Tune Change due to Ripple of Magnetic Field in a Synchrotron of Combined Function Lattice

Kazuo HIRAMOTO and Akira NODA*

Hitachi Research Laboratory, Hitachi,Ltd., 7-2-1 Omika-cho,
Hitachi-shi, Ibaraki-ken, 313, Japan

*Accelerator Research Facility, Institute for Chemical Research,
Kyoto University, Gokano-sho, Uji-shi, 606, Japan

Abstract

Tune change due to ripple of magnetic field in a synchrotron of combined function lattice is formulated. It is shown that the tune change is proportional to the ripple of the dipole magnetic field which is assumed to be the same as that of quadrupole magnetic field. The reductions of the betatron and dispersion functions are effective for decreasing the ripple of the tune. It is also shown that the tune change due to the ripple of the dipole magnetic field can be vanished by the sextupole magnetic field chosen adequately, the strength of which is equal to that for the chromaticity correction.

1 Introduction

High energy ion beam has been applied to cancer treatment because it shows rather sharp Bragg peak in human tissue. In ion synchrotrons for medical use, the treatment has been performed by the slowly extracted beam utilizing growth of the amplitude of the betatron oscillations caused by nonlinear resonance[1]. The nonlinear resonance of the betatron oscillations are generated when the amplitude of the betatron oscillations reaches the separatrix of the nonlinear resonance, which is determined by the tune, that is, the number of the betatron oscillations in a synchrotron, and the strength of the nonlinear magnetic field such as sextupole or octupole magnetic field. If the separatrix of the nonlinear resonance varies due to the ripple of the magnetic field, the beam extraction becomes intermittent. In the treatment by the beam scanning, the control to irradiate the target tissue homogeneously becomes complicated. Then, research and development to reduce the current ripple of the magnets have been performed to prevent the intermittent extraction, that is, beam spill ripple[3]. It has been considered that the ripple of the quadrupole magnetic field is the main cause of the beam spill ripple. However,it has been also shown that the ripple of the dipole magnetic field causes the beam spill ripple and the reduction of the dipole magnetic field further improves the time structure of the extracted beam[3].

On the other hand, we have proposed a synchrotron of combinefd function lattice for simplification of the operation for medical use[2]. In the lattice of the combined function, usually, the dipole and principal quadrupole magnetic fields are generated by common electric currents in combined function bending magnets. Then, it is expected that the ripple of the magnetic fields are also common for the dipole and principal quadrupole magnetic fields and improvement of beam spill ripple may be easier than synchrotrons of the separated function lattice. In the present paper, we formulate the tune change due to the ripple of the dipole and quadrupole magnetic fields in combined function lattice. Furthermore, according to the obtained formula, we show that the tune change due to the above ripple of the magnetic fields can be prevented by a sexupole magnetic field.

2 Tune Change by Ripple of Magnetic Field in Combined Function Lattice

2.1 Formulation of Tune Change

Figure 1 shows the coordinate system in the present paper. The distance along the reference orbit is denoted as s. x is the horizontal deviation from the reference orbit and $\rho(s)$, the curvature radius. Since the resonanat extraction is usuallly performed in the horizontal plane, the analysis in the present paper is focused on the horizontal tune change.

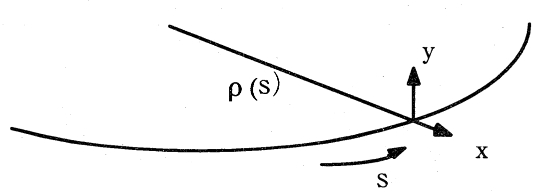


Fig. 1 Coordinate System

The equation for the betatron oscillations is designated as

$$\frac{d^2x}{ds^2} - \frac{\rho + x}{\rho^2} = -\frac{eB_y}{p}(1+\frac{x}{\rho})^2, \qquad (1)$$

where B_y is the magnetic field for the y direction. If higher order terms can be neglected, B_y is written as

$$B_y = B_d - B_q x - \frac{B_s}{2}x^2, \qquad (2)$$

where B_d, B_q and B_s are the dipole, quadrupole and sextupole magnetic fields,respectively. These magnetic fields are written as

$$B_d = B_{d0}(1-\delta_d), \qquad (3)$$
$$B_q = B_{q0}(1-\delta_q), \qquad (4)$$
$$B_s = B_{s0}(1-\delta_s), \qquad (5)$$

where δ_d, δ_q and δ_s are the ripple components, that is, temporal changing variables. Since the ripple frequency of the magentic field is usually lower than several kHz and the order of the revolution frequency of the beam in synchrotrons is about megahertz, the ripple component of the magnetic field can be treated as constant in Eq.(2). Then, Eq.(1) is written as

$$\frac{d^2x}{ds^2} - \frac{\rho + x}{\rho^2} = -\frac{e}{p}[B_{d0}(1-\delta_d) - B_{q0}(1-\delta_q)x - \frac{B_{s0}}{2}(1-\delta_s)x^2](1+\frac{x}{\rho})^2. \quad (6)$$

