BEAM TRACKING OF A SMALL STORAGE RING

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Abstract

Conventional linear and isomagnetic approximation of beam optics is not sufficient for designing a small storage ring using superconducting bending magnets. A new tracking program PROVIDENCE by numerical integration method was developed. In this program the Lorentz equations formulated in the curved coordinate were transformed to a simple system of equations without any approximation. As a result, the calculating speed was improved.

This program was applied to the SXLS lattice of BNL's compact storage ring. A banana shape of superconducting coils was assumed. The ring parameters calculated by a linear optics program were significantly different from the results obtained by this exact formulation. Beam tracking by this program showed that the ring with our bending magnet has a sufficient dynamic aperture.

1 Introduction

There are many computer programs for single-particle beam dynamics in cyclic particle accelerators. Most programs [1][2] make approximations in order to gain a calculating speed. For example, they use the linear trajectory equations of motion, they assume the magnetic field in bending magnets as isomagnetic, and the model of fringing fields of bending magnets is simple.

These approximations are adequate for the large rings where the betatron oscillations are negligible compared to the bending radius and where bending magnets are isomagnetic. But these approximations cannot be applied for a small storage ring with around ten meter circumference.

A new tracking computer code PROVIDENCE for a small storage ring was developed. The accuracy of this program was examined [3] by comparing its calculated tunes and chromaticities with experiment using the LEAR lattice.

This program was applied to an actual small storage ring. The calculated ring parameters were compared with the parameters obtained by a conventional program. The dynamic aperture of this ring was also calculated by this program.

In this paper the formulation of the PROVIDENCE and the calculated results will be discussed.

2 Method

The equations of motion were transformed to a simple form so as to fit to the numerical calculation. In the PROVIDENCE these equations are precisely integrated by using adjusted step sizes and the real magnetic fields of the bending magnet are accurately simulated. These features of this program will be described in the following sections.

2.1 Formulations

The Frenet-Seret coordinate system[4] was used. The distance s along the reference orbit is taken as the independent variable of the equations of motion. In this paper the reference orbit means a path of an on-momentum particle through idealized magnets with no fringing fields. The x and y coordinates measure the horizontal and vertical transverse deviation of the actual orbit from the reference orbit.

In the Frenet-Seret coordinate system the transverse equations are obtained from the Lorentz equation[4]:

$$\begin{aligned} x'' + \frac{\ddot{s}}{\dot{s}^2}x' - \frac{1+1/\rho}{\rho} &= \frac{e}{m\dot{s}}\left(y'B_s - \left(1+\frac{x}{\rho}\right)B_y\right) && (1)\\ y'' + \frac{\ddot{s}}{\dot{s}^2}x' &= \frac{e}{m\dot{s}}\left(-x'B_s + \left(1+\frac{x}{\rho}\right)B_x\right). && (2) \end{aligned}$$

From conservation of energy, the following equation holds

$$\dot{s}^2 = \frac{v^2}{x'^2 + y'^2 + (1+x/\rho)^2}, \qquad (3)$$

where ρ is the radius of curvature, $\dot{}$ and $'$ denote the differentiation with t and s, respectively.

By using Equation (3), and we finally obtain the following equations:

$$\begin{aligned} x'' &= \frac{C_1}{a}\left(ay'B_s + x'y'B_x - (x'^2+a^2)B_y\right) + \frac{1}{\rho a}[2x'^2 + a^2] && (4)\\ y'' &= \frac{C_1}{a}\left(-ay'B_s - x'y'B_x + (x'^2+a^2)B_x\right) + \frac{1}{\rho a}[2x'y']. && (5) \end{aligned}$$

where a and C_1 are defined as follows:

$$\begin{aligned} a &\equiv 1+\frac{x}{\rho} && (6)\\ C_1 &\equiv \frac{e}{mv}\sqrt{x'^2 + y'^2 + (1+\frac{x}{\rho})^2}. && (7) \end{aligned}$$

