An Idea of Dynamical Cooling of Electron Beam in SR Ring

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A beam cooling method which uses a stimulated emission of radiation by relativistic electrons in a magnetic field, is proposed to apply to an electron beam in synchrotron radiation rings. The stimulation is produced by a traveling RF electric field in a transmission line against the beam. Averaging the emission power over electron energy distributions, the net emission from the beam is taken place in a certain frequency region. Some conditions for the energy emission are discussed. The emission power is shown to be enough large to decrease the horizontal emittance of the beam to a very small value in typical synchrotron radiation rings.

I. Introduction

The equilibrium horizontal emittance of the electron beam in a synchrotron radiation (SR) ring is determined by competition of the quantum excitation and radiation damping [1]. The emittance scales like θ^3 where θ is a deflection angle per bending magnet [2]. Therefore, in principle, an arbitrarily small value of the emittance can be reached if the deflection angle is chosen small enough. This trend, however, is inevitably limited by the inability to correct chromaticities which restrict beam stability.

II. Principle of the method

As a possible alternate method to decrease the emittance, we propose a method based on a stimulated emission of radiation from a cyclotron motion of electrons in a magnetic field. The emission is caused by a relativistic quantum effect in a certain frequency condition of an incident radiation [3].

The cyclotron motion of an electron in a uniform magnetic field B is relativistically described by Dirac equation [4] which leads the kinetic energy levels

$$W_i = m_e c^2 \sqrt{1 + \frac{2\left(i + \frac{1}{2}\right)\hbar\omega_c}{m_e c^2}} - m_e c^2$$

for the motion in a perpendicular plane to the magnetic field, where m_e is the electron mass and $\omega_c = eB/m_e$ is the cyclotron frequency of the non-relativistic electron. The transition frequency $\omega_{i,i+1}$ between state i+1 and i can be obtained by expanding the square root up to the second order as

$$\omega_{i.i+1} = \omega_c \left(1 - \frac{i\hbar\omega_c}{m_e c^2}\right) .$$

The matrix element for the electric dipole transition between the states i and i+1 is

$$\mu_{i,i+1} = e\sqrt{\frac{(i+1)\hbar}{2 m_e \omega_c}}$$

The transition probability $w_{i,i+1} = w_{i+1,\,i}$ between the state i and i+1 is given as

$$w_{i,i+1} = \frac{E^2 (\mu_{i,i+1})^2}{\hbar^2} g(\omega, \omega_{i,i+1})$$

where E is the amplitude and ω the frequency of the applied electric RF field. The frequency response function $g(\omega, \omega_{i,\,i+1})$ of applied RF electric field can be assumed to be Lorentzian for a rather large debunching time τ as

$$g(\omega, \omega_{i,i+1}) = \frac{\tau}{1 + (\omega_{i,i+1} - \omega)^2 \tau^2} .$$

The net power transfer is

$$p_i = \hbar(\omega_{i,i+1} w_{i,i+1} - \omega_{i,i-1} w_{i,i-1}) \quad ,$$

which can be either plus or minus depending on the ω. The negative value means the emission of power from the electron. In the formula, the frequency between state i and i–1 is slightly larger than $\omega_{i,i+1}$ and can be written as $\omega_{i,i-1} = \omega_{i,i+1} + \alpha\omega_c$. Summarizing above relations, the net transfer of power per electron with kinetic energy W is written as

$$p(\omega) = \frac{e^2 \tau E^2}{m_e} \frac{W}{2\ \hbar\omega_c} \left(\frac{1 + \frac{\hbar\omega_c}{W} - \alpha}{1 + x^2} - \frac{1}{1 + (x + \alpha Q)^2} \right) \qquad (1)$$

where $Q = \tau\omega_c$, $\alpha = \hbar\omega_c / m_e c^2$ and

$$x = \tau\left(\omega_c \left(1 - \frac{W}{m_e c^2} \right) - \omega \right) .$$

Originally the power transfer (1) is not for single electron but for an ensemble of monoenergetic electron. However if electrons gyrate the cyclotron orbit in same phase, it may be used for single electron. A phase bunching mechanism is shown classically to arise from the frequency change due to the relativistic mass effect on the cyclotron gyration [5]. The mechanism is shown to be justifield by numerical calculation of the phase history of an electron gyration.The bunched electrons emit their energy coherently.

Hereafter we consider an electron in a beam bunch in an application of the phenomena to SR rings. In the application, the magnetic field is applied longitudinally and the RF field transversally to the electron beam. The quantities in the co-moving frame with the electron bunch are specified by the upper bar of them.