In the following, the relationship $\frac{e}{p} = \frac{1}{B_{d0}\rho}$ is applied. By expanding the above equation up to the second order terms for x and expressing $\frac{B_{q0}}{B_{d0}\rho}$ and $\frac{B_{s0}}{B_{d0}\rho}$ as B_{40} and B_{60}, respectively, Eq.(6) is written as follows,

$$\frac{d^2x}{ds^2} + x[\frac{1-2\delta_d}{\rho^2} - B_{40}(1-\delta_q)] + x^2[\frac{1-2\delta_d}{\rho^3} - \frac{2B_{40}(1-\delta_q)}{\rho} - \frac{B_{60}(1-\delta_s)}{2}] - \frac{\delta_d}{\rho} = 0. \quad (7)$$

Here, we insert $x = \bar{x} + \eta\delta_d$ into the above equation and obtain the following equations:

$$\frac{d^2\eta}{ds^2} + \eta(\frac{1}{\rho^2} - B_{40}) - \frac{1}{\rho} = 0, \quad (8)$$

$$\frac{d^2\bar{x}}{ds^2} + \bar{x}[\frac{1}{\rho^2} - B_{40} - \delta_d\{\frac{2}{\rho^2} - \frac{2\eta}{\rho^3} + \frac{4B_{40}\eta}{\rho} + B_{60}\eta\} + B_{40}\delta_q] = 0. \quad (9)$$

From Eq.(8), usual dispersion function is obtained. When obtaining Eq.(9), all the terms higher than the first order were neglected. When the equation for the betatron oscillations is expressed as $x'' + (k+\Delta k)x = 0$, the tune shift $\Delta\nu$ due to Δk is written as $\Delta\nu = \frac{1}{4\pi}\int \Delta k \beta ds$ where β is the betatron function of the unperturbed motion. Then, the tune shift $\Delta\nu$ is expressed as follows:

$$\Delta\nu = -\frac{\delta_d}{4\pi}\int(\frac{2}{\rho^2} - \frac{2\eta}{\rho^3} + \frac{4B_{40}\eta}{\rho})\beta ds - \frac{\delta_d}{4\pi}\int B_{60}\eta\beta ds + \frac{\delta_q}{4\pi}\int B_{40}\beta ds. \quad (10)$$

From the above equation, it is seen that the ripple of the sextupole magnetic field does not cause the tune change in the first order approximation. The effect of the ripple of the dipole magentic field decreases with an increase of the curvature radius ρ. Assuming that the ripple of the quadrupole magnetic field is equal to that of the dipole magnetic field including phase, the above equation is written as follows:

$$\Delta\nu = -\frac{\delta_d}{4\pi}\int(\frac{2}{\rho^2} - \frac{2\eta}{\rho^3} + \frac{4B_{40}\eta}{\rho} - B_{40})\beta ds - \frac{\delta_d}{4\pi}\int B_{60}\eta\beta ds. \quad (11)$$

2.2 Method of Reduction of Tune Change

Eq.(11) shows that the tune change can be decreased by the reduction of the ripple of the dipole magnetic field. Eq.(11) also shows that the tune change occurs due to the sextupole field and closed orbit displacement, the latter of which is caused by the ripple of the dipole magnetic field. It is seen that the small sextupole magnetic field, the small betatron and dispersion fucntions in the bending magnet of combined function lattice are generally effective for the reduction of the tune change.

On the other hand, according to Eq.(11), the tune change due to the ripple of the dipole magnetic field can be vanished by using a sextupole magnetic field. The ripple of the dipole magnetic field causes the temporal variation of the closed orbit, which makes the focusing or defocusing strength change at sextupole magnetic field. Then, by choosing adequate sextupole magnetic field,