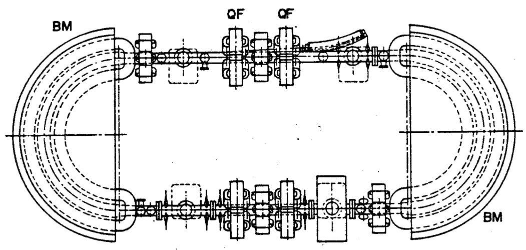

Figure 1: A schematic of the SXLS storage ring

Since these equations have simple forms which fit to the numerical integration, the CPU time was significantly reduced. The calculating speed was as fast as a two-dimensional tracking without this transformation.

2.2 Algorithm

The equations (4) and (5) are integrated with a fourth-order Runge-Kutta algorithm. A speed of the numerical integration would be slow if we used a constant integration step size which would be small in order to obtain the accurate results. Our program has a function of adjusting the variable step size to the local truncation error.

2.3 Magnetic Fields

The magnet fields B_s, B_x, B_y in equations (4) and (5) are calculated at every position of the particle. The field is interpolated by means of the three-dimensional spline functions. These coefficients are in advance calculated from the fileds at the sampled points computed with a 3D magnetic code.

3 Results and Discussions

Our program was applied to a small storage ring of the SXLS lattice[5]. A schematic of the SXLS storage ring is shown in figure 1. Following the original design, the energy and the bending radius are $696 MeV$ and $0.6m$, respectively.

3.1 Bending Field

Several parameters of the original design are not open. The most important unknown parameter is the bending field. We assumed the bending magnet consists of a pair of banana-shaped coils without an iron core.

The magnetic field of this bending magnet was computed by a 3D magnetic field analysis code. The calculated field on the reference orbit is shown in figure 2.

3.2 Twiss Parameters

Figure 3 shows the horizontal displacement of the closed orbit from the reference orbit calculated with the PROVIDENCE.

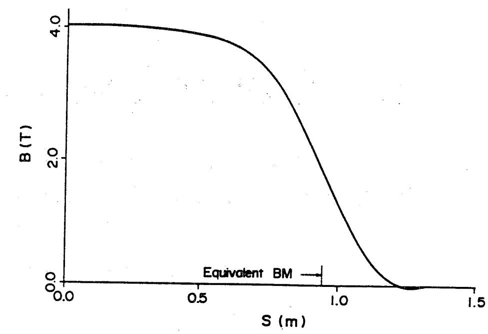

Figure 2: Field distribution along the reference orbit

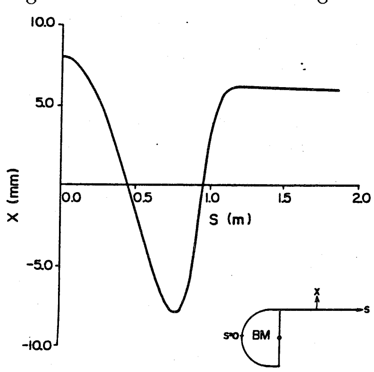

Figure 3: Displacement of the closed orbit from the reference orbit

When we design good field regions of superconducting bending magnets, we must take this difference into consideration.

The Twiss parameters of this ring were calculated by the PROVIDENCE and compared with the results by the MAD[1]. Since the MAD can only treat the isomagnetic field, the bending magnet was devided into subsections.

By the PROVIDENCE β functions were obtained from the ellipses of the phase space of the small amplitude oscillations. The betatron tunes were calculated with the fast fourier transformations.

Table 1 shows the horizontal and vertical tunes and the β functions at several points in the lattice. The maximum difference between the MAD and PROVIDENCE is 15 percent. Although the horizontal tunes agreed each other, the vertical tunes have significant discrepancy.

The differences of the lattice parameters between the results of the PROVIDENCE and the MAD may be attributed to the MAD's approximations of ignoring the B_x and B_s and identifing the closed orbit with the reference orbit.