III. Average over energy spreads

In order to obtain an effective power transfer of an electron in a synchrotron radiation ring, the power transfer have to be averaged over longitudinal and transverse energy spreads of the electrons in the beam. The longitudinal energy spread fields a spread $\Delta\overline{\omega}$ of the applied frequency $\overline{\omega}$. The frequency spread is given as $\Delta\overline{\omega}/\overline{\omega}=\Delta\gamma/\gamma$ where γ is the Lorentz factor for the central electron energy and $\Delta\gamma$ is the spread. Because of the energy spread, an electron feels an RF field with a frequency $\overline{\omega}'$ which distributes around $\overline{\omega}'$. We take a uniform distribution

$$f(\overline{\omega}')=1/2\Delta\overline{\omega} \qquad \text{for} \quad \overline{\omega}-\Delta\overline{\omega}<\overline{\omega}'<\overline{\omega}+\Delta\overline{\omega}$$

$$=0 \qquad \text{otherwise}$$

to make the calculation easy. The power transfer averaged over the frequency distribution is

$$\begin{aligned}<\overline{p}(\overline{\omega})>&=\int\overline{p}(\overline{\omega}')f(\overline{\omega}')d\overline{\omega}'\\
&=\frac{e^2\overline{E}^2}{2m_e}\frac{\overline{W}}{\hbar\overline{\omega}_c}\frac{1}{2\Delta\overline{\omega}}\left[\left(1+\frac{\hbar\overline{\omega}_c}{\overline{W}}-\alpha\right)\times\right.\\
&\times\left\{\tan^{-1}\overline{\tau}\left(\overline{\omega}_c\left(1-\frac{\overline{W}}{m_ec^2}\right)-\overline{\omega}+\Delta\overline{\omega}\right)\right.\\
&\left.-\tan^{-1}\overline{\tau}\left(\overline{\omega}_c\left(1-\frac{\overline{W}}{m_ec^2}\right)-\overline{\omega}-\Delta\overline{\omega}\right)\right\}\\
&-\tan^{-1}\overline{\tau}\left(\overline{\omega}_c\left(1-\frac{\overline{W}}{m_ec^2}\right)-\overline{\omega}+\Delta\overline{\omega}+\alpha\overline{\omega}_c\right)\\
&\left.+\tan^{-1}\overline{\tau}\left(\overline{\omega}_c\left(1-\frac{\overline{W}}{m_ec^2}\right)-\overline{\omega}-\Delta\overline{\omega}+\alpha\overline{\omega}_c\right)\right].\end{aligned}$$

Furthermore, the power transfer has to be average over the transverse kinetic energy $\overline{W}$. The distribution function of $\overline{W}$ in an electron ring is an exponential one as [1]

$$g(\overline{W})=(1/\overline{T})\exp(-\overline{W}/\overline{T}),$$

where $\overline{T}$ is some mean transverse kinetic energy of electrons in the beam. The average power transfer at $\overline{\omega}$ is

$$<\overline{p}>=\int_0^\infty<\overline{p}(\overline{\omega})>g(\overline{W})\,d\overline{W}.$$

The power transfer is function of the frequency of applied field $\overline{\omega}$ and can be plus or minus depending on the frequency. It is complicated to express the function at an arbitrary frequency. At the frequency $\overline{\omega}=\overline{\omega}_c+\Delta\overline{\omega}$, where $\overline{\omega}_c=eB_L/m_e$, B_L the longitudinal magnetic field, the power transfer is simplified to

$$<\overline{p}>=\frac{e^2\overline{E}^2}{2m_e}\frac{1}{\hbar\overline{\omega}_c}\frac{1}{2\Delta\overline{\omega}\overline{T}}\times$$

$$\times\int_0^\infty\overline{W}\left[\left(1+\frac{\hbar\overline{\omega}_c}{\overline{W}}-\alpha\right)\left\{-\tan^{-1}\frac{\overline{\tau}\overline{\omega}_c\overline{W}}{m_ec^2}+\tan^{-1}\overline{\tau}\left(2\Delta\overline{\omega}+\frac{\overline{\omega}_c\overline{W}}{m_ec^2}\right)\right\}\right.$$

$$\left.+\tan^{-1}\overline{\tau}\left(\frac{\overline{\omega}_c\overline{W}}{m_ec^2}-\alpha\overline{\omega}_c\right)-\tan^{-1}\overline{\tau}\left(2\Delta\overline{\omega}+\frac{\overline{\omega}_c\overline{W}}{m_ec^2}-\alpha\overline{\omega}_c\right)\right]\exp\left(-\frac{\overline{W}}{\overline{T}}\right)$$