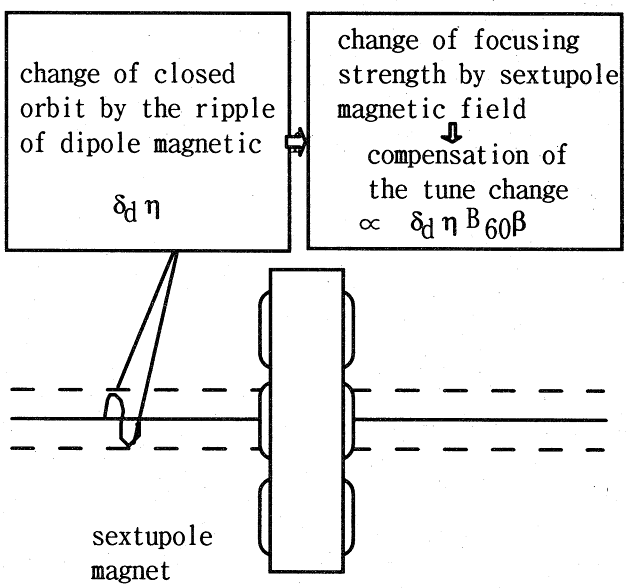


Fig. 2 Scheme of Compensation of the Tune Change

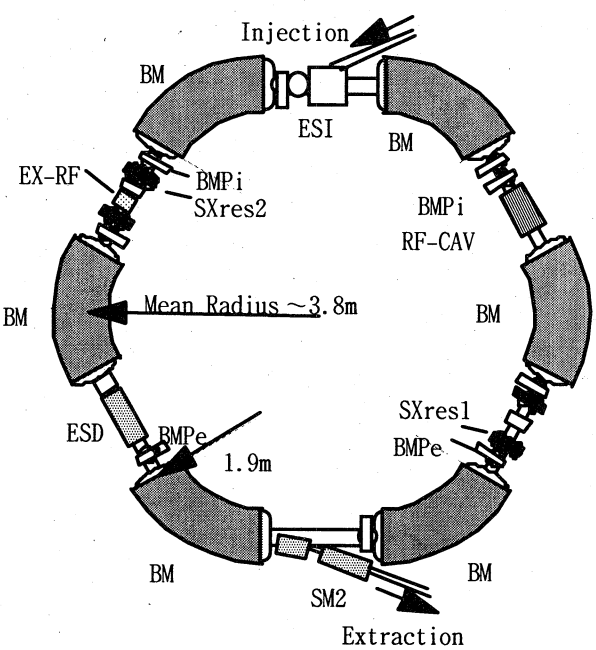
we can prevent the tune change due to the ripple of the dipole magnetic field. This method is depicted schematically in Fig.2. The needed strength of the sextupole field can be determined by calculation so that the right hand side of Eq.(11) may be vanished, because the distribution of the magnetic field along the reference orbit can be obtained. This strength of the sextupole field is equal to that for the chromaticity correction. As a reference, the tune change $\Delta\nu$ due to the momentum change $\frac{\Delta p}{p}$ is expressed as follows:

$$\Delta\nu = -\frac{\frac{\Delta p}{p}}{4\pi}\int(\frac{2}{\rho^2} - \frac{2\eta}{\rho^3} + \frac{4B_{40}\eta}{\rho} + B_{60}\eta - B_{40})\beta ds. \quad (12)$$

This equation can be obtained by the procedure similar to the above one.

In the case of the third order resonant extraction, the separatrix of the resonance is generated by sextupole magnetic field. Then, the sextupole magnets for the above correction of the tune change should be operated so that the separatrix for the nonlinear resonance may not be varied. This can be accomplished by choosing the positions, polarities and strengths of the sextupole magnets dedicated for the tune correction. When the tune is approximately $\frac{2m+1}{3}$ for the third order resonance, the separatrix size is determined by the difference of strength of the setupole magnetic field if the sextupole magnets are installed at the positions with 180 degrees rotational symmetry. Then, if these sextupole magnets are excited with the same strength and polarity, the tune change can be prevented without changing the separatrix. Furthermore, the prevention of the tune change and the adequate generation of the separatrix can be done by two sextupole magnets.

Figure 3 shows an example of the combined function



BM : Combined Function Bending Magnet
ESD:Electrostatic Deflector
ESI :Electrostatic Inflector
SM1,SM2 : Septum Magnets
SXres1,SXres2 : Sextupole Magnets

Fig.3 A Combined Function Synchrotron Using Two Sextupole Magnets

synchrotron, in which the sextupole magnets $SX_{res1}$and SX_{res2} for the resonance excitation are installed in the above configuration[2]. The difference of the strength of SX_{res1} and SX_{res2} determines the separatrix size. Then, by the resonance exciter, SX_{res1}, SX_{res2}, the correction of the tune change can be done, if the field strengths of $SX_{res1}$and SX_{res2} are determined so that they may vanishes the right hand side of Eq.(11) with keeping the difference of the strengths of SX_{res1} and SX_{res2} constant for the separatrix generation[4].

3 Conclusion

Tune change due to ripple of magnetic field in synchrotron of combined function lattice is formulated. The tune change is proportional with the ripple of the dipole magnetic field assuming that the ripples of the dipole and quadrupole magnetic field are the same both in amplitude and phase. The reductions of the betatron and dispersion functions are effective for decreasing the ripple of the tune. It was shown that the tune change due to the ripple of the dipole magnetic field can be compensated by the sextupole magnetic field chosen adequately, the strength of which is equal to that for the correction of the tune change due to the momentum deviation.

References

[1] K. Noda, et al., Nucl. Instrm. and Meth. A374 (1996) 269
[2] A. Noda, et al., this proceedings
[3] M. Kumada, et al., Proc. 5th EPAC, Barcelona, 1996,p2355
[4] K. Matsuda and K. Hiramoto, Proc. 10th Symp. on Accel. Sci. and Technol.,Ibaraki.Japan, 1995, p321