Mode Converters for Microwave-FEL application

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Abstract

Microwave mode conversion from TE_{10} to TE_{n0} is considered based on geometrical perturbations of the walls of a rectangular waveguide. Equations under a transmission-line approximation are derived, and design simulations for planar-FEL application are presented.

1 Introduction

The excitation of a specific electro-magnetic mode in a waveguide is a standard topic in microwave engineering. In Gyrotrons and helical-wiggler FELs, circular waveguides are used. The design method on mode converter of a circular waveguide has been confirmed as described in ref.(1). Maxwell's equations are transformed into an infinite set of coupled transmission-line equations as follows:

$$\frac{dV_n}{dz} = -j\omega\mu I_n + \frac{1}{a}\left(\frac{da}{dz}\right)\sum_{m=1}^{\infty}\frac{2k_n k_m}{k_m^2 - k_n^2}V_m ,$$

$$\frac{dI_n}{dz} = -j\frac{\beta_n^2}{\omega\mu}V_n + \frac{1}{a}\left(\frac{da}{dz}\right)\sum_{m=1}^{\infty}\frac{2k_n k_m}{k_m^2 - k_n^2}I_m . \qquad (1)$$

The designs can be performed numerically. Actually, the circular mode converter has already been used to heat magnetically confined plasmas at the electron cyclotron resonance frequency as described in ref.(2).

However, we can not find that of a rectangular waveguide which is used in a planar-wiggler FEL. In order to extend the application field of the microwave-FEL, a design study of a rectangular mode converter would be significant.

2 Rectangular waveguide mode converters

As a mode-excitation structure, the two-type geometry, which is shown in Fig.(1), is considered. One is a periodical taper-waveguide type, and the other is a periodical bend-waveguide type. The walls are assumed to be perfectly conducting and to have a sinusoidal variation in the x-direction.

2-1 Periodical Taper Waveguide Type

Mode Coupling Equations

In order to obtain mode-coupling equations, we adopt an approach which is similar to that present in reference 3, and confine our attention to TE_{n0} for the sake of simplicity.

By considering the geometrical structure of the waveguide, it is advantageous to separate the axial coordinate (z) from the transverse coordinates (x,y). The

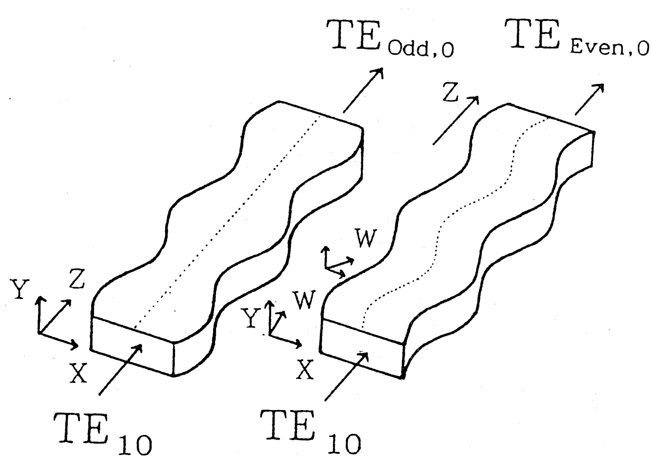

Fig.1 Two types of rectangular waveguide mode converters. The left side is of the periodical taper type, and the right side is of the periodical bend type.

component of a field in a rectangular waveguide can be represented as a superposition of mode function,

$$E_y = \sum_n V_n(z)E_n(x,y) \ , \quad H_x = \sum_n I_n(z)H_n(x,y) \ ,$$
$$H_z = \sum_n i_n(z)h_n(x,y) . \qquad (2)$$

In rectangular coordinates, the mode functions are given as

$$E_n = H_n = \sqrt{\frac{2}{ab}}\sin(k_n x) \quad , \quad h_n = \sqrt{\frac{2}{ab}}\cos(k_n x) . \qquad (3)$$

where $k_n = n\pi / a$ is the wave number; a and b are width and height of the rectangular waveguide. Subscript "n" means the TE_{n0} mode. These are orthogonal functions.