$$d\overline{W}.$$

Since in electron rings, the condition $\alpha\overline{\omega}_c\tau=\alpha Q<<1$ is always satisfied, we can expand the integrand function to the Taylor series. We take up to second order terms in the series, then power transfer is lead to for $\overline{\tau}\Delta\overline{\omega}>>1$

$$<\overline{p}>=-\frac{\pi e^2\overline{E}^2\hbar\overline{\omega}_c}{16m_e\overline{T}\Delta\overline{\omega}}\left[1-\frac{2m_ec^2}{\pi\hbar\overline{\omega}_c\overline{Q}}ze^zE_1(z)\right] \qquad (3)$$

where $\overline{Q}=\overline{\tau}\overline{\omega}_c$,

$$z=2m_ec^2\Delta\overline{\omega}/\overline{T}\overline{\omega}_c, \qquad (4)$$

$$E_1(z)=\int_z^\infty\frac{e^{-t}}{t}dt.$$

Otherwise for small value of $\overline{\tau}\Delta\overline{\omega}$, the power transfer at $\overline{\omega}=\overline{\omega}_c$ is calculated directly using eq. (1) to be

$$<\overline{p}>=-\frac{e^2\overline{E}^2}{4m_e}\frac{\overline{\tau}\hbar\overline{\omega}_c}{\overline{T}}\left[1-\frac{2\left(m_ec^2\right)^2}{\hbar\overline{\omega}_c\overline{T}\overline{Q}^2}\log\left(\frac{\overline{T}}{m_ec^2}-0.577\right)\right].$$

This is not in actual cases however. Either way, the characteristic cooling time of the betatron oscillation in the co-moving frame is

$$\overline{\tau}_c=\overline{T}/<\overline{p}>.$$

A negative value of $\overline{\tau}_c$ indicates the beam is cooled rather than heated.

IV. Estimation of the parameters

Now we express $\overline{T}$ as a function of some ring parameters. Here we take a usual coordinate system s, x and z to specify the position of an electron. Then the width of transverse kinetic energy $\overline{T}$ in the moving frame is given by

$$\overline{T}=T_x+T_z$$

In actual ring, T_z is much smaller compared to T_x. The typical value of T_x is expressed as [6]

$$T_x=m_ec^2\gamma^2\varepsilon_x/2\pi\beta_x$$

where β_x is the horizontal betatron function in the region of the RF field. We give typical values of the quantities for Advanced Light Source (ALS) ring at Lawrence Berkeley Laboratory [7]. The T_x is 8.6×10^2 eV for the beam energy $m_ec^2g=1.5$Gev for $\varepsilon_x=4.08\times10^{-9}\pi$m.radian and $b_x=11$m for the ring. The value of T_x is taken to be the mean energy $\overline{T}$ of the transverse motion in the beam.

The RF electric field is produced as a traveling wave in the parallel plate transmission line in the beam tube against the electron beam. The fields in the laboratory frame are, so to speak, compressed in the co-moving frame and the strengths

are amplified by a factor 2γ for extremely relativistic electron energy [8].
The frequency of the RF field in the laboratory frame ω is multiplied also by the same factor in the co-moving frame as $\overline{\omega}=2\gamma\omega$. The cooling time in the co-moving frame is elongated by a factor γ in the laboratory system by a relativistic effect.

Next we will consider the value of the debunching time $\overline{\tau}$ of the cyclotron motion in the transverse plane. The main source to reduce the phase breaking time comes from a spread of cyclotron rotation angle produced by energy spread of electron in the beam. The source is represented as the first term of the following equation of the breaking time $\overline{\tau}$ [9]

$$\frac{1}{\overline{\tau}}=\frac{\overline{\omega}_c}{\pi B_L L}\left\{-\frac{\Delta\gamma}{\gamma}\oint B_s(0,0,s)ds+\oint\Delta B_s(x,z,s)ds\right\} \quad (5)$$

for extremely relativistic electron beam, where L is the length of the longitudinal magnetic field and $B_S(x,z,s)$ the variation of the field. Since the first term in the bracket is too much large for usual relative energy width $\Delta\gamma/\gamma$ of electron beam in a ring in application to the master cooling, we propose to cancel out the term by introducing a compensating solenoidal magnet in the ring. We make the field direction of the compensating magnet to be opposite with that of the longitudinal magnet field shown as Fig. 1 of ref. [9].

