Evaluation of the Wake Field in Detuned Structure

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Abstract

The dispersion and loss parameter of a uniformly loaded structure were calculated by using a double chain of coupled circuit. The results agree well with those calculated by a direct electromagnetic field solver, MAFIA. Using this equivalent circuit model, the wake field in a detuned structure with lossy cells was calculated. It was found that the wake field in such a structure can be damped by two orders of magnitude during a very long period by reducing the Q value of every cell below 1000.

I. INTRODUCTION

In order to achieve a high luminosity of a few times 10^{34} $cm^{-2}sec^{-1}$, a goal of Japan Linear Collider (JLC)[1], a multi bunch operation is being planned. In this operation, the long-range transverse wake field during the passage of 90 bunches, between 1.4 nsec and 126 nsec, must be damped by two orders of magnitude compared with the amount excited by the preceding bunch. Two types of structures have been studied for this purpose. One of them is a damped structure[2,3] and the other a detuned structure[2]. The former is described in other paper while the latter is a main theme in this paper.

The wake field in a resonant cavity can be treated as the sum of the contributions from its resonant modes. In the case of the disk-loaded accelerating structure, it is really a good approximation that the long-range transverse wake field is expanded in terms of its dipole modes below the cut off of the beam pipe[4]. The amplitude of the wake field in each mode is in proportion to R/Q or loss parameter. If a proper frequency distribution of the loss parameters is realized, the wake field will be sufficiently cancelled after the period which is the inverse of the width of the frequency distribution. As an example to realize this idea, a detuned structure was studied in the present paper.

The code using a mesh method can not calculate the loss parameter of such a many-cell structure as a detuned structure consisting of more than a hundred cells whose dimensions change by a few tens μm from cell to cell. On the other hand, a coupled resonator model can be a good approximation because the coupling from cell to cell is not so large. Various authors have been trying to analyse such a structure using equivalent circuit model[5,6,7]. In the present paper, the wake fields were calculated by using a double chain of coupled circuit.

K. F. L. Bane et al. calculated the effect of the lossy cells on the wake field in a lossy detuned structure using a perturbation approach [5]. In the present calculation, however, the loss was treated explicitly using the circuit model with register in each cell. Thus even at very low Q value, the loss parameter can be calculated.

II. HOW TO CALCULATE THE WAKE FIELD

Expansion of wake field into modes

The long-range transverse wake field $W_\perp(t)$ is represented by the summation of the dipole modes as[8]

$$W_\perp(t) = \sum_m \frac{2cK_m}{a^2\omega_m} sin\left(\omega_m t\right) exp(-\frac{\omega_m t}{2Q_m}) \quad (1),$$

where subscript m represents the m'th mode, c the speed of light, a the beam hole radius, ω_m the angular frequency, Q_m the quality factor and K_m the loss parameter. The loss parameter K_m is defined as

$$K_m = \frac{\left|\int_{-\infty}^{\infty} dz E_{zm}(z) exp(\frac{i\omega_m z}{c})\right|^2}{4U_m} = \frac{1}{4}\frac{R_m}{Q_m} \quad (2),$$

where $E_{zm}(z)$ is the longitudinal electric field at beam hole radius, U_m the stored energy and R_m the shunt impedance.

Equivalent circuit model describing two lowest dipole modes

The loss parameter in the lowest dipole mode is five times as large as the second largest one in a typical disk-loaded structure at X-band[4]. Therefore, it seems a good approximation that the wake field is composed of the lowest dipole mode alone. However, the lowest dipole mode at 0-mode changes from TM110-like field for the beam hole radius below 3.9 mm to TE111-like one above, while the order at π-mode does not change[9]. Considering this situation, the field in the lowest dipole mode should be treated including both TM110-like mode and TE111-like one at the same time.

Therefore, it was assumed that the fields in two lowest dipole modes consisted of the mixing of both fields of TM110-like and TE111-like mode. These modes are coupled with each other and can be expressed by a double chain of coupled circuit model as shown in Fig.1[5].

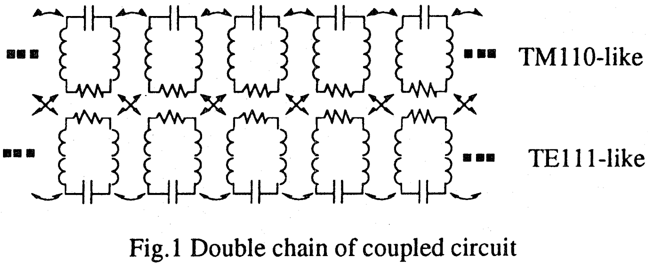


Fig.1 Double chain of coupled circuit

Each resonator in Fig.1 corresponds to a mode in a cell. It is coupled to the next resonator with some coupling coefficient. There is no coupling, however, between the two modes in the same cell because of the different symmetry of the electromagnetic fields of those modes.