In the TE_{n0} mode, Maxwell equations become

$$\frac{\partial E_y}{\partial z} = -j\omega\mu H_x , \qquad (4\text{-a})$$

$$\frac{\partial E_y}{\partial x} = j\omega\mu H_z , \qquad (4\text{-b})$$

$$\frac{\partial H_x}{\partial z} = -j\omega\varepsilon E_y + \frac{\partial H_z}{\partial x} . \qquad (4\text{-c})$$

A relationship is found by substituting the series (2) into Eq.(4-b), and comparing the coefficients,

$$V_n(z) = \frac{j\omega\mu}{k_n}i_n(z) . \qquad (5)$$

Inserting Eqs.(2) into Eq.(4-a) and Eq.(4-c), we should notice that the width (a) of a rectangular waveguide is a function of z when we differentiate mode functions with respect to z. Also k_n is dependent on z, and V_n is a function of z.

Multiplying both sides by $\sqrt{2/ab}\sin k_n x$ and integrating over the guide cross section, while making use of the orthogonal characteristics of the mode function, we obtain the following coupling equations in the form of ordinary differential-equation:

$$\frac{dV_n}{dz} = -j\omega\mu I_n + \frac{1}{2a}\frac{da}{dz}V_n + \sqrt{\frac{2b}{a}}\frac{1}{a}\frac{da}{dz}\sum_m \frac{2k_m k_n}{k_m^2 - k_n^2}V_m ,$$

$$\frac{dI_n}{dz} = -j\frac{\beta_n^2}{\omega\mu}V_n + \frac{1}{2a}\frac{da}{dz}I_n + \sqrt{\frac{2b}{a}}\frac{1}{a}\frac{da}{dz}\sum_m \frac{2k_m k_n}{k_m^2 - k_n^2}I_m ,\qquad (6)$$

where β_n is the propagation constant for the TE_{n0} mode and the summations are extended under the condition that the integer n-m is an even integer, except for n = m. The mode convertion between the n mode and the m mode occurs under the condition that the integer n-m is an even integer.

Forward wave equation

The mode voltage (V_n) and the current (I_n) are related to the wave amplitude (A_n and B_n), which correspond to the forward and backward waves,

$$V_n = \sqrt{K_n}\left(A_n + B_n\right) ,$$

$$I_n = \frac{\left(A_n - B_n\right)}{\sqrt{K_n}} ,\qquad (7)$$

where K_n is the wave impedance of the n-mode. In order to obtain an equation on the forward wave, both equations are summed to eliminate the backward wave .

We design the mode converter between the TE_{10} and the TE_{30} . If the sinusoidal perturbation of the waveguide wall is defined in term of the wave number (k_W), we can expect that the largest growth of the mode conversion would occur with $k_W = \bar{k}_1 - \bar{k}_3$; $\bar{k}_1$ and $\bar{k}_3$ are the average of k_1 and k_3 , respectively . The interaction must be characterized by the "beating" of the three waves.

Assuming that the waveguide has a slow taper, or $-j\beta_m >> (1/2a)da/dz$, and assuming that the reflected modes (B_n) are negligible, we obtain

$$\frac{dA_1}{dz} = -j\beta_1 A_1 + \sqrt{\frac{2b}{a}}\left(\frac{1}{a}\frac{da}{dz}\right)\frac{2k_1k_3}{k_3^2 - k_1^2}\left[\sqrt{\frac{K_1}{K_3}} + \sqrt{\frac{K_3}{K_1}}\right]A_3 ,$$

$$\frac{dA_3}{dz} = -j\beta_3 A_3 + \sqrt{\frac{2b}{a}}\left(\frac{1}{a}\frac{da}{dz}\right)\frac{2k_1k_3}{k_3^2 - k_1^2}\left[\sqrt{\frac{K_1}{K_3}} + \sqrt{\frac{K_3}{K_1}}\right]A_1 ,\qquad (8)$$

where

$$K_n(z) = \frac{\omega\mu}{\beta_n(z)} = \frac{\omega\mu}{\sqrt{k^2 - k_n^2(z)}} = \frac{\omega\mu}{\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n\pi}{a(z)}\right)^2}} .$$

Design simulation

The results concerning mode conversion were obtained by numerically integrating the above 1st-order equations. The simulation parameters are as follows: the frequency is 35GHz, the waveguide size is WRJ-10 (a = 22.9mm, b = 10.2mm), and the wall period is 5.54 cm. Letting the excitation at z=0 be, $A_1(0) = 1, A_3(0) = 0$, the simulation results on the width modulation of 5% show that complete energy transfer from the TE_{10} mode to the TE_{30} mode is accomplished at z = 82 cm.