This comparison suggested that the existing simulation codes like MAD cannot be applied to a small storage ring with superconducting bending magnets.

Table 1: Comparison of Twiss Parameters

	s[m]	PROV.	MAD	diff.
	0.0	0.486	0.489	+0.6%
	0.930	0.837	0.864	+3.0%
β_x	1.400	1.35	1.525	+13.0%
	1.931	2.372	2.729	+15.0%
	1.994	2.611	2.728	+4.0%
	0.0	6.729	6.335	-5.9%
	0.930	4.342	4.282	-1.4%
β_y	1.400	2.848	2.502	-12.0%
	1.931	1.549	1.326	-15.0%
	1.994	1.425	1.323	-7.2%
ν_x		1.409	1.397	-0.85%
ν_y		0.398	0.425	6.8%

3.3 Beam Tracking

The dynamic aperture of this ring was calculated by beam tracking. Since the PROVIDENCE can accurately track particles with large amplitude oscillations, this program is suitable for examining dynamic apertures.

The required good field region was obtained by calculating $10\sigma + COD$ by the SYNCH[2]. The good field region at the center of the bending magnet is $-4mm < x < 20mm$ [1] and $-20mm < y < 20mm$.

Figure 4 shows the phase spaces at the center of the bending magnet. The initial point of tracking is $x = 20mm$ and $y = 20mm$ which locates on the boundary of the needed good field region. Considering the forms of ellipses of figure 4, we conclude this ring has a sufficient dynamic aperture.

4 Conclusion

A new tracking program PROVIDENCE by numerical integration method was developed. The Lorentz equations in the Frenet-Seret coordinate system were transformed into simple forms without any approximation. As a result of these simple formulations the calculating speed was significantly improved.

This program was applied to a small storage ring with superconducting bending magnets whose coils were assumed to have banana-shape. The calculated twiss parameters calculated with this program were different from the values calculated by a linear optics code. The differece may comes from the consideration of s and x components of the bending field and from the large discrepancy between the closed orbit and the reference orbit.

The beam tracking by this code showed that this ring may have a sufficient dynamic aperture.

[1]The required good field region in x-coordinate is not symmetric due to the difference between the closed orbit and the reference orbit

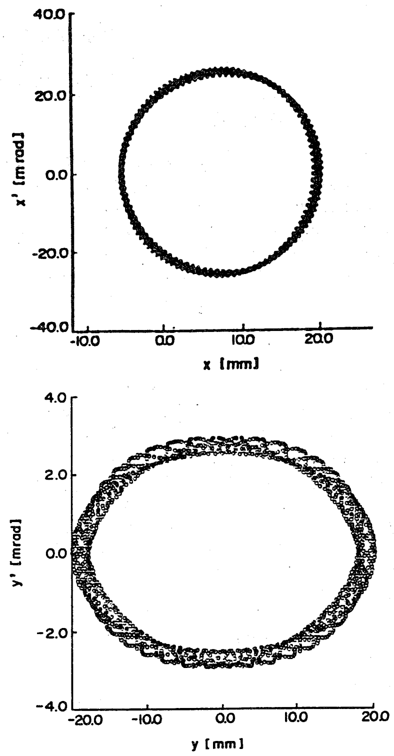

Figure 4: Phase spaces of the particle with maximum amplitude at the center of the bending magnet

References

[1] F.Christoph Iselin, **CERN-LEP-TH/85-15**, 1985.

[2] A.A. Garren et al., FNAL, **FN-420**, 1985.

[3] H.Tanaka and T.Nakanishi, Proc. Ann. Meet. of Phys. Soc. of Japan, 1989.

[4] K.G.Stefen, *High Energy Optics*, **Vol.17**, Interscience Publishers, New York, 1965.

[5] M.Q.Barton et al., **BNL 38789**, 1986.