This circuit model is expressed by the following equation

$$\left([M]\omega^2+[C]\omega+[K]\right)\{x\}=\{0\} \qquad (3).$$

The eigen value ω and the eigen vector $\{x\}$, which represent the frequency and the current in the circuit, correspond to the resonant frequency and the intensity of the field of a mode, respectively. If the structure consists of N cells, the matrices $[M]$, $[C]$, $[K]$, representing the coupling, the Q value and the resonant frequencies, respectively , become 2N×2N matrices as

$$M_{ij}=\begin{array}{cc} i\le N & i>N \\ \left\{\begin{array}{cc} 1 & j=i \\ -\dfrac{\kappa_{M\pm}}{2} & j=i\pm 1 \\ \mp\dfrac{\sqrt{\kappa_{M\pm}\kappa_{E\pm}}}{2} & j=N+i\pm 1 \\ 0 & \text{others} \end{array}\right. & \left\{\begin{array}{cc} 1 & j=i \\ \dfrac{\kappa_{E\pm}}{2} & j=i\pm 1 \\ \pm\dfrac{\sqrt{\kappa_{M\pm}\kappa_{E\pm}}}{2} & j=i\pm 1-N \\ 0 & \text{others} \end{array}\right. \end{array} \qquad (4),$$

$$C_{ij}=\frac{\Omega_i}{Q_i}\delta_{ij} \qquad (5),$$

$$K_{ij}=\Omega_i^2\delta_{ij} \qquad (6),$$

where the κ_{M-}, κ_{M+}, κ_{E+} and κ_{E-} are the coupling coefficients. The subscripts $_M$ and $_E$ denote the TM110-like mode and TE111-like mode, respectively, while the subscripts $_-$ and $_+$ the coupling to the left side cell and to the right, respectively. The parameter Ω is the resonant frequency without coupling and Q the Q Value. These parameters were calculated by using URMEL[10].

III. CALCULATION OF INFINITELY PERIODIC CASE

To examine the above modeling, the single cell behaviors were calculated and compared to the direct calculation using the field calculated by MAFIA[11]. In this calculation, the boundary condition was set to be periodic and the Q value was assumed to be infinite. The beam hole radia of a=5 mm, 4 mm and 3mm correspond to the cells near input, middle, and output region in a detuned structure, respectively. The calculated dispersion and the loss parameters of the forward and backward component for each standing wave mode were shown in Fig.2. It was found from the figure that the results of the present circuit model agreed very well with those by direct calculation in all over the phase advance per cell from 0 to π. Similar calculation was performed by K. L. F. Bane et al.[5], where the kick factors (or the loss parameter) does not agree well with the calculation by TRANSVRS[12] at far from the synchronous mode. This disagreement comes from the neglection of the loss parameter from TE111-like mode whose loss parameter is small compared to TM110-like's one. In the present calculation of the loss parameter in eq. (2), the field in the TE111-like mode was also included.

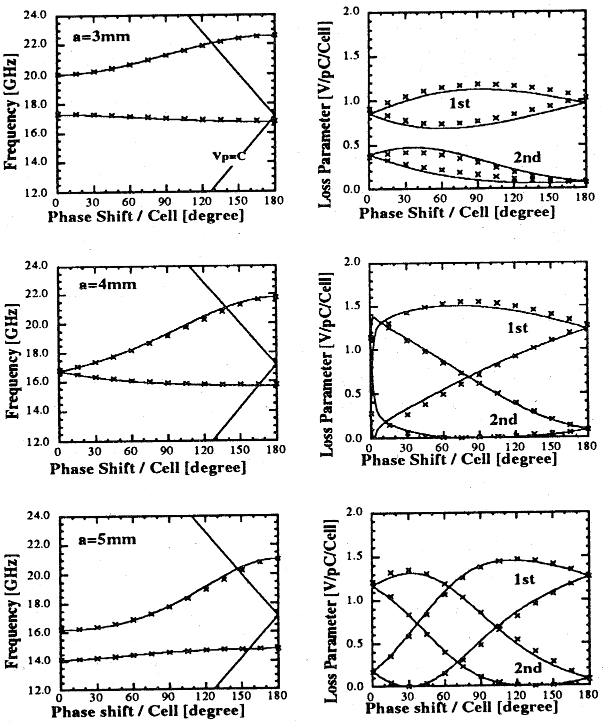


Fig.2 The dispersion relation and the loss parameter for single cell. Solid lines show the equivalent circuit model, while crosses the direct calculation using the field by MAFIA.

IV. WAKE FIELD IN A LOSSY DETUNED STRUCTURE

The long-range transverse wake fields in a detuned structure with lossy cells was calculated by using the double chain of coupled circuit model with register in Fig.1. In this calculation, the frequency distribution of the lowest dipole mode of all the cells were set to be a truncated Gaussian distribution, the same as reference [7], where the center frequency, the standard deviation and the full width were 15.6 GHz, 0.35 GHz and 1.7 GHz, respectively. In addition, the Q value of the normal cell was assumed to be 6000. The result is shown Fig.3(a).