2-2 Periodical Bend Waveguide Type

Mode Coupling Equations

In the curvilinear coordinate system as shown in Fig.1, the element of length is,

$$ds^2 = dx^2 + dy^2 + e_3^2 dw^2 .$$

Maxwell equations are

$$\frac{1}{e_3}\left(\frac{\partial}{\partial w}E_y\right) = -j\omega\mu H_x ,\qquad (9\text{-a})$$

$$\frac{\partial}{\partial x}E_y = j\omega\mu H_w ,\qquad (9\text{-b})$$

$$\frac{1}{e_3}\left[\frac{\partial}{\partial w}H_x - \frac{\partial}{\partial x}e_3 H_w\right] = -j\omega\varepsilon E_y ,\qquad (9\text{-c})$$

where $e_3 = 1 + \frac{x}{\rho}$.

We assume the following fields as shown in ref.(4):

$$E_y = \sum_m \sqrt{\frac{2}{ab}}\sin k_m x \cdot V_m(w) ,$$

$$H_x = \sum_m \sqrt{\frac{2}{ab}}\sin k_m x \cdot I_m(w) ,$$

$$e_3 H_w = \sum_m \sqrt{\frac{2}{ab}}\cos k_m x \cdot U_m(w) .\qquad (10)$$

Inserting Eqs.(10) into Eq.(9-a), multiplying by $\sqrt{2/ab}\cdot\sin k_n x$ and integrating over the cross section (x-y plane), we can easily obtain an equation,

$$\frac{dV_n}{dw} = -j\omega\mu I_n - \sum_m j\omega\mu\frac{2}{ab}\iint\frac{x}{\rho}\sin k_n x \sin k_m x dS \cdot I_m .\qquad (11)$$

Substituting Eqs.(10) into Eq.(9-c), and multiplying by $\sqrt{2/ab}\cdot\sin k_n x$ and integrating over the cross section (x-y plane), we obtain

$$\frac{dI_n}{dw} = -j\omega\varepsilon V_n$$
$$-j\omega\varepsilon\sum_m \frac{2}{ab}\iint \frac{x}{\rho}\sin k_m x \sin k_n x dS\cdot V_m - k_n U_n \tag{12}$$

In order to obtain U_n, we need some approximations, as below. If we substitute Eqs.(10) into Eq.(9-b) , we obtain

$$\sqrt{\frac{2}{ab}}\sum_n k_n \cos k_n x\cdot V_n = j\omega\mu\sqrt{\frac{2}{ab}}\sum_n \frac{\cos k_n x}{e_3}\cdot U_n$$

Multiplying by $\sqrt{2/ab}\cos k_m x$ and integrating over the cross section (x-y plane), and assuming $\frac{1}{e_3}\cong 1-\frac{x}{\rho}$, we obtain

$$k_m V_m = j\omega\mu\sum_n \Big(\frac{2}{ab}\iint \cos k_m x \cos k_n x dS$$
$$-\frac{2}{ab}\iint \frac{x}{\rho}\cos k_m x \cos k_n x dS\Big)\cdot U_n$$
$$= j\omega\mu\sum_n \left(\delta_{m,n}-\xi_{m,n}\right)\cdot U_n$$

This equation can be written in matrix form, and we obtain an approximate inverse matrix as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ \cdot \\ U_n \end{bmatrix} = \left(\frac{-j}{\omega\mu}\right)\begin{bmatrix} 1+\xi_{1,1} & \xi_{2,1} & \cdot & \xi_{m,1} \\ \xi_{1,2} & 1+\xi_{2,2} & \cdot & \xi_{m,2} \\ \cdot & \cdot & \cdot & \cdot \\ \xi_{1,n} & \xi_{2,n} & \cdot & \delta_{m,n}+\xi_{m,n} \end{bmatrix}\begin{bmatrix} k_1 V_1 \\ k_2 V_2 \\ \cdot \\ k_m V_m \end{bmatrix}.$$

