case H=25 mm and W=10 mm, respectively. The error of this approximation is known to be less than 10 % for 0.3<W/H<3. The electric focusing is taken into consideration with the axial component of the electric field to be

$$E_z=(yz/\sigma^2)E_y \quad (6).$$

The isochronous average field $\bar{B}_i(r)$ was fixed for r>b as described in the previous section. The injection radius was also fixed at r=20 mm and the condition to accelerate injected protons stably from this point to the isochronous region was searched for various values of initial condition of injection, dee voltage and injection phase. The boundaries a and b and the factor c were also searched and the values in the previous section were chosen for the center plug #3. No successful condition was obtained for the plugs #1, #2, #4 and #5.

An example of parameters for a stable acceleration orbit with the plug #3 is listed in Table 1.

Table 1

| Parameters for acceleration orbit | | |
|---|---|---|
| Orbit frequency | 23.3 | MHz |
| Injection radius | 20 | mm |
| Injection angle | 0 | deg |
| Injection energy | 60 | keV |
| Acceleration voltage | 50 | kV |
| Injection phase | -37 | degree |

The orbit with these parameters is also shown in Fig. 6. The energy and the phase are shown in Figs. 7A and 7B. The particle come to the isochronous region after 8 turns for this case. Fig. 8A shows the variation of the axial oscillation of an accelerated off-center particle. Only the focusing by the electric field is effective for the region r<50 mm. This effect can be seen in the variation of the axial momentum shown in Fig. 8B. Small spike peaks correspond to the variation of $p_z$ at the passage of gaps. The effect is shown in Fig. 8C with the extended scale for the first three turns. Twelve times gap passage causes the first 1/4 period of the axial oscillation.

Though the transition from the central region to the isochronous region should be treated in more appropriate way, the program is conveniently used to find the proper condition for axial injection of other particles and expected to be connected with the one for phase space calculation.

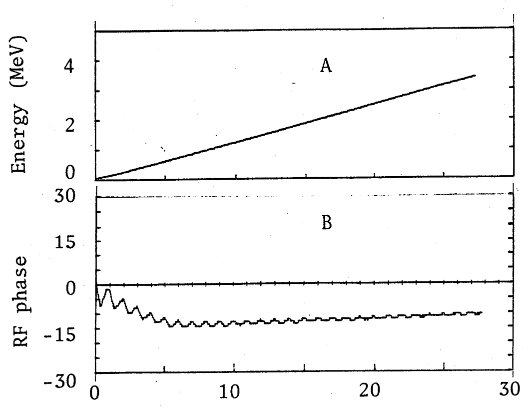


Fig. 7A Particle energy vs. turn number.
Fig. 7B RF phase vs. turn number.

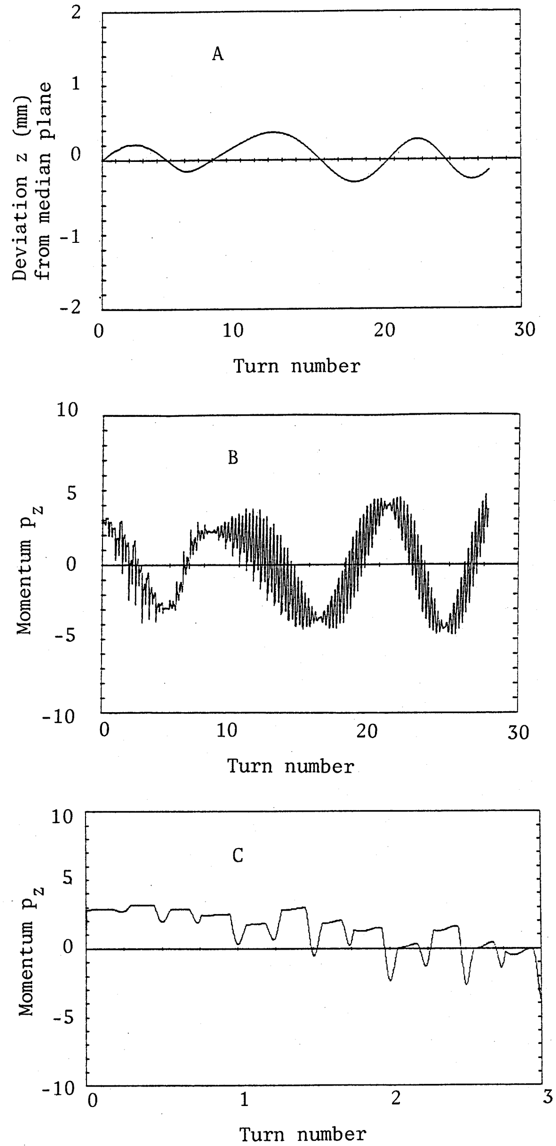


Fig. 8A Axial oscillation of an accelerated off-center particle.
Fig. 8B Momentum $p_z$ of axial motion of the particle of Fig. 8A.
Fig. 8C The above effect is shown in extended scale for first three turns.


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References


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