The second term denotes the small field variation of longitudinal field components of the two magnet. The field variation has an appreciable effect on the debunching of the cyclotron motion. However any beam dynamical effects of the field may not cause the pile up of the field integral for a ring which has a symmetric lattice structure. Debunching of the motion may occur from the longitudinal component in the fringing fields of the solenoidal and lattice magnets. However the electron receives the fields within a very short time duration in the co-moving frame and the effect on the cyclotron motion may be much attenuated by an adiabatic consideration of the motion. Further, the sum of the effect cancels out for each magnet. Intra-beam scattering in the bunch is estimated to give an insignificant effect on the debunching.

Finally we will estimate the lower limited of the $\overline{\tau}$ required. In the first place, the parameter z of eq. (4) is calculated to be 0.358 for the ALS ring parameters $\overline{T}$=860 eV, $\Delta\overline{\omega}/\overline{\omega}_c=3\times10^{-4}$ and $m_ec^2=5.11\times10^5$ eV and it leads the value of the function $ze^zE_1(z)$ of 0.4. We adopt 10 Tesla for B_L. It leads the cyclotron frequency $\overline{\omega}_c=1.76\times10^{12}$ rad/sec and the corresponding quantum energy $\hbar\overline{\omega}_c$ = 1.16×10^{-3} eV. The condition of energy emission is that the value in the square bracket of eq. (3) is positive. It leads a lower limit of $\overline{\tau}$. The lower limited of $\overline{\tau}$ which leads energy emission can be derived from eq. (3) with above parameters to be

$$\overline{\tau}>10^{-4}\text{ sec}$$

for the typical example. The condition may be fulfilled when the compensation less than 10^{-4}. This may be technically possible. The condition is almost independent of $\overline{T}$ and $\Delta\overline{\omega}/\overline{\omega}_c$ for $z \gtrsim 0.5$.

V. Conclusions and remarks

We are now in position to estimate the cooling time $\overline{\tau}_c$ of the betatron oscillation. A rough estimate of the time is the first factor of $<\overline{p}>$ in eq. (3) as

$$\overline{\tau}_c \sim -\frac{16m_e\overline{T}^2\Delta\overline{\omega}}{\pi e^2\overline{E}^2\hbar\overline{\omega}}$$
$$= -1.1\times10^4\,\frac{\overline{T}^2(\text{eV})}{\gamma^2E^2}\frac{\Delta\overline{\omega}}{\overline{\omega}}\ \text{sec} \quad (6)$$

where E is the amplitude of RF electric field in laboratory system in V/m. In our example, the values of quantities in eq. (6) are $\overline{T}$=860 eV, $\gamma=3\times10^3$, $\Delta\overline{\omega}/\overline{\omega}_c=3\times10^{-4}$ and the cooling time is estimated to be 2.7×10^{-9} sec for $E=10^4$ V/m. The time is enogh short to decrease the transveerse emittance in particle electron rings. For the electron field, the power transmitted through the parallel plate transmission line is some 10 0W and the loss is about 0.01 W/m in usual design of the line. The loss of this extent will be tolerated in the superconducting solenoidal magnet.

Though the cooling time of the dynamic method seems to be able to be shorrt as much as one like using as RF field of large amplitude, the time in the laboratory frame cannot decrease below an inverse electron frequency $1/\omega_c=2\gamma/\overline{\omega}_c$ which is order of 10^{-9} sec in this example by an adiabatic condition.

The electron beam is cooled only in the RF field with the longitudinal magnetic field. The electron motion in the remainder of the ring does not affect the transition between the Landau levels in the solenoid. Consequently the transition rate reduce by a factor L/C where L is the solenoid length and C the circumference of the ring. Thus the cooling time constant $\overline{\tau}_c$ has to be multiplied by a factor C/L to obtain the effective cooling time constant. The factor is 100 if the length L is assumed to be 1% of the C. In order to continue the cyclotron rotation successively, a suitable phase locking consideration is necessary to the betatron motion in the ring.

Though the cooling time constant in the co-moving frame has to be multiplied by the product of two factors γ (3×10^3) and C/L (10^2) to obtain the laboratory one, the time is yet much shorter than the radiation damping time which is typically some 10 msec in usual SR rings. Since the transverse emittance of the beam decrease in proportional to the cooling time [1], we can obtain an electron beam of very small emittance in an electron ring by the dynamical method.

VI. References

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