We substitute this U_n into Eq.(12), and obtain an equation;

$$\frac{dI_n}{dw} = -j\omega\varepsilon V_n + \frac{jk_n^2}{\omega\mu}V_n$$
$$-j\omega\varepsilon\sum_m \frac{2}{ab}\iint \frac{x}{\rho}\sin k_n x \sin k_m x dS\cdot V_m$$
$$+\frac{jk_n}{\omega\mu}\sum_m k_m \frac{2}{ab}\iint \frac{x}{\rho}\cos k_n x \cos k_m x dS\cdot V_m \quad . \tag{13}$$

We should notice that mode conversion occurs under the condition that the integer n+m is an odd number.

Forward wave equation

Using the relation of Eqs.(7), we convert the mode voltage and the mode current in eq.(11) and eq.(13) into the forward-wave and backward-wave amplitude. we consider only the case of coupling between TE_{10} and TE_{20}. The mode-coupling equations, neglecting reflected modes, can be written in the following form

$$\frac{dA_1}{dw} = -j\left[\beta_1 A_1 + C_1^2 A_2\right] ,$$
$$\frac{dA_2}{dw} = -j\left[\beta_2 A_2 + C_2^1 A_1\right] , \tag{14}$$

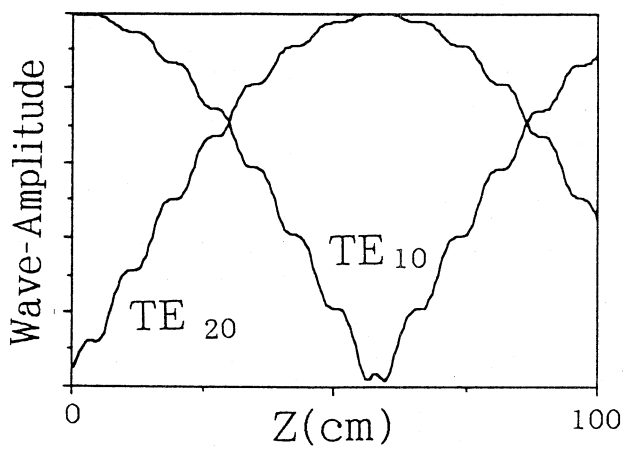

Fig.2 Simulation result in a periodical bend type converter.

where

$$C_2^1 = C_1^2 = -\frac{a}{\rho}\left[\left(\frac{k^2}{\sqrt{\beta_1\beta_2}}+\sqrt{\beta_1\beta_2}\right)\frac{8}{9\pi^2} - \frac{k_1 k_2}{\sqrt{\beta_1\beta_2}}\frac{10}{9\pi^2}\right]$$

and $k=\omega/c$, $k_1=\pi/a$, $k_2 = 2\pi/a$, $\beta_1=\sqrt{k^2-k_1^2}$, $\beta_2=\sqrt{k^2-k_2^2}$.

In numerical simulations, we introduce a z-axis which is normal to the x-y plane. The variable w is converted to z. The simulation parameter is as follows: the frequency is 35GHz, waveguide size is WRJ-10, the wall period is 15.59 cm, and the modulation of the wall is 5%.

Fig.2 shows the amplitude evolution of the TE_{20} versus the z-position. Complete mode conversion results at z = 60 cm. After that, the inverse conversion occurs.

3 Conclusion

We obtained coupled-mode differential equations in the rectangular waveguides and designed two mode converters of 35 GHz .

In the next step simulation, we must take account of spurious higher modes and TM mode. Further we must mention that this perturbation scheme is no longer valid for a large wall variation.

The actual output mode from planar-FELs is TE_{01} mode. The conversion from TE_{01} to TE_{10} is possible. We had already experience using a taper-waveguide.

Acknowledgments

We would like to thank Dr. T. Kikunaga of Mitsubishi Electric Corp. and Dr. K. Takayama of KEK for discussions on the Schelkunoff's generalized telegraphist equations.

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