It is difficult to keep the wake field low in a single detuned structure during a long period of a bunch train, 126 nsec. It is because the wake field is recovered at a recurrence time, at 120 nsec in the figure, which is roughly the inverse of the frequency spacing. In this case, the intrinsic Q values of 6000 does not help much to damp the wake field even at 120 nsec later. One of the possible solutions to get rid of this problem is to use the same type of structures with slightly different frequencies to make the effective frequency spacing narrow and the recurrence time long. Another possible solution is to make the detuned structure lossy. In the latter structure, the wake field at relatively short time range is damped owing to the detuning while that at long range the loss in a cell becomes effective.

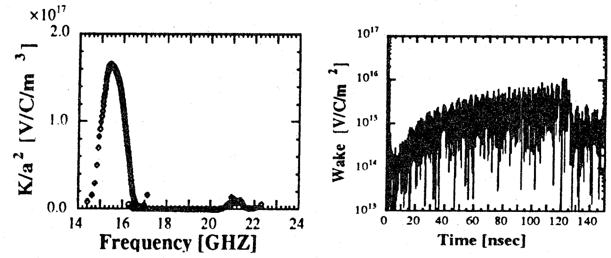
The wake field and the loss parameter distribution in various detuned structures with lossy cells were calculated. Three examples are shown in Figs.3(b), (c) and (d). Following characteristics were found from these results.

(1) If the structure consists of only lossy cells with the Q value larger than 1200, the largest wake is determined by the damping due to the lossy cells. On the other hand, in the case of the Q value less than 1200, the maximum is at 1.4 nsec, corresponding to the next bunch, and determined by the frequency distribution. This situation is shown in the solid circles Fig.4.

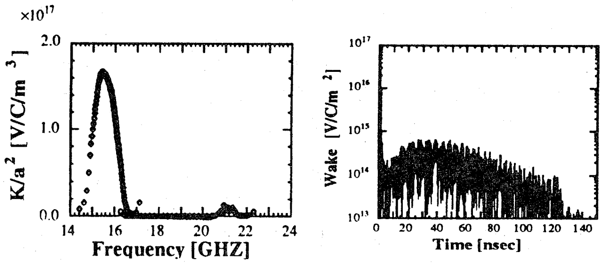
(2)At very low Q value, below a few tens, the initial wake changes from the normal case of 1×10^{17} V/C/m^2 as shown in Fig.3(c). This implies the limitation of the present circuit model approach.

(3) The wake field in a structure with very lossy cells in every several cells can not be damped enough as shown in Fig.3(d). It is because the frequency distribution of the loss parameter is largely perturbed due to these lossy cells.

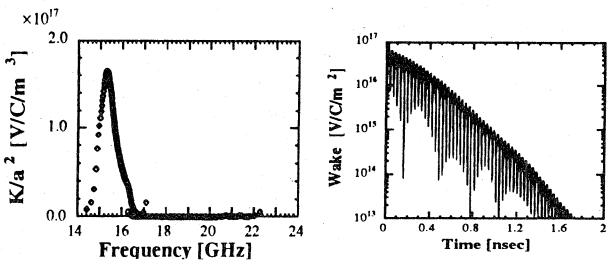
It was found from these analyses that the best structure with lossy cells is to make all the cells lossy with the Q value around 1000. The loss parameter distribution and the wake field in this structure are shown in the Fig.3(b). The wake field in such a structure can easily be damped by two orders of magnitude during the long period of 126 nsec.



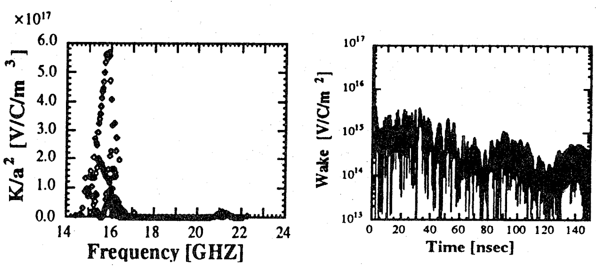
(a)



(b)



(c)



(d)

Fig.3 The loss parameter distribution and the wake field. (a): all the cells with Q values of 6000, (b): all the cells with Q values of 1000, (c) all the cells with Q values of 20 and (d): every fifth cell with Q values of 50.

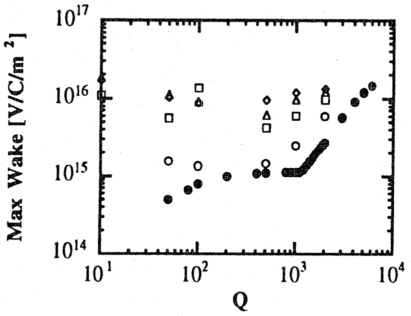


Fig.4 Dependence of wake field on Q value. Vertical axis is the largest wake field from 1.4 nsec to 126 nsec. Solid circle shows the structure consisting of only lossy cells. Open circle shows the structure with every second cell lossy, square every fifth, triangle every tenth and diamond the every twentieth.

V. CONCLUSION

By including the loss parameter of TE111-like mode, the loss parameter of a uniformly loaded structure can be estimated very precisely over the phase advance per cell from 0 to π using a double chain of coupled circuit.

The wake field can be damped by two orders of magnitude over 90 bunches in a detuned structure by making the Q values of all the cells around 1000.

VI. REFERENCES

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